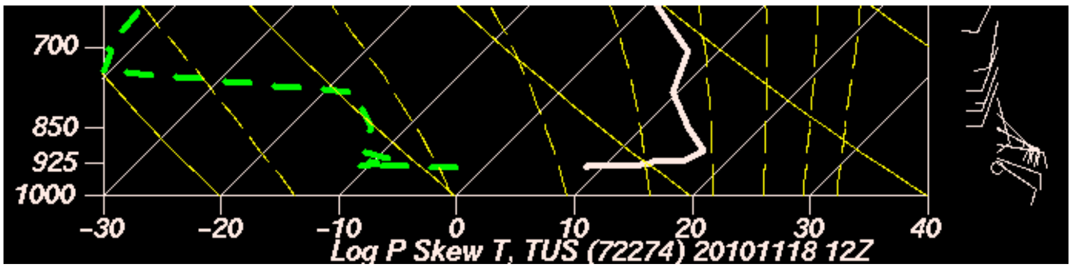
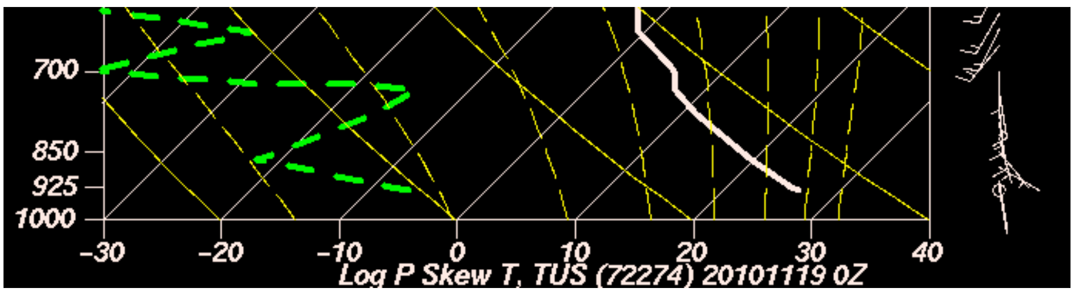
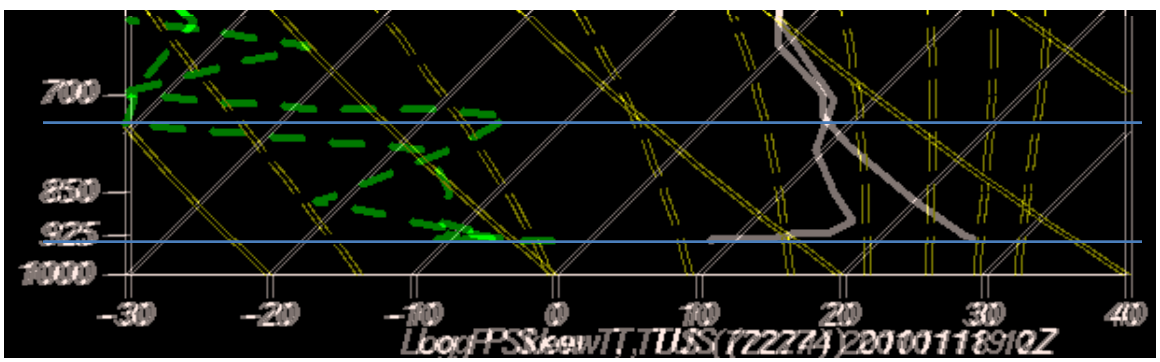
1. Sensible heat flux

Follow the steps below to determine the surface sensible heat flux by calculating the daytime heating of the planetary boundary layer (PBL)

Below are three figures. The first two show recent radiosonde profiles at 5 am and 5 pm in Tucson. The third figure shows the two figures superimposed on top of one another to show the day versus night difference in the PBL thermal structure.







a. Based on the difference between these two profiles, how deep is the daytime convective boundary layer (measured relative to the surface), in both pressure and in altitude?

The pressure at the top is about 730 mb. The pressure at the surface is about 930 mb.

Taking the height at the surface as 0, the height at the top depends on the pressure scale height. The average temperature in the PBL is about 15C. So the pressure scale height is RT/mg = 8.4 km. The height between the surface and the PBL top is therefore ln(930/730)*8.4 = 1.9 km.

b. What is the temperature lapse rate in the PBL at 5 PM?

Approxiately the dry adiabat

c. Based on the difference between the radiosonde temperature profiles in the afternoon and morning, and assuming that all of this difference is due to sensible heat flux from the surface into the atmosphere, how much energy has been added to the lowermost troposphere from morning to late afternoon.

The integral of the change in energy per unit area over the depth of the boundary layer is in terms of pressure via the hydrostatic balance

$$\frac{\overline{\Delta E}}{A} = \int_{surf}^{PBLtop} \rho C_p \left(T_{afternoon} - T_{morning}\right) dz = C_p \int_{surf}^{PBLtop} \rho \left(T_{afternoon} - T_{morning}\right) dz = -\frac{C_p}{g} \int_{surf}^{PBLtop} \left(T_{afternoon} - T_{morning}\right) dP$$

For simplicity, assume the temperature difference increases linearly with pressure over the depth of the boundary layer. So $\Delta T/\Delta P = a$ where a = 10K/20000 Pa = 5e-4 K/Pa.

$$\Delta E_{SH}(per\ m^2) = -\frac{C_p}{g} \int_{surf}^{PBLtop} \left(T_{day} - T_{night}\right) dP = \frac{C_p}{g} \int_{P_{PBLtop}}^{P_{surf}} a\left(P - P_{PBLtop}\right) dP$$

$$= \frac{C_p}{g} a \left[\frac{\left(P_{surf}^{\ 2} - P_{PBLtop}^{\ 2}\right)}{2} - P_{PBLtop}\left(P_{surf} - P_{PBLtop}\right)\right] = \frac{C_p}{g} a \left[\frac{P_{surf}^{\ 2}}{2} + \frac{P_{PBLtop}^{\ 2}}{2} - P_{PBLtop} P_{surf}\right]$$

$$= \frac{C_p}{g} a \frac{\left[P_{surf} - P_{PBLtop}\right]^2}{2}$$

because T_{day}-T_{nit} at the top of the PBL is 0 and it increases approximately linearly down to the surface. The answer is ~ 7.5e6 J/m^2. NOTE: the units are NOT W/m^2.

