Review of Linear Algebra

2-D data can be expressed in matrix form. For this course:

1^{st} dimension : Space
2^{nd} dimension : Time. $\Big\}$ Space-time array.

Example for HW #3:

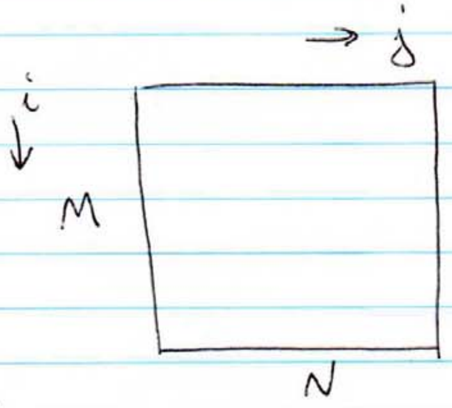
Spatial dimension: $81 \times 51 = 4131$ datapts.
Time dimension: 660 time intervals.

Other possibilities:

- Parameter-time array
- Parameter-space array

Space time arrays, as we'll see a bit later, are used for EOF/PC analysis.

2-D Matrix ($\underset{\sim}{X}$) of $M \times N$



Matrix $\underset{\sim}{X}$

$\underset{\sim}{X} = x_{ij}$

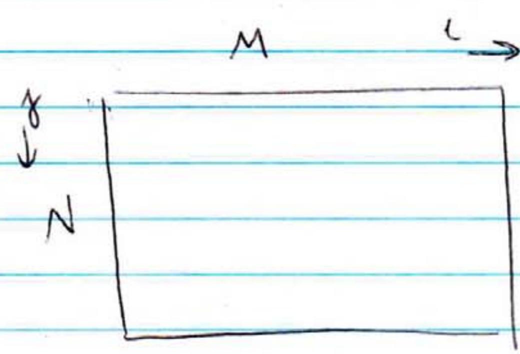
Matrix notation

Tensor notation

$i = 1 \ldots M$ (rows)
$j = 1 \ldots N$ (columns)

Transpose of Matrix ($\bar{X}^T$)

Transpose: Reverse the order of rows and columns.



Matrix

$$\bar{X}^T = x_{ji}$$

Matrix notation

Tensor notation.

$i = 1 \ldots N$ (rows)

$j = 1 \ldots M$ (columns)

Wilk's example (p. 412)

$$\bar{X} = \begin{array}{cc} & (N) \\ (M) & \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} \\ a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} \\ a_{3,1} & a_{3,2} & a_{3,3} & a_{3;4} \end{bmatrix} \end{array} = a_{ij}$$

Dimensions $M \times N$

$$X^T = \begin{array}{cc} & (M) \\ (N) & \begin{bmatrix} a_{1,1} & a_{2,1} & a_{3,1} \\ a_{1,2} & a_{2,2} & a_{3,2} \\ a_{1,3} & a_{2,3} & a_{3,3} \\ a_{1,4} & a_{2,4} & a_{3,4} \end{bmatrix} \end{array} = a_{ji}$$

Dimensions $N \times M$

Inverse of a Matrix: Defined as the matrix which, when multiplied by the original matrix yields the identity matrix I.

$$X X^{-1} = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Identity matrix.

For a 2×2 matrix

$$X^{-1} = \frac{1}{\det[X]} \begin{bmatrix} a_{2,2} & -a_{1,2} \\ -a_{2,1} & a_{1,1} \end{bmatrix}$$

$$= \frac{1}{(a_{1,1} a_{2,2}) - (a_{2,1} a_{1,2})} \begin{bmatrix} a_{2,2} & -a_{1,2} \\ -a_{2,1} & a_{1,1} \end{bmatrix}$$

Types of matrices

Symmetric $\rightarrow$ $X = X^T$
Orthogonal $\rightarrow$ $X^T = X^{-1}$

Additional properties of arithmetric operations with matrices given in Table 9.1 of Wilks. (We're just going over the ones we'll need to get the concepts for ~~what we~~ this part of the course).

Inner product - Vectors (1-D)

Two vectors of the same dimension can be multiplied using the dot product:

$$x^T y = [x_1, x_2, x_3 \ldots x_K] \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_K \end{bmatrix}$$

$$= x_1 y_1 + x_2 y_2 + x_3 y_3 + \cdots + x_K y_K$$

$$= \sum_{K=1}^{K} x_K y_K$$

If x and y have a mean of zero, then the inner product is the covariance of x and y (times K) (i.e., $x' y'$)

This is communative

Now let's extend the idea to matrices (2-D)

X and Y are M x N matrices.
The product of these two matrices is:

$$C = X \cdot Y = c_{ij}$$

where C_{ij} is the the inner product between row $x(i,:)$ and column $y(:,j)$

$$C_{M \times N} = X_{M \times K} Y_{K \times N}$$

K is the inner dimension consumed by the multiplication.

* This is <u>not</u> commutative $(xy \neq yx)$

Wilks' <u>example</u> of matrix multiplication (p. 413)

$$\begin{array}{ccccc} X & & Y & & C \\ \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \end{bmatrix} & & \begin{bmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \\ b_{3,1} & b_{3,2} \end{bmatrix} & = & \begin{bmatrix} c_{1,1} & c_{1,2} \\ c_{2,1} & c_{2,2} \end{bmatrix} \\ 2 \times 3 & & 3 \times 2 & & 2 \times 2 \end{array}$$