

**Enthalpy and the Moist Adiabats**

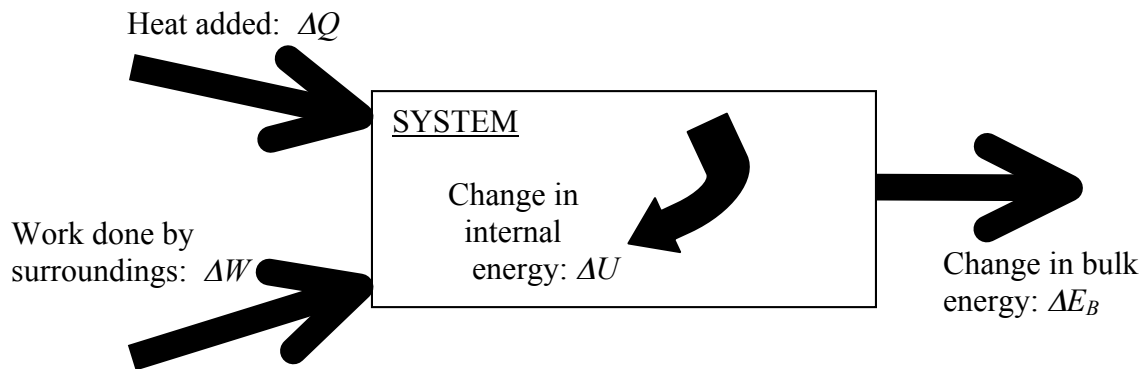
We have described the dry adiabat where an air parcel is lifted rapidly causing the air parcel to expand as the environmental pressure decreases and the air parcel does work on the environment but no heat is exchanged with the environment. As a result some of the air parcel’s internal energy is converted to doing work on the environment. The result is the dry adiabatic temperature gradient

$$\frac{dT}{dz} = -\frac{g}{C_p} \tag{1}$$

Now we discuss the effect of transferring the latent of water vapor to increase the temperature of the air. This includes discussion about various measures of energy or heat, and highlight what the concept of enthalpy is.

Recall that when heat enters or is added to a parcel, this heat can go to the internal energy of the parcel, as well as to work the parcel does on the environment. Remember that energy added to the system goes into some combination of the bulk energy of the system,  $E_B$ , (which is large scale mechanical energy: kinetic plus potential energy) plus the internal energy of the system,  $U$ . We write this as

$$\Delta(U + E_B) = \Delta Q + \Delta W \tag{2}$$



where  $\Delta W$  is defined as positive when the environment does work on the parcel (**not** when the parcel does work on the environment). Remember also that  $\Delta W = -p \Delta V$ .

Now we ignore changes in bulk energy and express the properties of the parcel in “specific” units (i.e. per unit mass), yielding

$$\begin{aligned} dq &= du - dw \\ &= du + p d(1/\rho) \end{aligned}$$

Note that  $q$  here is the heat per unit mass, not the specific humidity. Remember that when heat is added to the system (=air parcel) when volume (i.e. density) is kept constant, the only place for the energy to go is into the kinetic energy of the molecules:

$$dq|_v = du = c_v dT.$$

If we consider heating up the gas from some absolute zero temperature, we can define the total specific internal energy as

$$u = c_v T$$

where we have assumed  $c_v$  is independent of  $T$  and that no phase changes have occurred. We'll address the effects of phase changes shortly.

Now, when heat is added to the air parcel when it is held at constant pressure (which is the typical situation in the atmosphere) we can rewrite  $d(1/\rho) = d(R^*T/p m_A)$ , and end up with

$$\begin{aligned} dq|_p &= c_v dT + (R^*/m_A) dT \\ &= c_p dT \end{aligned}$$

Here, there is a work term, because as the gas heats up at constant pressure, it expands and does work on the environment. To keep track of this process, we define the enthalpy,  $h$

$$\begin{aligned} h &= u + p/\rho \\ &= c_p T \end{aligned}$$

Note that Enthalpy is not a measure of the energy contained in a parcel. It is a hybrid that contains both the internal energy and some of the energy that the air parcel transferred to the environment that the environment stores to make room for the parcel. In a hydrostatic atmosphere, this second form of energy is stored as the extra gravitational potential of the overriding air that has been lifted to make room for the expanded parcel.