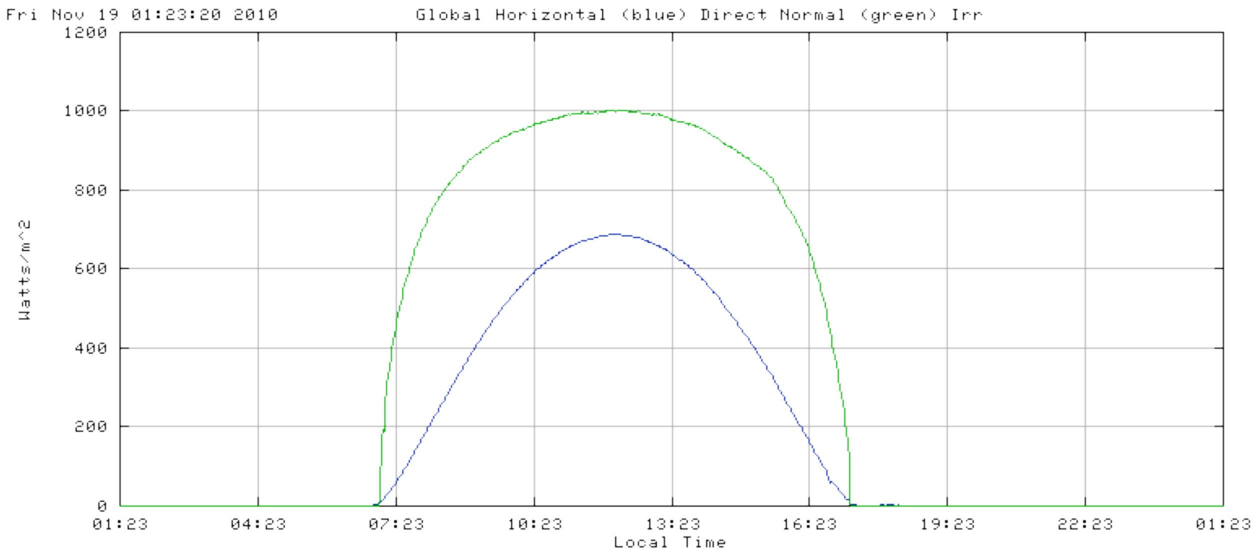
d. Divide this by the length of time over which the atmospheric heating took place to determine the approximate average sensible heat flux from the surface into the atmosphere from sunrise to the late afternoon? (your answer should be around 200 W/m^2).

Late afternoon is 3 PM = 15 hours. Early morning is 6 hours. The time between them is 9 hours which is 32,400 seconds. The energy flux is therefore 7.5e6 J/m^2 /32000 sec = 230 W/m^2.

2. Approximate Daytime Surface Energy Budget

For this problem, you will estimate an approximate energy budget at the surface. From sunrise to late afternoon, the energy into the surface is absorbed solar flux and absorbed IR flux from the atmosphere. Some of this energy is lost via sensible heat flux into the atmosphere. Some is lost via IR flux out of the surface. Some goes into raising the temperature of the surface as a diffusive boundary layer forms and thickens in the soil.

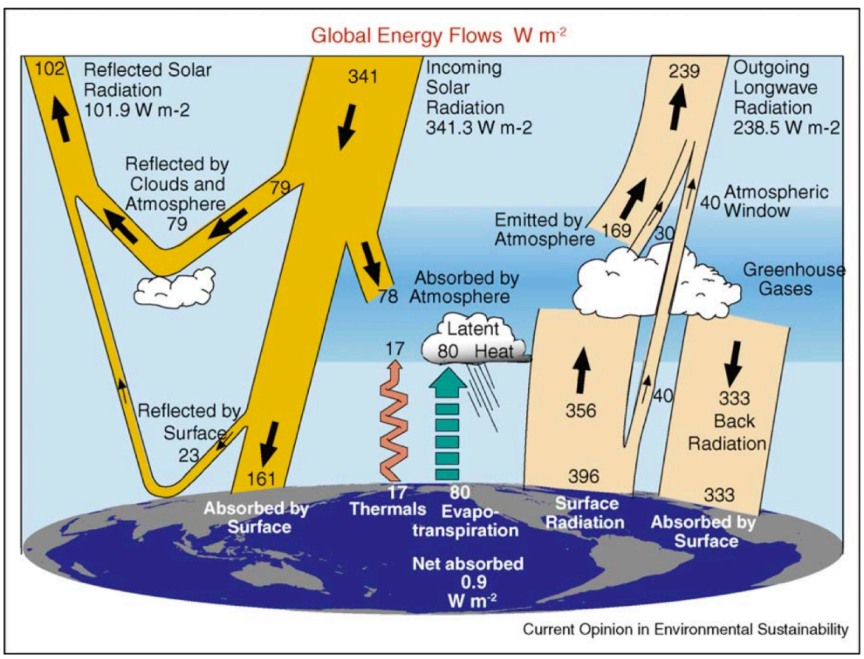
The solar flux measured on the roof of PAS is given below.



