Introduction to Scattering

We now generalize radiative transfer to involve the effects of particles suspended in the atmosphere.

What is scattering?

Whenever electromagnetic radiation, which is what we call light, is traveling in one medium and hits a boundary into another medium defined as a sharp change in the index of refraction, some of the light passes across the boundary and some is reflected. If the object has finite size then some of the light may pass through it.

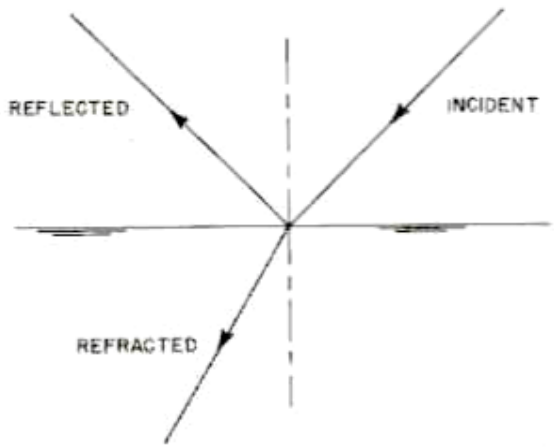


Figure 1.2 Reflection and refraction at an optically smooth interface.

The portion of the light that is reflected and transmitted through the object is the ***scattered*** radiation. The remaining portion of the light that strikes the obstacle is absorbed by the obstacle.

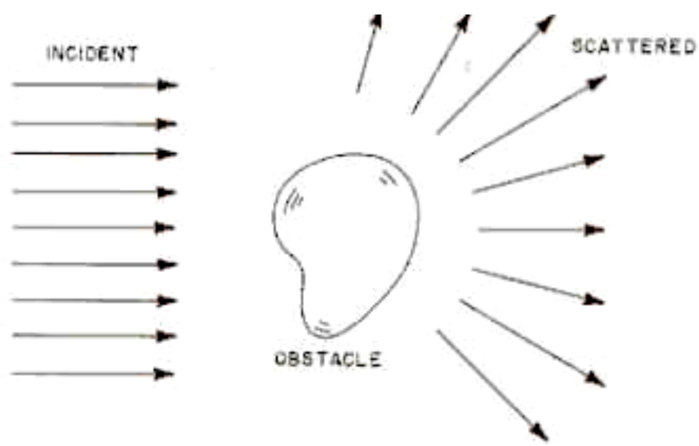


Figure 1.1 Scattering by an obstacle.

The scattering can be thought of as the superposition of the radiation from many microscopic electric dipoles on the obstacle that have been excited by the incident radiation field.

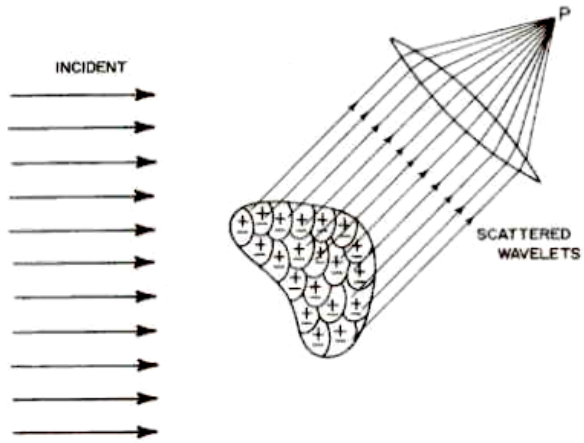


Figure 1.4 The total scattered field at *P* is the resultant of all the wavelets scattered by the regions into which the particle is subdivided.