Now consider the most general form of  $dq$  again, where we take advantage of our expression for enthalpy

$$\begin{aligned} dq &= du - dw \\ &= du + pd(1/\rho) \\ &= du + p d(R^*T/pM_A) \\ &= dh - \rho^{-1} dp \end{aligned}$$

In hydrostatic equilibrium, we have

$$dq = dh + gdz = dh + d\Phi$$

We call the term

$$h + \Phi$$

the “dry static energy”, and keep track of it with the potential temperature

$$\begin{aligned} \theta &= T + \Phi/c_p \\ &= T + \Gamma_d z \end{aligned}$$

What goes into the dry static energy is the gravitational potential energy of the parcel, the internal energy of the parcel, and the gravitational potential of the air overlying the parcel caused by the parcel's presence (volume). We keep track of dry static energy because it is constant

during adiabatic processes, when  $dq = 0$ . Dry static energy goes to zero as a parcel is adiabatically dropped to the ground and then cooled at constant pressure to zero.

### *Moist static energy*

This has all been review, but I'm now using the notation and terms in W+H to be consistent. Now let's consider what happens when we throw phase changes into the mix.

Water vapor releases heat when it condenses. This is called the latent heat of vaporization in W+H, although you will also see it called the latent heat of condensation. The amount of heat released is a weak function of temperature, with a value of about  $2.5 \times 10^6$  J/kg at  $0^\circ\text{C}$ , and  $2.25 \times 10^6$  J/kg at  $100^\circ\text{C}$ .

The amount of heat released depends on the amount of mass of water vapor condensed

$$dQ = -L_v dm_v$$

or in units of specific heat, we have

$$dq = -L_v dq_v$$

Note that there's a minus sign because vapor content goes down when condensation occurs. Relating this to the heating of our parcel, we have

$$dq = dh + d\Phi = -L_v dq_v$$

If there is no heat exchange besides this latent process, we can write

$$0 = dh + d\Phi + L_v dq_v$$

This is called a "moist adiabatic" process – the only heat exchange is the conversion of latent heat of vapor into sensible heat as the vapor condenses. Since no heat or mass actually crosses the boundary of our parcel, this is really considered an interconversion of internal energy. We can even consider the un-condensed vapor as a form of internal energy, not as a form of heat. In this case, we define the "moist static energy" of a parcel as,

$$h + \Phi + L_v q_v$$

To keep track of moist static energy, we define the equivalent potential temperature

$$\theta_e = \theta + (L_v/c_p)q_v$$

$\theta_e$  is always larger than  $\theta$ , as it includes the internal potential energy of the vapor. It is also conserved when cloud water condenses or evaporates. You can think of equivalent potential temperature as the temperature a parcel would have if you first condensed all the vapor out, and then dropped it to surface pressure adiabatically.

Exercise: What is the potential temperature outside right now? ( $T = 34.7$ ;  $T_d = 1.6$ ;  $p = 926$  mb)

In the general adiabatic case for a rising air parcel,  $dz > 0$ ,  $dT < 0$  and  $dq < 0$ . We need an additional constraint on the problem and this is provided by the assertion that  $dq = dq_s(T)$ , which is the same thing as saying that RH will max out at 100%. (We will learn later that this is merely an approximation, but it is a good one for coarse purposes). For RH to equal 100% as

temperature is decreasing, there must be a corresponding decrease in saturation vapor pressure, which is achieved by condensation.

The saturation specific humidity is the ratio of the saturation density to air density.

$$q_s = \rho_s / \rho_a$$

The saturation density is obtained from the saturation vapor pressure using the ideal gas law.

$$\rho_s = n_{sv} M_w$$

$$n_{sv} = e_s(T) / (R^* T)$$

Using the ideal gas law to relate air density to air pressure, we get

$$\rho_a = p M_w / (R^* T)$$

These lead to the following simplification for  $q_s$

$$q_s = (M_w / M_a) (e_s(T) / p)$$

Now our goal is to find  $dq_s$ , which is more easily manipulated through its log...

$$dq_s = q_s d \ln q_s = q_s (d \ln e_s - d \ln p)$$

[Note that we have conveniently neglected the sensitivity of molecular weight of air to vapor pressure, which will contribute a very minor term.]