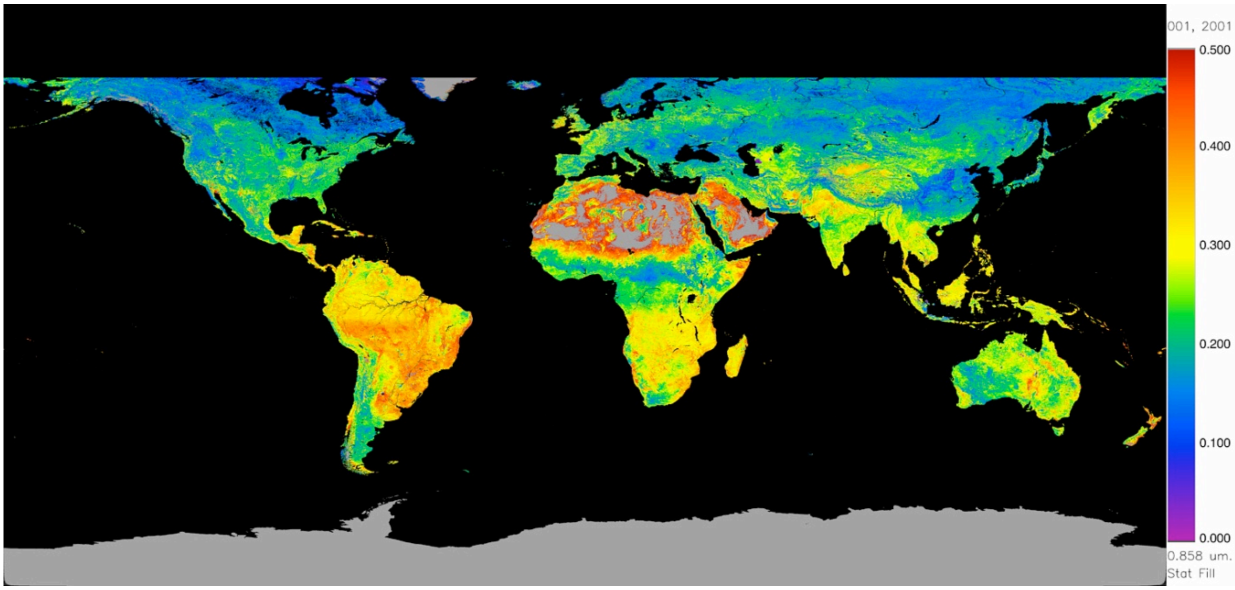
a. From sunrise to late afternoon, what is the average solar radiation absorbed by the surface?

Use the blue curve which includes the angle between the sunlight and the incident solar radiation. Based on the blue curve, divide the day into 3 intervals of 3 hours each. Approximately $(300 + 650 + 400)/3 = 450\ W/m^2$.

Multiply this by (1 – albedo). Now note that the albedo has two contributions, one from the atmosphere and one from the surface.



According to the figure, most of the albedo is due to the atmosphere ($79\ W/m^2 \sim 78\%$ of the albedo) vs only ($23\ W/m^2 \sim 22\%$ of the albedo) from the surface. The blue curve already includes the portion of the albedo due to the atmosphere. The atmospheric albedo is lower over Tucson than the global average because there are few clouds. Now the low contribution of the surface to the albedo is low because most of Earth's surface is covered by water which is dark. So the albedo over semi-arid regions is much higher than over the oceans.



A guess of what the surface albedo is around Tucson is it is 0.15. This is consistent with the albedo map which has Tucson as a blue to green area.

Using this value the solar energy absorbed is 450*(1-0.15)=360 W/m^2.

b. From sunrise to late afternoon, what is the average IR flux upward out of the surface? Be careful with your average because the Stephan-Boltzmann equation is quite a nonlinear function of temperature.

Again, divide the 9 hours into three 3 hour segments. I get 63, 70 and 77 F as the temperatures of those segments. The fluxes are 402, 425 and 447 W/m^2 for an average of 425 W/m^2.

c. Assuming the downward IR comes approximately from an altitude of 3 km (global average is ~2 km but Tucson is quite dry so the downward IR comes from higher altitudes), what is the approximate average downward IR flux from sunrise to late afternoon?

The temperature at 3 km is about 0 C. Therefore the average downward radiation is about 315 W/m^2.

d. Energy absorbed and stored in the soil: You can estimate the energy that flows into the soil based on the increase in soil temperature multiplied by the diffusive depth of penetration into the soil. You can estimate the depth of the diffusive boundary layer from equation (16) of the Diffusion Lecture and the diffusivity and heat capacity of soil which you can find on line.

The heat capacity of soil is about 800 J/K/kg. The density of the soil is 1600 kg/m^3. The energy per unit volume stored in the soil is

$$\rho_E = \rho c_p \Delta T$$

The next question is how deep is the thermal boundary layer in the soil. The relation between the diffusion depth and time is related via the equation

$$\tau_\lambda = n\tau_\lambda = n\frac{\lambda}{v_t} = \left(\frac{X}{\lambda}\right)^2 \frac{\lambda}{v_t} = \frac{X^2}{v_t \lambda} \quad (16)$$

Therefore, $X = \sqrt{\tau_\lambda v_t \lambda} = \sqrt{\tau_\lambda D_t}$ where D_t is the thermal diffusivity of the soil. The time of solar heating we already know. The thermal diffusivity of soil is about 0.2x10^{-6} m^2/s. So the thickness of the diffusive boundary layer is about 8 cm. So the energy per m^2 in the soil is

$$\rho_E X = \rho c_p \Delta T X$$

The temperature increase is from 46 to 79F which is 18C. which is 1,900,000 J/m^2. Dividing this by the time gives the watts per m^2. The answer is 58 W/m^2.

e. Use the answers to parts a-d and the previous question to show what the surface energy budget is. (I get ~340 W/m^2 in which is balanced approximately by the energy out and the energy stored in the soil)

Energy into surface from top:
The measured 450 W/m2 solar includes the effect of the atmospheric albedo but does not include the effect of the surface albedo. Guess that the surface albedo is 15%.

