1. Calculate the dry adiabatic temperature lapse rate of the atmosphere of Titan in K/km. Assume the atmosphere is made of 97% N_2 and 3% methane.

g = 1.352 m/s^2.
$C_p = 0.97\ C_{p\text{-}N2} + 0.03\ C_{p\text{-}CH4} = 0.97\ 1039 + 0.03\ 1941 = 1066$ J/Kg/K
$dT/dz = -g/C_p = 1.352/1066 = -1.268$ K/km.

So Titan's dry adiabat is much small in magnitude that that of Earth's, even though they are both N_2 atmospheres, because Titan has a much lower gravitational acceleration because it is largely made of ice rather than rock.

2. Gravity calculations
   a. calculate the gravity at the surface and at 5 km altitude at the pole

The equation for gravity is

$$g = -\frac{GM}{r^2} + \frac{3GMa^2 J_2}{2r^4}\left(3\sin^2\phi - 1\right) + \omega^2 r\cos^2\phi$$

First, the radius of the Earth as a function of latitude:

$$r_0 = a\left[1 + \frac{\left(2f - f^2\right)}{\left(1-f\right)^2}\sin^2\phi\right]^{-1/2}$$

with a = 6378.139 km and f = 1/298.256.

	0 km	5 km	
Radius	6356.90	6361.90	m
g	-9.832	-9.817	m/s^2.

   b. calculate the gravity at the surface and at 5 km altitude at 45° latitude

	0 km	5 km
Radius	6367.491138	6372.491138
g	9.788799261	9.773417313

   c. calculate the gravity at the surface and at 5 km altitude at the equator

	0 km	5 km
Radius	6378.139	6383.139
g	9.780715793	9.765294477

SUMMARY

lat	**r (alt=0)**	**g total**	**1/r^2**	**J$_2$**	**centrifugal**
0	6378.14	-9.781	-9.799	-0.0159	0.03392
45	6367.49	-9.807	-9.832	0.0080	0.01693
90	6356.90	-9.832	-9.864	0.0323	0

lat	**alt=5km**	**g total**	**1/r^2**	**J$_2$**	**centrifugal**
0	6383.14	-9.765	-9.783	-0.01586	0.03394
45	6372.49	-9.791	-9.816	0.00798	0.01694
90	6361.90	-9.817	-9.849	0.03215	0

The magnitudes of the latitudinal *variations* of the three terms at a constant altitude are comparable, in the range: 0.03 to 0.06 m/s^2. The J_2 and centrifugal variations approximately cancel one another so the $1/r^2$ term ends up being quite close by itself (but too strong by about 0.02 m/s^2).

d. What is the percentage change in the gravity between the surface and 5 km in each of the cases?
0.16% (0.158% 0.157% 0.157%) The 3 are virtually identical. The change is small. A 7 km scale height at the surface would be a 7.011 km scale height at 5 km.

Consider 2 atmospheric cases. In both cases, assume the surface pressure is 1000 mb and the latitude is 45°. To make things simple, assume the air is completely dry and contains no water.

- In Case 1, the surface temperature is 280.5 K and temperature decreases with altitude at 5 K/km.
- In Case 2, the surface temperature is 288K and temperature decreases with altitude is 6.5 K/km.

3. Calculate the pressure at 5 km altitude

Use the equation below from page 10 of the notes entitled "Physical Properties of the Atmosphere"

$$\frac{P_2}{P_1} = \exp\left[-\frac{gm}{R^*\dot{T}}\ln\left(\frac{T_2}{T_1}\right)\right] = \exp\left[\ln\left(\left[\frac{T_1}{T_2}\right]^{\frac{gm}{R^*\dot{T}}}\right)\right] = \left[\frac{T_1}{T_2}\right]^{\frac{gm}{R^*\dot{T}}}$$

and the average gravity to calculate the pressure at 5 km altitude.

a. sum the values of gravity at the surface and 5 km and divide by 2 to get the approximate average gravity between the surface and 5 km altitude to use in the equation
-9.781

b. Calculate the temperature and the pressure at 5 km altitude for Case 1
255.5 K and 529.2 hPa

c. Calculate the temperature and the pressure at 5 km altitude for Case 2
255.5 K and 533.7 hPa

d. Which pressure is higher? Explain why.
(*hint: Think in terms of the pressure scale height*)
The pressure is higher in the second case because the surface pressure is the same in both cases but the average temperature is higher in the second case so the pressure scale height is larger in the second case so pressure decreases with altitude more slowly in the second case. So pressure at 5 km altitude is higher in the second case than in the first case

4. Determine the potential temperature, θ.

$$\theta = T_0 = T_1\left(\frac{P_0}{P_1}\right)^{\frac{R}{C_p}} = T_1\left(\frac{P_0}{P_1}\right)^{\frac{R^*}{C_p'}} \qquad (22)$$

where T_1 is 255.5K, P_1 is the pressure at 5 km altitude in the two cases and P_0 is the approximately surface pressure of 1000 mb.

a. Calculate the potential temperature at 5 km altitude for Case 1
306.03

b. Calculate the potential temperature at 5 km altitude for Case 2
305.29

c. What would the temperature of the air parcel be if it were lowered to the surface in each Case?
306.03 and 305.29

d. Which value is higher? Explain why

The one with the lower pressure at 5 km will be compressed more when it is raised to 1000 mb. Therefore, Case 1's temperature will increase more when it is compressed adiabatically to 1000 mb and will therefore have a higher potential temperature.

