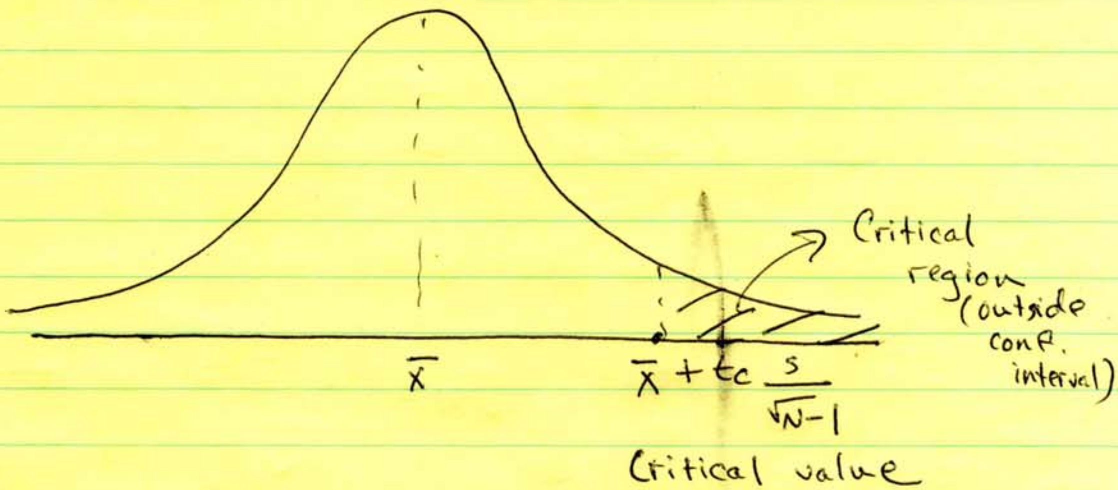
Lect. 4

Graphical interpretation



A common test is a difference of means. This compares whether or not 2 sample populations drawn from a larger population are statistically significantly different.

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_p \sqrt{\frac{1}{N_1} + \frac{1}{N_2}}}$$

$$s_p = \sqrt{\frac{N_1 s_1^2 + N_2 s_2^2}{N_1 + N_2 - 2}} \rightarrow \text{Pooled variance.}$$

$$\nu = N_1 + N_2 - 2$$

Example like what will be used in homework assignment:

- Have approximately 50 years of winter precipitation data in Tucson (Dec - Mar)
- Use Niño 3.4 index to classify ENSO pos. or neg. winters using some threshhold for the index
- Say get 12 ENSO+ winters, 13 ENSO- winters.
- Perform a difference of means test to see if the average precip. for each composite is different.
- Any idea what the answer might be?? Why?

We can also test whether variances are different.

Chi-Squared Dist. - Test of Variance. (χ^2)

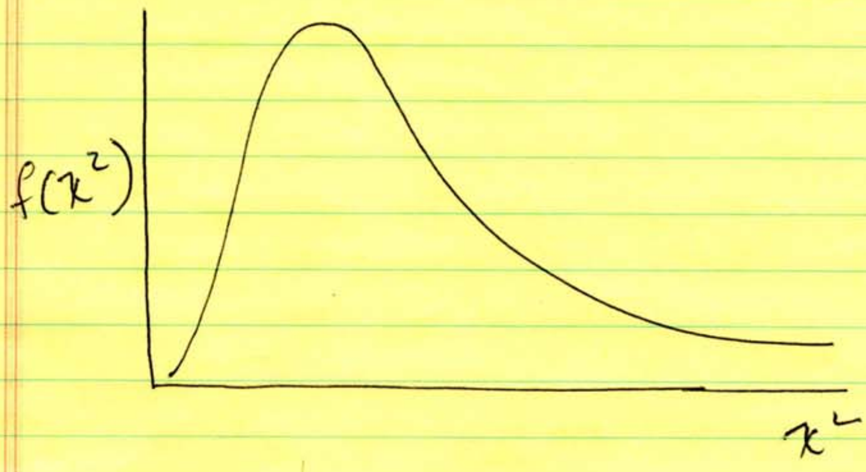
This is a special case of the gamma distribution (we'll talk about that one later)

$$f(\chi^2) = f_0(\nu)\,(\chi^2)^{\left(\frac{1}{2}\nu - 1\right)}\, e^{-\frac{1}{2}\chi^2}$$

$$\chi^2 = (n-1)\frac{s^2}{\sigma^2}$$

ν = degrees of freedom.

This distribution is positively skewed:



Like t-distribution, χ^2 is function of the degrees of freedom:

Test for confidence interval (e.g. 95%)

$$\frac{(N-1)s^2}{\chi^2_{0.025}} < \sigma^2 < \frac{(N-1)s^2}{\chi^2_{0.975}}$$

Read these values off a table.

To compare variances of different samples from the same population, ~~can~~ use the F-statistic.

$$F = \frac{s_1^2}{s_2^2}$$ with N_1 and N_2 ~~degrees~~ being the sample sizes.

$$\left.\begin{aligned} \nu_1 &= N_1 - 1 \\ \nu_2 &= N_2 - 1 \end{aligned}\right\} \text{degrees of freedom.}$$

Relationship to χ^2 test: F statistic is the same as χ^2 with $\nu_1 = N-1$ and $\nu_2 = \infty$.

These distributions are important when doing spectral analysis, as we'll see later in the course.

Brief note on degrees of freedom.

Degrees of freedom = # indep. samples − numbers of parameters in stat. to be est.

t-stat. $(\nu = N-1)$ $\qquad \dfrac{\bar{x}-\mu}{\frac{s}{\sqrt{N-1}}}$ $\qquad \mu$ est. (1 param)

χ^2 $(\nu = N-1)$ $\qquad \chi^2 = \dfrac{(N-1)\,s^2}{\sigma^2}$ $\qquad \sigma^2$ estimated (1 param)

CAVEAT IN DETERMINING DEGREES OF FREEDOM!!

- The samples N <u>must</u> be "independent"
- Independence means samples are not related, or autocorrelated, in <u>time</u> and <u>space</u>.

More on this later...