- The solar energy absorbed is about (1-0.15)*450 = 383 W/m^2.
- The IR in is 316 W/m^2.
- TOTAL IN= 698 W/m^2.

Energy out of surface:

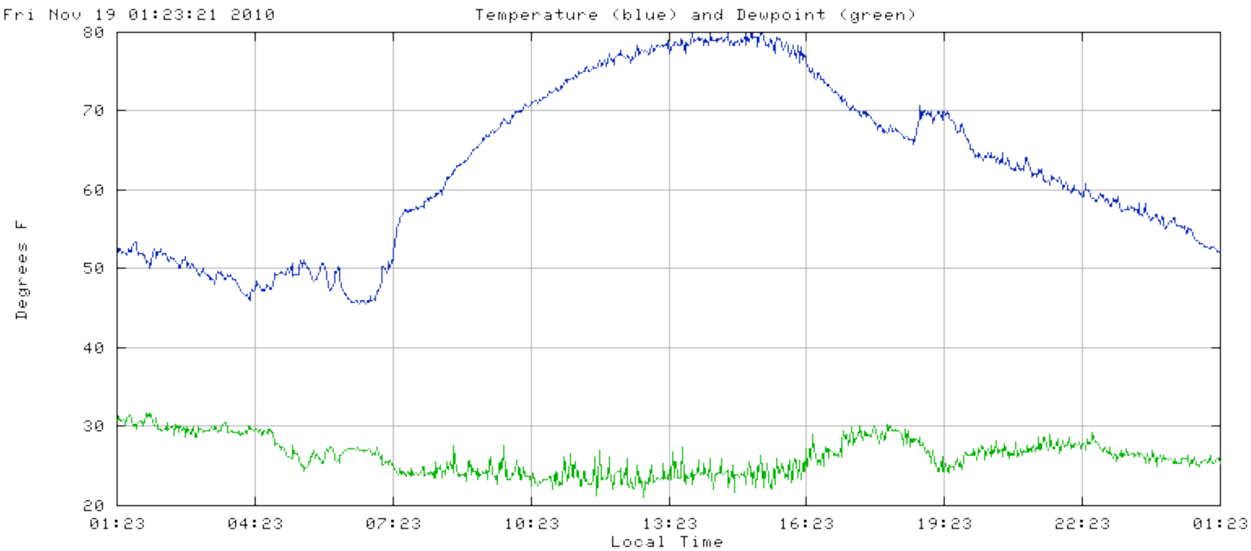
- The IR out is 425 W/m^2.
- The sensible heat flux is about 233 W/m^2.
- The energy into diffusion layer 58 W/m^2.
- TOTAL OUT = 716 W/m^2.

The net is IN-OUT = -18 W/m^2. This is closer than I expected.

Alternatively, one can solve for the albedo.

Note that we did not include a latent heat flux. So the real energy flux out of the surface is actually higher than we estimated.

Below is the temperature and dew point over the same day.



3. Nocturnal Boundary Layer Thickness

A sharply defined nocturnal boundary layer (NBL) is evident in the 5 AM radiosonde profile. You can estimate the depth of that shallow boundary layer from how long the sensible heat flux takes to heat it up and make it disappear after the sun comes up.

In the figure above, the surface temperature increased very quickly around 7 AM. The sun heated up the surface which then heated up the NBL via sensible heat transfer. Based on the rapid change in the surface temperature shortly after sunrise centered approximately on 7:23 AM in the figure, what is the depth of the nocturnal boundary layer?

To keep things simple, assume the thermal inversion is such that the atmospheric temperature across the depth of the NBL increases linearly with altitude up to the depth of the nocturnal boundary layer at sunrise. Assume an albedo and that the absorbed portion of the incident solar flux is converted directly to a sensible heat flux. (*I got an answer of about 30 m.*)

$$\overline{F_{SH}} = C_p \frac{\int_{surf}^{PBLtop} \rho \left(T_{end} - T_{begin}\right) dz}{t_{end} - t_{begin}}$$

Assuming the nocturnal BL temperature increases linearly with altitude, then

$$\overline{F_{SH}} = C_p \rho \frac{\left(T_{end} - T_{begin}\right)}{\left(t_{end} - t_{begin}\right)} \frac{Z_{NBL}}{2}$$

Assume the solar flux is converted to a sensible heat flux. $F_{SH} = F_{solar}$.

$$Z_{NBL} = 2 \frac{\overline{F_{solar}}}{C_p \rho} \frac{\left(t_{end} - t_{begin}\right)}{\left(T_{end} - T_{begin}\right)}$$

F_{solar} is about 50 W/m^2. Assume a surface albedo of 0.15. The solar energy absorbed is about 42 W/m^2.

The increase is from 46°F to 57°F for a change of 11 F or about 6 C over a period of about 30 minutes or 2000 seconds. The rate of surface temperature increase is therefore about 3/1000 = 0.003 C/sec.

The height of the nocturnal boundary layer is therefore about 25 m.

4. Occasional temperature gradient extremes in Tucson

Occasionally, there will be extremely different overnight temperature minima across Tucson. For example, on a clear night, minimum temperatures 4 miles north of campus near the Rillito Wash can be below freezing while the minimum temperature on campus is 50F. Thinking in terms of topography and turbulent sensible heat fluxes, explain how this is possible.

The Rillito Wash is a somewhat lower altitude than campus. The cold air therefore flows downward into the wash. On the very clear nights, a strong thermal inversion can set up at the surface because of the radiative cooling from the surface. Warmer air is aloft. If there is turbulence aloft, it will mix the warmer air downward toward the surface. On these unusual nights where the wash is much colder than campus, the vertical mixing has reached down to the surface of campus but has not managed to reach down to the surface of the somewhat lower wash.