5. Calculate the stability ($d\theta/dz$) at 5 km altitude for each Case.

From equation (32) of the dry adiabatic lapse rate notes,

$$\frac{\partial \theta}{\partial z} = -\frac{\theta}{T_1}\left[\frac{\partial T_{adiabatic}}{\partial z} - \frac{\partial T_1}{\partial z}\right] \tag{32}$$

g(5 km) = -9.773 m/s^2.
C_p = 1012 J/K/kg
Dry adiabatic lapse rate = 9.773 K/km

Plugging in the values for θ, T and dT/dz at 5 km altitude for cases 1 and 2 yields
5.717 and 3.911 K/km.

6. At what frequency would a parcel oscillate if it were displaced at 5 km altitude in each Case?

$$\omega = N = \left(\frac{g}{\theta}\frac{\partial \theta}{\partial z}\right)^{1/2} \tag{38}$$

Case 1: 0.01337 radians/sec. Case 2: 0.01001 radians/sec
Case 1: 0.00213 cycles/sec. Case 2: 0.00176 cycles/sec
The period of the oscillation is
Case 1: 469 seconds Case 2: 567 seconds (about 10 minutes)

7. a. What is the restoring acceleration for a vertical displacement of 100 m in each Case?

From the dry adiabatic lapse rate notes, the acceleration due to a vertical displacement, z, is

$$a = \frac{d^2 z}{dt^2} = -\frac{k}{m}z \tag{36}$$

and

$$\frac{k}{m} = \frac{g}{\theta}\frac{\partial \theta}{\partial z} \tag{34}$$

so the vertical acceleration is

$$a = -\frac{g}{\theta}\frac{\partial \theta}{\partial z}z$$

Plugging in values for a vertical displacement of 100 m yields a = 0.0179 and 0.010 m/s^2.

b. How large is this compared to g?
Very small: 0.183% and 0.102%

Even with a 1 km displacement, the acceleration will only by 2% and 1% of g.

8. **Venus Surface Temperature**: Along the lines of the argument given in class, estimate the surface temperature of Venus assuming the atmospheric temperature equals the radiative equilibrium temperature at 300 mb and the surface pressure is 92 bars. State any assumptions you make. Account for the fact that the heat capacity of CO_2 changes with temperature

Assume the atmosphere is opaque to IR and that sunlight reaches the Venus surface and drives convection. Assume a dry adiabat. Use the potential temperature to estimate the surface temperature: $T_{surf} = \theta(300\text{mb})$

$$T(P_{surf}) = T(P_{ref})\left(\frac{P_{surf}}{P_{ref}}\right)^{\frac{R^*}{mC_p}}$$

Venus is largely a CO_2 atmosphere so we know m and C_p. The atmosphere has a composition of 96.5% CO_2 and 3.5% N_2. m = 43.44 g/mole. 44.0096 g/mol. The heat capacity of CO_2 varies with temperature which turns out to be an important effect here. I found the variation in the heat capacity of CO_2 at
http://www.engineeringtoolbox.com/carbon-dioxide-d_974.html

T(K)	*CO_2 C_p(J/kg/K)*	*N_2 C_p(J/kg/K)*	*C_p(J/kg/K)total*
225	0.763	1.039	
250	0.791	1.039	
275	0.819	1.039	
300	0.846	1.040	
325	0.871	1.040	
350	0.895	1.041	
375	0.918	1.042	
400	0.939	1.044	
450	0.978	1.049	
500	1.014	1.056	
550	1.046	1.065	
600	1.075	1.075	
650	1.102	1.086	
700	1.126	1.098	
750	1.148	1.110	

A simple approach is to take the C_P at the average temperature. An average C_P (500K) = 1014 J/kg/K.
R*/m/Cp = 8.3145/0.004344/1014 = 0.1886.
The radiative temperature is 232K. The dry adiabatic surface temperature would then be 683K = 410ºC. The actual surface temperature is 735K = 460ºC. So this estimate is a bit simplistic but does show that the surface temperature must be quite high just because the surface pressure is so high.

A more sophisticated approach is break the atmosphere into layers and use an appropriate value for the hear capacity for each layer. To do this and take advantage of the table above (which is in terms of temperature) we rearrange the adiabatic pressure temperature relation:

$$P_{i+1} = P_i \left(\frac{T(P_{i+1})}{T(P_i)} \right)^{\frac{mC_p(\bar{T})}{R^*}}$$

Here are the results:

T	P(bars)	Cp	mCp/R*
232	0.3	0.77084	
250	0.406935455	0.791	4.080000577
275	0.6075965	0.819	4.205809129
300	0.887107139	0.846	4.349485838
325	1.270268242	0.871	4.485325636
350	1.788026529	0.895	4.613328522
375	2.47903686	0.918	4.736106801
400	3.390405723	0.939	4.851048169
450	6.115226939	0.978	5.007786397
500	10.58086693	1.014	5.203709183
550	17.67134503	1.046	5.381345842
600	28.61834724	1.075	5.540696374
650	45.11657668	1.102	5.686985387
700	69.44769692	1.126	5.820212881
735.3	92.9441564	1.141532	5.923482475

So the surface temperature from this more accurate dry adiabat is 735K. The actual surface temperature is 735K. Note I said in the problem that the surface pressure is 92 bars but it is actually 93 bars. So this is remarkable agreement! Note that the heat capacities that I used above are actually for pure CO_2 which is not quite right because of the N_2 in the atmosphere.

The point is that the very high surface temperature of Venus is a direct consequence of its high surface pressure combined with its radiative equilibrium temperature and its high IR opacity and its letting at least some sunlight through to heat the surface. That solar energy absorbed at the surface must be transported back out of the atmosphere and since the IR opacity is so high, the heat has to be transported primarily by convection => dry adiabat.