1. *Gravity calculations*
   a. *calculate the gravity at the surface and at 5 km altitude at the pole*
   b. *calculate the gravity at the surface and at 5 km altitude at 30^o latitude*
   c. *calculate the gravity at the surface and at 5 km altitude at the equator*

Use the approximate equation for gravity from the notes

$$g = -\frac{GM}{r^2} + \frac{3GMa^2J_2}{2r^4}\left(3\sin^2\phi - 1\right) + \omega^2 r\cos^2\phi$$

The next question is what to use for the radius f the Earth as a function of latitude. For this you use the approximate equation of the geoid

$$r_0 = a\left[1 + \frac{\left(2f - f^2\right)}{\left(1-f\right)^2}\sin^2\phi\right]^{-1/2}$$

with a = 6378.139 km and f = 1/298.256.

	Pole	30°N	equator
latitude	90	30	0
r surf	6356.75	6372.77	6378.14
g point mass	-9.865	-9.815	-9.799
g bulge	0.0323	-0.0040	-0.0159
g centrifugal	0.0000	0.0293	0.0339
g total	-9.8325	-9.7899	-9.7807

	Pole	30°N	equator
latitude	90	30	0
r surf + 5	6361.75	6377.77	6383.14
g point mass	-9.849	-9.800	-9.783
g bulge	0.0322	-0.0040	-0.0159
g centrifugal	0.0000	0.0294	0.0339
g total	-9.8171	-9.7745	-9.7653

   d. *What is the percentage change in the gravity between the surface and 5 km in each of the cases?*

% change	-0.157%	-0.158%	-0.158%

Consider 2 atmospheric cases. In both cases, the surface pressure is 1000 mb, the latitude is 30^o and the air is completely dry and contains no water.

- *In Case 1, the surface temperature is 280.5 K and temperature decreases with altitude at 5 K/km.*
- *In Case 2, the surface temperature is 288K and temperature decreases with altitude is 6.5 K/km.*

2. *Calculate the pressure at 5 km altitude*

Use the equation below from page 10 of the notes entitled "Physical Properties of the Atmosphere"

$$\frac{P_2}{P_1} = \exp\left[-\frac{gm}{R\dot{T}}\ln\left(\frac{T_2}{T_1}\right)\right] = \exp\left[\ln\left(\left[\frac{T_1}{T_2}\right]^{\frac{gm}{R\dot{T}}}\right)\right] = \left[\frac{T_1}{T_2}\right]^{\frac{gm}{R\dot{T}}}$$

and the average gravity to calculate the pressure at 5 km altitude.

a. *sum the values of gravity at the surface and 5 km and divide by 2 to get the approximate average gravity between the surface and 5 km altitude to use in the equation*

$g_{avg}(30^oN)$ = (-9.7899-9.7745)/2 = -9.7822 m/s^2.

b. *Calculate the temperature and the pressure at 5 km altitude for Case 1*

c. *Calculate the temperature and the pressure at 5 km altitude for Case 2*

	Case 1	Case 2	
T (5km)	255.5	255.5	K
Tsurf	280.5	288	K
dT/dz	-5	-6.5	K/km
T avg	268	271.75	K
Psurf	1000	1000	mb
P(5km)	529.2192715	533.7301802	mb

d. *Which pressure is higher? Explain why.*
(hint: Think in terms of the pressure scale height)

The average temperature for Case 2 is slightly higher than the average temperature in Case 1 so the pressure scale height in Case 2 is larger so the pressure decreases more slowly with altitude in case 2 so the pressure aloft is higher in case 2.