5. Aerodynamic Form

Set the daytime sensible heat flux you got in problem 1 equal to the aerodynamic form of the surface flux and determine the temperature difference θ(z) – θ(0). Use a representative value of the drag coefficient from the table below.

$$F_{SH} = \rho_a c_P \overline{w'\theta'} = -K_H \rho_a c_P \frac{\partial \bar{\theta}}{\partial z} = -\rho_a c_p C_H |U| [\theta(z) - \theta(0)] \quad (8)$$

$$[\theta(z) - \theta(0)] = -\frac{F_{SH}}{\rho_a c_p C_H |U|}$$

The wind speed is 11 mph ~ 5 m/s . The drag coefficient is ~0.05. The surface density is 1.1 kg/m3. The heat capacity is 1000. The temperature difference is

230/(1.1*1000*0.05*5) = 0.9 K.

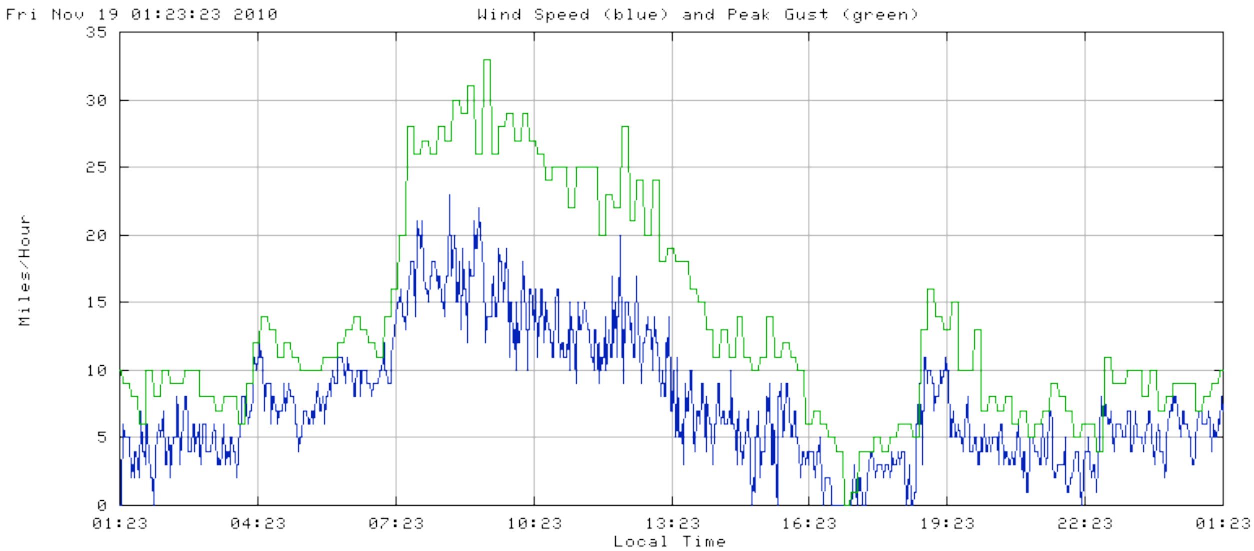


TABLE 1. Relevant parameters as determined from previously published data. All data are based on a 10-m standard height except where noted: C_m is the new drag coefficient in the MABL, b is the neglected constant in the traditional definition of C_D, and u_{*_0} is the friction velocity at the onset of aerodynamically rough flow. In the determination of these linear regression coefficients, data below $U_{10} = 8$ m s^{-1} and $u_* = 0.27$ m s^{-1} have been neglected. The correlation coefficient (Corr coef) is taken as that between the u_* and U_{10} distribution. The value R_{nr} is described further in the text and is the ratio of the norm of residuals as found from a linear and quadratic fit to data [see Eq. (21)]. A negative sign leading this value (−) here denotes a quadratic model that is concave down.