The first term shows that saturation mixing ratio will increase if saturation vapor pressure increases. Note the second term that shows that saturation specific humidity will increase with a decrease in ambient pressure. This might seem counterintuitive, since we know that  $q$  doesn't change with changes in pressure. However,  $q_s$  is a different beast than  $q$ . Whereas  $q$  keeps track of the mass of vapor in a given mass of air,  $q_s$  tracks the mass of water vapor that would be in equilibrium with a flat surface of water per unit mass of air. As pressure goes up and down, saturation vapor pressure doesn't notice. It is only sensitive to temperature. However, the dry air mass is fluctuating with pressure, causing the constant saturation vapor pressure to be a relatively larger fraction of the air pressure as pressure goes down. To explore this further, consider a chamber that is partially filled with water and is held at temperature  $T$ . In equilibrium, the total pressure will be  $p_d + e_s(T)$ , where  $p_d$  is the pressure due to the dry air. Now we evacuate the chamber of half the air. At this very instant the pressure will be  $p_d/2 + e_s(T)/2$ , and  $q$  will be the same as before since there has been no condensation or evaporation (yet). But the water surface will still have a vapor pressure of  $e_s(T)$ . So over time, water will evaporate and  $q$  will increase to a new, higher equilibrium value,  $q_s(T, p_f)$ .

Now we go back to the moist parcel that is ascending adiabatically. From Clausius-Clapyron, we have

$$d \ln e_s = L_c M_w / (R^* T^2) dT$$

And hydrostatic pressure for ideal gases can be written as:

$$d \ln p = -dz/H$$

Where  $H$  is the scale height of the atmosphere  $= R^*T/(M_Ag)$ . Thus our expression for  $dq_s$  can be written in terms of  $dT$  and  $dz$ , which are the same independent variables used in our dry adiabatic equation.

$$dq_s = q_s(L_c M_w / (R^* T^2)) dT + dz/H$$

Subbing all this into the energy conservation equation yields,

$$0 = c_p dT + g dz + L_c q_s [L_c M_w / (R^* T^2)] dT + dz/H$$

Now we can solve for  $dT/dz = -\Gamma_M$ , which is called the moist adiabatic lapse rate...

$$dT/dz = - (g + L_c q_s / H) / [c_p + L_c q_s (L_c M_w / (R^* T^2))]$$

At this point, it is useful to note the  $g/c_p$  is the dry adiabatic lapse rate,  $\Gamma_D$ , and that  $H$  is a function of gravity. After some algebra, we get,

$$\Gamma_M = \Gamma_D \frac{1 + \frac{L_c q_s M_A}{R^* T}}{1 + \frac{L_c q_s}{c_p} \frac{L_c M_w}{R^* T^2}}$$

When all is said and done, we see that there are two terms that make the moist adiabatic lapse rate different from the dry value. The numerator contains a term that came from the pressure dependence of saturation specific humidity (as discussed above). The denominator has a term that came from the temperature dependence of saturation specific humidity through the Clausius Clapyron relation, and this is the dominant term. While both terms are important to getting the right value, the CC term dominates.

At 0°C and mean sea level pressure (MSL) the denominator is 1.68 and the numerator is 1.12, with that the moist lapse rate is (6.5 K/km), about 2/3 the value of the dry lapse rate. At 15°C, the numerator is 1.312 and the denominator increases to 2.630, for a moist lapse rate of 4.9 – half the value of the dry lapse rate.

This addition of the 2<sup>nd</sup> term in the denominator signifies that that both kinetic and latent energy sources are used to do work on the environment as it expands, thus slowing the cooling relative to the case when only kinetic energy is used. The modifier in the numerator accounts for the evaporation (and hence cooling) that happens independently of temperature as air depressurizes, due to the drop in vapor pressure at fixed specific humidity. Because it represents a cooling with height, it increases the magnitude of the lapse rate, but only slightly relative to the CC effect.