1. Verify the following vector identities, where Φ is any scalar and $\vec{A}$ is arbitrary vector field:

   (a) $\vec{\nabla} \times (\vec{\nabla}\Phi) = 0$

   (b) $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{A}) = 0$

   (c) $(\vec{A} \cdot \vec{\nabla})\vec{A} = (\vec{\nabla} \times \vec{A}) \times \vec{A} + \frac{1}{2}\vec{\nabla}(\vec{A} \cdot \vec{A})$

   * (d) $\vec{\nabla} \times (\vec{A} \times \vec{B}) = \vec{A}(\vec{\nabla} \cdot \vec{B}) - \vec{B}(\vec{\nabla} \cdot \vec{A}) + (\vec{B} \cdot \vec{\nabla})\vec{A} - (\vec{A} \cdot \vec{\nabla})\vec{B}$

   (Note: It is usually sufficient to prove just one Cartesian component.)

2. Suppose there is 2-dimensional, steady motion of an ideal, incompressible fluid that is characterized by a velocity potential of the form:

$$\phi = K(x^2 + y^2)$$

   where K is a positive constant.

   a) Derive an expression for the fluid velocity and sketch the streamlines of this flow.

   b) Does the flow satisfy the equation of continuity? Discuss, if necessary.

*ATMO 551b -Extra credit ATMO 451b.

3. Consider the 3-dimensional velocity of a fluid at some point (x, y, z) where

$$\hat{v} = u\hat{i} + v\hat{j} + w\hat{k}$$

Show that the changes in the velocity components in going to a neighboring point (x + dx, y + dy, z + dz) can be described by

$$du = \epsilon_x dx + \Upsilon_{xy} dy + \Upsilon_{xz} dz + \omega_y dz - \omega_z dy$$

$$dv = \Upsilon_{xy} dx + \epsilon_y dy + \Upsilon_{yz} dz + \omega_z dx - \omega_x dz$$

$$dw = \Upsilon_{xz} dx + \Upsilon_{yz} dy + \epsilon_z dz + \omega_x dy - \omega_y dx$$

where the coefficients are:

<u>lineal strain rates</u>

$$\epsilon_x \equiv \frac{\partial u}{\partial x} \qquad \epsilon_y \equiv \frac{\partial v}{\partial y} \qquad \epsilon_z \equiv \frac{\partial w}{\partial z}$$

<u>rates of shear deformation</u>

$$\Upsilon_{xy} \equiv \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) = \Upsilon_{yx}$$

$$\Upsilon_{yz} \equiv \frac{1}{2}\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right) = \Upsilon_{zy}$$

$$\Upsilon_{zx} \equiv \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) = \Upsilon_{xz}$$

and

<u>angular velocity components</u>

$$\omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)$$

$$\omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)$$

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

Note: Only six independent quantities describe the deformation and these can be grouped into a symmetrical matrix

$$\begin{pmatrix} \epsilon_x & \Upsilon_{xy} & \Upsilon_{xz} \\ \Upsilon_{xy} & \epsilon_y & \Upsilon_{yz} \\ \Upsilon_{xz} & \Upsilon_{yz} & \epsilon_z \end{pmatrix}$$