Author	C_m	$-b$ (m s^{-1})	u_{*_0} (m s^{-1})	Corr coef	R_{nr}	ΔU_{10}[a] (m s^{-1})	Location	Method[b]	Data source[c]
Avg (over all data)	0.051	0.14	0.27	0.93	(−)1.000[d]	8–30	—	—	—
Open ocean									
Large and Pond (1982)	0.048	0.14	0.24	0.97	0.982	8–18	North Pacific [Storm Transfer and Response Experiments (STREX)]	PV, ID	Fig. 8b
Banner et al. (1999)	0.052	0.13	0.29	0.84	0.988	8–20	Southwest Tasmania [Southern Ocean Waves Experiment (SOWEX)]	A, ECA	Table 2
Persson et al. (2005)	0.057	0.18	0.27	0.86	0.969	8–20	Mid-Atlantic [Fronts and Atlantic Storm Track Experiment (FASTEX)]	S, ECM	Fig. 7a[e]
Black et al. (2007)	0.047	0.12	0.25	0.87	(−)0.993	10–29	Atlantic [Coupled Boundary Layer Air–Sea Transfer (CBLAST)]	A, ECA	Fig. 5[f]
Open ocean–coastal site									
Smith and Banke (1975)	0.053	0.16	0.27	0.99	0.952	8–21	Sable Island, Canada	T, EC	Table 1
Smith (1980)	0.055	0.25	0.19	0.95	0.927	8–22	Halifax Harbour	T, EC	Table 1
Large and Pond (1981)	0.049	0.16	0.23	0.95	0.995	8–19	Halifax Harbour	PV, ID	Fig. 3
Dobson et al. (1994)	0.050	0.13	0.27	0.97	0.927	8–17	Grand Banks	PV, ID	Table 1
Donelan et al. (1997)[g]	0.061	0.25	0.24	0.94	0.980	8–14	Virginia coast [Surface Wave Dynamics Experiment (SWADE)]	PV, ECM	Table 1
Drennan et al. (1999a)[g]	0.042	0.06	0.28	0.84	(−)0.977	8–17	Virginia coast (SWADE)	PV, ECM	Fig. 12a[h]
Sea/limited fetch									
Smith (1980)	0.044	0.06	0.29	0.96	(−)0.938	8–20	Halifax Harbour	T, EC	Table 2
Geernaert et al. (1987)	0.058	0.21	0.26	0.97	0.974	8–25	North Sea	S, EC	Table 2
Anderson (1993)	0.050	0.16	0.25	0.99	0.982	8–19	Georges Bank–Labrador Sea	PV, ID	Fig. 5a
Janssen (1997)	0.065	0.27	0.25	0.98	0.848	8–20	North Sea (HEXOS)	S, EC	Appendix A
Johnson et al. (1998)	0.047	0.10	0.27	0.97	0.852	8–16	Vindeby Island, Denmark (RASEX)	S, EC	Table 1
Bumke et al. (2002)	0.046	0.10	0.27	0.94	(−)0.982	8–15	Labrador Sea	S, ID	Fig. 7[i]
Larsen et al. (2003)[g]	0.049	0.13	0.26	0.94	0.942	8–17	Ostergarnsholm, Sweden	S, EC	Fig. 4c
Drennan et al. (2003)[g]	0.055	0.20	0.24	0.96	0.964	8–19	Gulf of Lion, Mediterranean [Flux, sea state, and remote sensing in conditions of variable fetch (FETCH)]	S, ECM	Fig. 4
Petersen and Renfrew (2009)	0.050	0.06	0.34	0.90	0.999	9–25	Denmark Strait [Greenland Flow Distortion Experiment (GFDex)]	A, ECA	Fig. 7a
Lake									
Graf and Prost (1980)	0.040	0.06	0.25	0.94	0.977	8–16	Lake Geneva	P	Table 1
Graf et al. (1984)	0.059	0.14	0.33	0.93	0.935	8–17	Lake Geneva	P	Table 2

Foreman and Emeis (2010), Revisiting the Definition of the Drag Coefficient in the Marine Atmospheric Boundary Layer, p. 2325, DOI: 10.1175/2010JPO4420.1

This is the temperature difference between the surface and atmosphere typically taken as 10 m altitude that is driving the heating of the atmosphere.

Use this vertical temperature structure to determine the eddy diffusivity.

$$\overline{w'\theta'} = -K_H \frac{\partial\overline{\theta}}{\partial z} = -z|U|C_H \frac{\partial\overline{\theta}}{\partial z} \tag{9}$$

$$F_{SH} = -K_H \rho_a c_P \frac{\partial\overline{\theta}}{\partial z} \tag{8}$$

$$K_H = -\frac{F_{SH}}{\rho_a c_P \frac{\partial\overline{\theta}}{\partial z}} = z|U|C_H$$

The answer is 2.4 m^2/s.

This is 5 orders of magnitude larger than the molecular diffusivity which is 2e-5 m^2/s. So eddy diffusivity indeed dominates typically over molecular diffusivity.

6. Near surface nighttime sensible heat flux

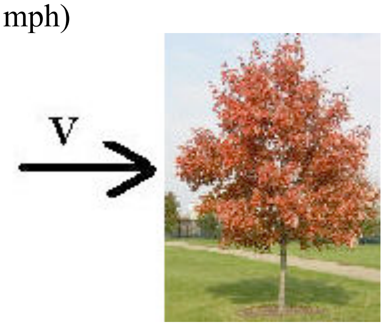
Consider a citrus tree grove where the trees are about 5 m in height and the wind at 10 m is blowing at 2 m/sec with a surface temperature of 0^oC and pressure of 1000 mb. Assume the temperature at 10 m is 2^oC warmer than the surface temperature

a. Calculate the vertical sensible heat flux using the aerodynamic form.

$$F_{SH} = -\rho_a c_p C_H |U| \left[\theta(z) - \theta(0)\right] \tag{8}$$

The question is what to use as C_H. I allows 0.05 from the previous problem or something in the range of 0.2 to 0.43 based on the info below.

Tree (C values from Munson et al., 1998)
A=Tree frontal area
C=0.43 if V=10 m/s (36.0 km/h, 22.4 mph)
C=0.26 if V=20 m/s (72.0 km/h, 44.7 mph)
C=0.20 if V=30 m/s (108 km/h, 67.1 mph)



Munson, Bruce R., Donald F. Young, and Theodore H. Okiishi. 1998. Fundamentals of Fluid Mechanics. John Wiley and Sons, Inc. 3ed.

Plugging in C_H=0.05 and assuming the temperature difference at 10m and the wind velocity is 2 m/s is 1K yields -224 W/m^2. Plugging in C_H=0.33 as many people used yields 1477 W/m^2.

b. Is the flux up or down?

Down

c. Calculate an approximate eddy diffusivity.

$$K_H = -\frac{F_{SH}}{\rho_a c_P \frac{\partial \bar{\theta}}{\partial z}} = z|U|C_H$$

Plugging in the numbers yields 10m 2 m/s 0.05 = 1 m^2/s.

d. Assuming the scale of the eddies is the size of the trees, use the eddy diffusivity to determine what the typical vertical velocity of the eddies is.

$K_H = vL$ So $v = K_H/L$ = 1/5 = 0.2 m/sec

e. For the nighttime conditions in the figures in Problem 1, how large is the sensible heat flux in comparison to the net (up minus down) IR radiative flux at the surface? Assume the downward IR flux from the atmosphere comes from a level where the temperature of -10C. Is the sensible heat flux sufficient to keep the trees from freezing overnight?

