

The Temperature of Planets: Radiative Equilibrium

Any object with a finite temperature emits radiation according to Planck's law. The radiative *emission* from a blackbody per unit surface area with a temperature, T , in Kelvin is given by the Stephan Boltzmann law (which is the spectral integral of Planck function):

$$F = \sigma T^4 \quad (\text{W/m}^2) \quad (1)$$

where σ is the Stephan-Boltzmann constant: $5.67 \times 10^{-8} \text{ W/m}^2/\text{K}^4$. So the total radiative emission from a blackbody depends only on its temperature, a rather remarkably simple relation.

The Sun has a blackbody temperature of 5,780 K. Therefore the flux at the Sun's surface is $6.3 \times 10^{23} \text{ W/m}^2$.

This flux is the energy passing through each square meter of the Sun's outer "surface" each second. To obtain the total emission from an object we multiply this watts per unit area by the object's surface area which for a sphere is $4\pi r^2$ where r is the radius of the sphere.

$$F_{\text{tot}} = \sigma T^4 4\pi r^2 \quad (\text{W}) \quad (2)$$

The Sun's radius is $6.96 \times 10^5 \text{ km}$ (quick memory point: Earth's average radius is 6,378 km, Jupiter's radius is about 10 times that, and the Sun's is about 10 times that). So the total power radiated by the sun is $3.9 \times 10^{42} \text{ W}$.

The Sun's radiation spreads out over space until it reaches a planet such as Earth. This emission spreads over a surface area of the sphere whose radius is the distance from the sun to the planet. The solar flux at the planet is the total flux radiated by the Sun divided by a sphere of radius the planet's orbital radius.

$$F_{\text{planet}} = \frac{F_{\text{sun-total}}}{4\pi R_{\text{planet-orbit}}^2} = \frac{\sigma T_{\text{sun}}^4 4\pi r_{\text{sun}}^2}{4\pi r_{\text{planet-orbit}}^2} = \sigma T_{\text{sun}}^4 \frac{r_{\text{sun}}^2}{r_{\text{planet-orbit}}^2} \quad (\text{W/m}^2) \quad (3)$$

The amount of radiation striking the planet is this flux times the crosssectional area of the planet:

$$F_{\text{tot-planet}} = \sigma T_{\text{sun}}^4 \frac{r_{\text{sun}}^2}{r_{\text{planet-orbit}}^2} \pi r_{\text{planet}}^2 \quad (\text{W}) \quad (4)$$

Only a portion of this radiation striking the planet is absorbed. The fraction that is reflected is called the albedo, A . So the amount of solar radiation power that is absorbed by the planet is

$$F_{tot-planet-absorbed} = F_{tot-planet} (1 - A_{planet}) = \sigma T_{sun}^4 \frac{r_{sun}^2}{r_{planet-orbit}^2} \pi r_{planet}^2 (1 - A_{planet}) \quad (W) \quad (5)$$

So (5) defines the solar energy absorbed by the planet. In radiative equilibrium, this absorbed energy is balance by the energy emitted by the planet. This balance is written as

$$\sigma T_{sun}^4 \frac{r_{sun}^2}{r_{planet-orbit}^2} \pi r_{planet}^2 (1 - A_{planet}) = \sigma T_{planet}^4 4\pi r_{planet}^2 \quad (W) \quad (6)$$

So the *radiative equilibrium* temperature of the planet, T_{planet} , is given as

$$T_{planet} = T_{sun} \left[\frac{r_{sun}}{2r_{planet-orbit}} \right]^{1/2} (1 - A_{planet})^{1/4} \quad (K) \quad (7)$$

Notice that it does not depend on the size of the planet. Rather it depends on the size and temperature of the sun and the planet's distance from the sun and albedo.

Note that the temperatures of the gas giant planets tend to be somewhat higher than this equilibrium temperature because as the planet contracts over time, the energy gained by compression is radiated out to space.

If the planet has no atmosphere, this radiative equilibrium temperature equals the surface temperature of the planet. If the planet has a substantial atmosphere (surface pressure > 100 mb), then this is the temperature near the tropopause of the planet approximately the level in the atmosphere where the radiation from the planet is emitted to space.

Tsun 5780 K
Rsun 6.960E+05 km

	Mercury	Venus	Earth	Mars	Jupiter	Triton	Units
Albedo	0.1	0.7	0.3	0.25	0.45	0.7	

Rsun-planet	5.500E+07	1.100E+08	1.495E+08	2.278E+08	7.700E+08	4.485E+09	km
Rsp(AU)	0.367892977	0.735785953	1	1.52366	5.150501672	30	AU

Solar flux		10134.15		2533.54		1371.61		590.82		51.70		1.52		W/m^2
Solar flux absorbed		9120.74		760.06		960.13		443.11		28.44		0.46		W/m^2
Tequilibrium		448		241		255		210		106		38		K
Tsurf		448		740		288		140-300				38		K
Tdiff		0		499		33						0		K
Greenhouse		no		YES		yes		some				no		
observed Te										124.4				K
Finterior/solar										0.909987129				