Impulse and Work

Normally Newton's second law is written as

$$\boldsymbol{F} = m\,\boldsymbol{a}$$

which means the force on an object is equal to the object's mass times its acceleration, where bold indicates a vector. Consider gravity. The ***acceleration*** of gravity is independent of the object being pulled. The ***force*** of gravity is proportional to the mass of the object being pulled. So two objects with very different masses will accelerate identically but the forces on the two objects will be quite different.

The $\boldsymbol{F}=m\boldsymbol{a}$ equation can be written in terms of momentum, $\boldsymbol{p}$. $\boldsymbol{p}$ is a vector defined as

$$\boldsymbol{p} = m\,\boldsymbol{v}$$

where m is mass and $\boldsymbol{v}$ is the velocity (vector). Therefore,

$$\boldsymbol{F} = m\,\mathbf{a} = m\,\mathrm{d}\boldsymbol{v}/\mathrm{d}t = \mathrm{d}\boldsymbol{p}/\mathrm{d}t$$

So force equals the time rate of change of momentum of an object.

Impulse

For understanding the pressure exerted by a gas, we need to think in terms of impulses. An impulse, $\boldsymbol{I}$, is defined as a finite change in momentum. So

$$\boldsymbol{I} = \Delta\boldsymbol{p}$$

But

$$\Delta\vec{p} = \vec{p}_2 - \vec{p}_1 = \int_{t_1}^{t_2} d\vec{p} = \int_{t_1}^{t_2} \frac{d\vec{p}}{dt}\,dt = \int_{t_1}^{t_2} \vec{F}\,dt$$

so

$$\vec{I} = \int_{t_1}^{t_2} \vec{F}\,dt$$

Now notice that the time average of a quantity, A, is given as

$$\overline{A} = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} A\,dt$$

from which it follows that

$$\overline{A}(t_2 - t_1) = \overline{A}\int_{t_1}^{t_2} dt = \int_{t_1}^{t_2} A\,dt$$

So

$$\vec{I} = \int_{t_1}^{t_2} \vec{F} dt = \overline{\vec{F}}(t_2 - t_1) = \overline{\vec{F}} \Delta t = \Delta \vec{p}$$

So an impulse is equal to the *average* force over a time interval, Δt, times the length of that time interval. We will use this relation later on understanding the pressure exerted by a gas at the microscopic level.

Work

Kinetic energy, E_K, is

$$E_K = \frac{1}{2} m v^2$$

Now consider the following quantity, the dot product of the force and infinitesimal change in position:

$$\vec{F} \bullet d\vec{x} = F_x \, dx + F_y \, dy + F_z \, dz$$

Now consider just one component or consider the direction, $\boldsymbol{x}$, as the direction along which the force is pushing. So the direction, $\boldsymbol{x}$, is aligned with $\boldsymbol{F}$.

$$F \, dx = ma \, dx = m \frac{dv}{dt} dx = m \frac{dv}{dt} \frac{dx}{dt} dt = m \frac{dv}{dt} v \, dt = m \frac{d}{dt}\left(\frac{v^2}{2}\right) dt = \frac{m}{2} d\left(v^2\right)$$

So $\vec{F} \bullet d\vec{x}$ is equal to the change in kinetic energy, dE_K. This quantity is defined as the change in work, dW.

Now integrate $F \, dx$ along the path, Δx, where $\boldsymbol{x}$ is in the direction of the acceleration.

$$\Delta E_K = \Delta W = \int_{x_1}^{x_2} F \, d\xi = \int_{v(x_1)}^{v(x_2)} m \, d\left(\frac{v^2}{2}\right) = m\left[\frac{v^2(x_2)}{2} - \frac{v^2(x_1)}{2}\right] = E_K(x_2) - E_K(x_1)$$

So the force on an object times the distance over which it pushes on the object equals the change in the kinetic energy of an object and this process is called Work.