

Kinetic pressure of a gas

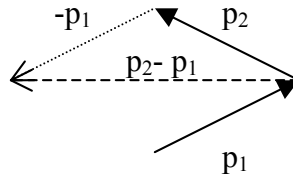
Here we relate the pressure of a gas to the collisions of the molecules on the walls of the container holding the gas.

Consider a container that is a rectangular parallelepiped. The dimension in the x direction is L . The area of the yz side is A . The volume is therefore LA . We focus on any molecule in the gas. Its x component of its velocity is v_x . Consider a time interval, Δt . What is the probability that a gas molecule will hit the right wall and bounce back? We choose Δt to be small enough that the molecule is very unlikely to hit any other molecule before it hits the wall. So $\Delta l = v_x \Delta t$ must be \ll the mean free path, mfp , of the molecules, that is, the average distance traveled by molecules between collisions.

So if the molecule is farther from the wall than Δl , then the molecule will not hit the wall within the time interval, Δt . Of the molecules that have an x component of velocity equal to v_x , half of them are moving toward the wall and half are moving away from the wall. So the probability of molecules hitting the wall is

$$f = \frac{1}{2} \frac{\Delta l}{L} = \frac{|v_x| \Delta t}{2L}$$

f is the probability that a randomly chosen molecule with a velocity, v_x , hits the wall and rebounds within a time interval, Δt .



Momentum exchange

The figure above shows the momentum vectors of the molecule, before (p_1) and after (p_2) the collision with the wall. The change in the momentum of the **molecule** is

$$p_2 - p_1 = -2 p_x \mathbf{i} = -2 m v_x \mathbf{i}$$

By Newton's 3rd law, the momentum impulse absorbed by the **wall** from the collision of the single molecule is equal and opposite to that experienced by the molecule and is therefore

$$I_x = p_1 - p_2 = +2 p_x \mathbf{i} = +2 m v_x \mathbf{i}$$

The *expected value* of the impulse, I_1 , is the probability of a collision by a molecule with the wall times the impulse associated with that collision.

$$I_1 = f I_x = \frac{|v_x| \Delta t}{2L} 2 m v_x = \frac{\Delta t}{L} m v_x^2$$

I_1 is the average impulse delivered by a molecule collision in the time interval, Δt . Averaging over all molecules yields

$$\bar{I}_1 = \frac{\Delta t}{L} m \overline{v_x^2}$$

The overbar designate averages over all molecules.

Multiplication by N , the total number of molecules in the container, gives the total impulse, I , delivered to the right wall in time interval, Δt .

$$I = N\bar{I}_1 = \frac{N}{L} m\bar{v}_x^2 \Delta t$$

Since an impulse can be expressed as the average force over a time interval, Δt ,

$$I = \bar{F} \Delta t$$

the average force on the wall of the container is

$$\bar{F} = \frac{N}{L} m\bar{v}_x^2$$

The pressure on the wall is $P = \bar{F}/A$ (force per unit area),

$$P = \frac{N}{LA} m\bar{v}_x^2 = \frac{N}{V} m\bar{v}_x^2$$

where V is the volume of the container.

The next step is to recognize that the mean-square velocity of the molecules, \bar{v}^2 , can be written as

$$\bar{v}^2 = \bar{v}_x^2 + \bar{v}_y^2 + \bar{v}_z^2$$

Assuming no bulk motion of the gas, all directions are equally likely. Therefore, $\bar{v}_x^2 = \bar{v}_y^2 = \bar{v}_z^2$ and $\bar{v}^2 = 3\bar{v}_x^2$ so

$$\bar{v}_x^2 = \frac{\bar{v}^2}{3}$$

So

$$P = \frac{N}{3V} m\bar{v}^2$$

$$PV = \frac{1}{3} Nm\bar{v}^2$$

but

$$PV = nR^*T$$

$$PV = nk_B N_A T$$

$$\frac{1}{3} Nm\bar{v}^2 = nk_B N_A T$$

$$\frac{1}{3} m\bar{v}^2 = k_B T$$

$$\frac{1}{2} m\bar{v}^2 = \bar{K} = \frac{3}{2} k_B T$$

So temperature is related to translational energy (and only the translational energy).