1. The C_p of water vapor at 100^oC is 2.080 kJ/kg/K.

a. What is C_p' in J/mole/K?

C_p' = 2080 J/kg/K * 0.018 kg/mole = 37.44 J/mole/K

b. What are C_v and C_v'?

Cv' = 29.13 J/mole/K Cv=1618 J/kg/K

c. How many degrees of freedom of internal energy storage does water vapor have?

= Cv'/R *2 = 7

2. If Hurricane Katrina rained an average of 6 inches of rain fall over a swath that was 400km wide and 1600 km long,

a. How much energy did it release into the atmosphere via the release of latent heat.

6 inches = 15 cm = 0.15 m of rainfall

total volume of rainfall: 0.15 * 4e5 * 1.6e6 = 9.75e10 m^3.

Mass of rainfall: 9.75e10 m^3 * 1000 kg/m^3 = 9.75e13 kg.

Energy released: 9.75e13 kg * 2.5e6 J/kg = 2.4e20 J

b. If Katrina lasted about a week, what was its average power generation in watts?

One week is 86400*7 604800 seconds

Power = 2.4e20 J / 604800 seconds = 4e14 W

Context: The average power usage of the US is 3.3 TW = 3.3e12 w. So Katrina was generating power at a rate 100 times that of the entire U.S. Equivalently, in ~7 days, Katrina generated enough power to power the US for ~ 2 years.

3. Cloud base: Assuming an atmospheric boundary layer with clouds at the top of the layer,

a. if the dew point temperature at the surface were increased by 4K, how much would the altitude of the cloud base increase or decrease?

Cloud base is also known as

$$z_{LCL} = -\frac{(T - T_d)}{\left(\frac{dT}{dz}_{dry-adia} - \frac{dT_d}{dz}_{mixed}\right)} \cong -\frac{(T - T_d)}{(-9.8 - (-1.7))} \cong \frac{(T - T_d)}{8} \quad \text{(in km)}$$

So if Td were increased by 4 K such that T-Td decreased by 4 K, then cloud base lowers by 4/8 = 0.5 km.

4. Given the air goes over the mountain scenario, briefly explain why Death Valley is even hotter and drier than other locations along the east side of the Sierra Nevada mountains in California?

The temperature that the air on the east or dry side of the mountain depends on the compressional heating as the air moves downward according to the dry adiabat. Death Valley is below sea level so the surface pressure there is higher than anywhere else along the east side of the Sierra Nevada mountains. Therefore the compressional heating is larger than any other location along the east side of the mountains.

5. CAPE: Consider a convective updraft in the core of a thunderstorm. Assume the air in the core of that thunderstorm is following a moist adiabat. Suppose that on average the air temperature in the core is 10°C warmer than the environmental air surrounding the thunderstorm.

a. What is the approximate vertical buoyancy acceleration of the air in the core feels?

$a = -g\,\delta\rho/\rho = g\,\delta T/T \sim g$ 10/250 = 4% g in the upward direction = 0.4 m/s^2.

Assume the thunderstorm core is 10 km in height.

b. When work creates kinetic energy, show that acceleration creates energy per unit mass

dW = F dz

So *dW/m = F/m dz = a dz*

which has units of energy per unit mass

c. What is the kinetic energy per unit mass of the air in the core after the air has been accelerated over 10 km vertically? (this is called the Convective Available Potential Energy, CAPE)

$$\frac{W}{m} = \int_{z_0}^{z_0+10} \frac{F_g}{m}\frac{\Delta T}{T}dz = \int_{z_0}^{z_0+10} g\frac{\Delta T}{T}dz \cong g\frac{\Delta T}{T}\int_{z_0}^{z_0+10} dz \approx g\,\frac{10}{250}\left(z_0 + 10{,}000 - z_0\right) = \frac{10^4}{25}g \sim 4000 \text{ m}^2/\text{s}^2.$$

d. Given the rising air parcel's kinetic energy per unit mass at the top of the core, what is the approximate vertical velocity of the air at the top of the core?

KE = ½ mv^2. So KE/m = ½ v^2.

v^2 = 2 * 400 g ~8000 m^2/s^2.

v ~ 90 m/s

This magnitude of updraft velocity is achieved in real life in strong thunderstorms