3. *Determine the potential temperature, θ.*
   a. *Calculate the potential temperature at 5 km altitude for Case 1*
   b. *Calculate the potential temperature at 5 km altitude for Case 2*

$$\theta = T_0 = T_1\left(\frac{P_0}{P_1}\right)^{\frac{R}{C_p}} = T_1\left(\frac{P_0}{P_1}\right)^{\frac{R^*}{C_p'}}$$

	Case 1	Case 2
P 5km	529.2192715	533.7301802
P surf	1000	1000
T 5 km	255.5	255.5
θ	306.444723	305.7024876

   c. *What would the temperature of the air parcel be if it were lowered to the surface in each Case?*

It would just be equal to the potential temperature because the surface pressure is 1000 mb which is a standard reference pressure to use in defining the potential temperature.

If the surface pressure were not equal to 1000 mb then the potential temperature would not tell you what the temperature of the air parcel would be if the air parcel were lowered to the surface. Instead, in the slot for the higher pressure in the potential temperature equation, you would put whatever the actual surface pressure is.

d. Which value is higher? Explain why

As a result of the average temperature between 5 km and the surface being higher in case 1, the pressure at 5 km has decreased less relative to the surface than in case 2. Therefore in Case 1 there is a smaller pressure difference between 5 km and the surface. Therefore when the air parcel is lowered from 5 km to the surface and compressed such that the environment is doing work on it and transferring energy into its internal energy and raising its temperature, it is being compressed less in Case 1 and therefore its temperature increase is less in Case 1 than in Case 2.

4. Calculate the stability ($d\theta/dz$) at 5 km altitude for each Case.

The easiest way to do this is

- Calculate the temperature at a slightly different altitude for each of the two Cases. I used 5.1 km.
- Calculate the potential temperature at the new altitude for each of the cases.
- $d\theta/dz$ is the difference between the potential temperatures at the two altitudes divided by the difference in altitudes

		T(K) Case 1	T (K) Case 2	θ(K) Case 1	θ (K) Case 2	dθ/dz (K/km) Case 1	dθ/dz (K/km) Case 2
z_{upper}(km)	5.1	255	254.85	307.01	306.09		
Z_{lower}(km)	5	255.5	255.5	306.44	305.70	5.69	3.88

5. At what frequency would a parcel oscillate if it were displaced at 5 km altitude in each Case?

The frequency is the Brunt Vaisala frequency: $\omega = N = \left(\frac{g}{\theta}\frac{\partial\theta}{\partial z}\right)^{1/2}$

We know all the variables from the previous questions so we can plug in values.
Note: be careful of units. The value of $d\theta/dz$ was in units of K/km not K/m. The units of distance must be the same as the units of distance used for g.

	Case 1	Case 2	
Brunt Vaisala frq, N	0.01348	0.01115	radians/sec
period	466.1	563.8	seconds

The period is $2\pi/\omega$ and represents the oscillation period of a simple displaced air parcel in each of the two environments and is closely related to gravity waves in the atmosphere somewhat like waves on the ocean surface

6. *a. What is the restoring acceleration for a vertical displacement of 10 m in each Case?*
   b. How large is this compared to g?

From the notes, the acceleration is $a = N^2 \Delta z$.

displacement	10	10	m
acceleration	0.001817552	0.001242161	m/sec^2
acc/g	0.019%	0.013%	

So thwaw accelerations is quite small meaning it does not take much energy to displace an air parcel by 10 meters under typical tropospheric conditions. If we increased the displacement to 100 m would increase the acceleration by a factor of 10 to 0.2% and 0.13% of g. A kilometer of vertical displacement requires quite a bit of work and air flowing over mountains generates significant waves as a result.

$dW = F\,dx$ so $\frac{dW}{m} = \frac{F}{m} dz = a\,dz = N^2 z\,dz$

$$\frac{\Delta W}{m} = N^2 \frac{\Delta z^2}{2}$$

The energy per unit mass to displace the air parcel different distances for case 1 is given below in J/kg = m^2/s^2.

	10 m	100 m	1000 m	Units
Energy/mass	0.00909	0.909	90.9	m^2/s^2
Velocity	0.095	0.95	9.5	m/s

The velocity is the velocity of the air parcel when it goes through the altitude of 0 displacement during its oscillations.