Surface IR up = 316 W/m^2. Surface IR down = 272 W/m^2. Net IR up = 44 W/m^2. This is much less than 224 W/m^2 down. So this downward sensible flux will keep the trees warm and above freezing under these conditions.

7. Increase in the surface evaporative flux with global warming

Suppose Earth's surface were to warm by 2°C while the relative humidity of the air and the winds were to remain the same. Using the aerodynamic formula for latent heat flux, determine the ratio of the new surface latent heat flux to the present surface latent heat flux.

The aerodynamic surface latent heat flux is

$$F_{LH} = \rho_a L_v C_W U(q_s - q_a)$$

and the ratio of the new to the present (or old) latent heat flux is

$$\frac{F_{LH-new}}{F_{LH-old}} = \frac{\rho_{a-new} L_v C_{W-new} U_{new}(q_{s-new} - q_{a-new})}{\rho_{a-old} L_v C_{W-old} U_{old}(q_{s-old} - q_{a-old})}$$

We expand the specific humidity ratio in terms of what the specific humidity is

$$\frac{(q_{s-new} - q_{a-new})}{(q_{s-old} - q_{a-old})} = \frac{\left(\frac{m_v e_{s-new}}{(m_d P_{s-new} - m_v e_{s-new})} - \frac{m_v e_{a-new}}{(m_d P_{a-new} - m_v e_{a-new})}\right)}{\left(\frac{m_v e_{s-old}}{(m_d P_{s-old} - m_v e_{s-old})} - \frac{m_v e_{a-old}}{(m_d P_{a-old} - m_v e_{a-old})}\right)}$$

$$\frac{\left(q_{s-new}-q_{a-new}\right)}{\left(q_{s-old}-q_{a-old}\right)} \cong \frac{\left(\frac{m_v e_{s-new}}{\left(m_d P_{s-new}\right)}-\frac{m_v e_{a-new}}{\left(m_d P_{a-new}\right)}\right)}{\left(\frac{m_v e_{s-old}}{\left(m_d P_{s-old}\right)}-\frac{m_v e_{a-old}}{\left(m_d P_{a-old}\right)}\right)} \cong \frac{\left(\frac{m_v e_{s-new}}{\left(m_d P_{s-new}\right)}-\frac{m_v e_{a-new}}{\left(m_d P_{s-new}\right)}\right)}{\left(\frac{m_v e_{s-old}}{\left(m_d P_{s-old}\right)}-\frac{m_v e_{a-old}}{\left(m_d P_{s-old}\right)}\right)}$$

$$\frac{\left(q_{s-new}-q_{a-new}\right)}{\left(q_{s-old}-q_{a-old}\right)} \cong \frac{\left(\frac{m_v e_{s-new}-m_v e_{a-new}}{\left(m_d P_{s-new}\right)}\right)}{\left(\frac{m_v e_{s-old}-m_v e_{a-old}}{\left(m_d P_{s-old}\right)}\right)}$$

Now we use $e = e_s\, RH$ and the differential form of the Clausius Clapeyron equation

$$\frac{de_s}{e_s} = \frac{L_v}{R_v T^2} dT$$

So when RH is constant,

$$\frac{de}{e} = \frac{d\left(RH\, e_s\right)}{RH\, e_s} = \frac{RH\, d\left(e_s\right)}{RH\, e_s} = \frac{de_s}{e_s} = \frac{L_v}{R_v T^2} dT$$

So

$$\frac{e_{new}}{e_{old}} = \frac{\left(RH\, e_s\right)_{new}}{\left(RH\, e_s\right)_{old}} = \frac{\left(e_s\right)_{new}}{\left(e_s\right)_{old}} = \frac{\left(e_s\right)_{old}+\Delta e_s}{\left(e_s\right)_{old}} = \frac{\left(e_s\right)_{old}+\left(e_s\right)_{old}\frac{L_v}{R_v T^2}\Delta T}{\left(e_s\right)_{old}} = 1+\frac{L_v}{R_v T^2}\Delta T$$

so

$$\frac{\left(q_{s-new}-q_{a-new}\right)}{\left(q_{s-old}-q_{a-old}\right)} \cong \frac{\left(\frac{m_v e_{s-new}-m_v e_{a-new}}{\left(m_d P_{s-new}\right)}\right)}{\left(\frac{m_v e_{s-old}-m_v e_{a-old}}{\left(m_d P_{s-old}\right)}\right)} = \frac{\left(\frac{RH_s e_{s-s-new}-RH_a e_{s-a-new}}{\left(P_{s-new}\right)}\right)}{\left(\frac{RH_s e_{s-s-old}-RH_a e_{s-a-old}}{\left(P_{s-old}\right)}\right)}$$

$$\frac{\left(q_{s-new}-q_{a-new}\right)}{\left(q_{s-old}-q_{a-old}\right)} \cong \frac{P_{s-old}}{P_{s-new}}\left(\frac{RH_s e_{s-s-old}\left(1+\frac{L_v}{R_v T^2}\Delta T\right)-RH_a e_{s-a-old}\left(1+\frac{L_v}{R_v T^2}\Delta T\right)}{RH_s e_{s-s-old}-RH_a e_{s-a-old}}\right)$$

$$\frac{\left(q_{s-new}-q_{a-new}\right)}{\left(q_{s-old}-q_{a-old}\right)} \cong \frac{P_{s-old}}{P_{s-new}}\left(\frac{RH_s e_{s-s-old}-RH_a e_{s-a-old}}{RH_s e_{s-s-old}-RH_a e_{s-a-old}}\right)\left(1+\frac{L_v}{R_v T^2}\Delta T\right) = \frac{P_{s-old}}{P_{s-new}}\left(1+\frac{L_v}{R_v T^2}\Delta T\right)$$

Notice that with these simplifications we didn't even need to know what the current relative humidity is. We simply had to assume it remains unchanged in a warmer climate. This is approximately what the global climate models do. (Is this correct? I don't know. There are some observations that are consistent with this type of constant RH behavior with warming).

We then assume the surface pressure does not change with global warming to simplify this further. There actually will be a slight increase in surface pressure as the amount of water vapor in the atmosphere increases but we ignore that subtle change. So

$$\frac{(q_{s-new} - q_{a-new})}{(q_{s-old} - q_{a-old})} \cong \left(1 + \frac{L_v}{R_v T^2}\Delta T\right)$$

We also need to consider potential changes in $\rho_{a-new} L_v C_{W-new} U_{new}$. We will ignore changes in the latent heat although there will be a slight decrease as the temperature changes (the water molecules are more active when they are warmer and therefore it is easier for them to fly off the surface. Therefore there is not as much energy associated with them changing phase and flying off the surface as temperature increases). The temperature dependence of the molar latent heat, L_m, is

$$L_m = \alpha\,(T_{crit} - T)^{0.375}$$

where T_{crit} is the critical temperature which is the temperature where the distinction between vapor and liquid ceases to exist (647.096 K for water) and α is a constant that depends on the liquid. From this we see that

$$\frac{L_{m-new}}{L_{m-old}} = \frac{(T_{crit} - T_{new})^{0.375}}{(T_{crit} - T_{old})^{0.375}} = \frac{(647.096 - 292)^{0.375}}{(647.096 - 288)^{0.375}} = 0.996$$

We will ignore any possible changes in the wind speed. In some general sense the heat engine concept says there should be some increase in the winds with global warming but we ignore this. The drag coefficient does not change if the surface roughness does not change. If the plants start changing in a given area due to global warming then there could be a change. Over the oceans, C_w depends on the wnds speed which creates surface roughness over the oceans. We have assumed the winds don't change so C_w over the oceans won't change.

The density of the air changes slightly

$$\frac{\rho_{air-new}}{\rho_{air-old}} = \frac{P_{surf-new}}{P_{surf-old}}\frac{T_{old}}{T_{new}} = \frac{T_{old}}{T_{new}}$$

Plugging these last two equations into the surface latent heat flux ratio equation yields

$$\frac{F_{LH-new}}{F_{LH-old}} = \frac{\rho_{a-new} L_v C_{W-new} U_{new} (q_{s-new} - q_{a-new})}{\rho_{a-old} L_v C_{W-old} U_{old} (q_{s-old} - q_{a-old})} = \frac{T_{old}}{T_{new}}\left(1 + \frac{L_v}{R_v T^2}\Delta T\right)$$

So for a 2 K increase in surface temperature, the latent heat flux will increase by 12%.

$$\frac{F_{LH-new}}{F_{LH-old}} = \frac{288}{290}\left(1 + \frac{2.5e6}{462 * 288^2}2\right) = 1.12$$

The present globally averaged latent heat flux is 78 W/m^2. So a 12% increase would raise it to 87 W/m^2.

So even though water vapor is a greenhouse gas that enhances and roughly doubles the global warming due to increased CO_2 concentrations, the increase in the surface latent heat flux with global warming will actually cool Earth's surface.

Compare the increases in the upward surface radiative flux and surface latent heat flux in terms of W/m^2. Which increase is larger?

The upward IR flux changes from 390.08 to 401.0 which is a change of 10.95 W/m^2. The latent heat flux increase is 9.4 W/m^2. The latent heat flux is almost as large as the increase in the upward IR flux. The downward flux also increases perhaps faster than the upward IR flux. So the net IR upward flux increases perhaps a bit more slowly than. The nonlinear dependence on temperature of the latent heat is larger than that in the Stephan Boltzmann equation.

Why is this important to how much surface temperatures will increase as the downward IR flux from the atmosphere into the surface increases as GHG concentrations increase?

Roughly half of the increase in downward IR flux into the surface is offset by an increase in the latent heat flux from the surface. The other half is made up primarily by the increase in IR emitted by the surface due to the higher surface temperatures. This will hold the increase in the surface temperature to about half of the increase required radiatively.

8. Diffusion scaling:

The time to cook a hard-boiled egg is ~12 minutes. Based on your understanding of diffusion (see eq. (16) of the diffusion lecture), approximately how long should it take for a watermelon to cool down to the refrigerator temperature?

$$\tau_\lambda = n\tau_\lambda = n\frac{\lambda}{v_t} = \left(\frac{X}{\lambda}\right)^2 \frac{\lambda}{v_t} = \frac{X^2}{v_t\lambda} \qquad (16)$$

The point here is that the mean free path and velocity of molecules in the egg and in the watermelon are about the same. The watermelon is much larger meaning that X is much larger and the time to cook should therefore be about the ratio of their linear dimension size squared. If the watermelon is about 5 times larger than the egg in each dimension then the watermelon will take about 25 times as long to cool down.

One can do this more precisely by accounting for the fact that the velocities in the egg will be higher because of the higher temperatures near boiling in the egg and the cooler temperatures in the watermelon near freezing. The ratio of the velocities (ignoring differences in the masses of the average molecules which are both probably primarily water) will be

$$v_{egg}/v_{watermelon} = (T_{egg}/T_{watermelon})^{1/2} = (373/273)^{1/2} = (1.37)^{1/2} = 1.17.$$

So the velocity effect is there but the sensitivity to the square of the relative sizes of the objects is much larger. So taking into account the velocity dependence on temperature, the watermelon that is 5 times larger than the egg in terms of their linear dimensions will take about 1.17 times 25 which is 29 times as long to cool as the egg will take to cook.

Since the egg takes 12 minutes or 1/5 of an hour, the watermelon will take 5-6 hours to cool down.

You can use this physics cookbook to figure out how long your Christmas turkey, ham or roast beast will take to cook.