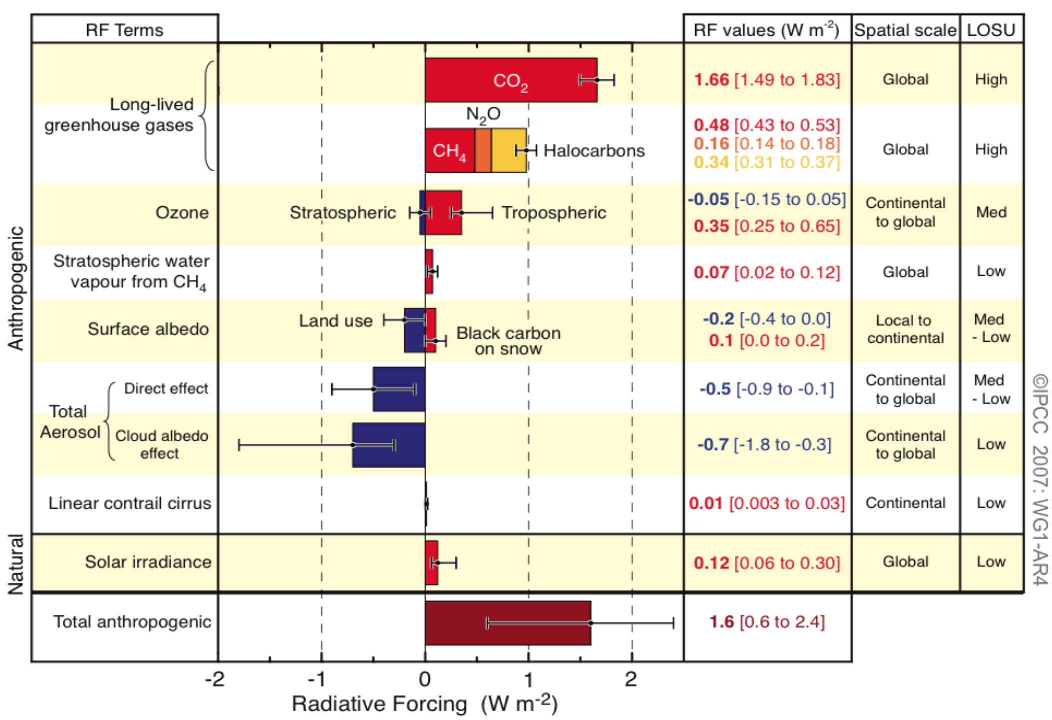
Why do we care about particle scattering?

Examples of scattering and light interaction with

- aerosols (note: ugly looking air when the relative humidity > 80%)
- clouds,
- rain
- hail particles
- rainbows

Scattering depends on the wavelength because

1. the size of the particle relative to the wavelength is important and
2. the index of refraction of the particle depends on wavelength.



Aerosols are important for climate and the uncertainties in the properties of aerosols, anthropogenic generation of aerosols, changes in aerosols *due to* changes in climate. There are questions about the magnitude and perhaps even the sign of the aerosol effect. Different types of aerosols absorb radiation while others scatter radiation depending largely on their composition and how large the imaginary part of the index of refraction is.

Direct and indirect effects

- The "direct effect" of radiative forcing on climate relates to the changes in net radiative fluxes in the atmosphere caused by the modulation of atmospheric scattering and absorption properties due to changes in the concentration and optical properties of aerosols.
- The "indirect effect" of radiative forcing on climate relates to the changes in net radiative transfer in the atmosphere caused by the changes of cloud properties due to changes in the concentration of cloud condensation nuclei, CCN. An issue of current concern is whether such a forcing might be caused by particle emissions from anthropogenic activities. These fall into two broad categories: a) changes in cloud albedo caused by the increases in the number of cloud droplets, and b) changes in cloud lifetime caused by modification of cloud properties and precipitation processes.

Active remote sensing

In Radio Detection And Ranging (radar) and Light Detection And Ranging (lidars), we use scattering to measure precipitation and clouds and aerosols. Radars rely on scattering to measure precipitation and estimate rain rate. Shorter wavelength radars around 94 GHz are used to sense cloud droplets. Still shorter wavelength lidars are used to sense ice clouds and aerosols.

Four categories of scattering theory

1. Mie (*spherical* particles of any size, published in 1908)
2. T-matrix (non-spherical particles of any size)
3. Rayleigh ($\lambda >> r$) (original derivation did not include absorption)
4. Geometric Optics ($r >> \lambda$)

The wave equation and the real and imaginary parts of the index of refraction

In a vacuum, Maxwell's equations can be combined to give the wave equation for the electric field:

$$\nabla^2 \vec{E} + \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial^2 t} = 0 \qquad (1)$$

where μ_o is the permeability of vacuum (related to the magnetic field) and ε_o is the permittivity of vacuum (related to the electric field). The left hand term is called the Laplacian and represents 2nd spatial derivative of spatial variations of the electrical field and the right hand term represents the 2nd derivative of the temporal variations. There is an analogous equation for the magnetic field.

The Laplacian in a variety of different coordinate systems (where $\Delta f \equiv \nabla^2 f$)…

Cartesian coordinates

$$\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}. \tag{2}$$

Cylindrical Coordinates

$$\Delta f = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial f}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 f}{\partial z^2}. \tag{3}$$

Spherical coordinates

$$\Delta f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial f}{\partial r}\right) + \frac{1}{r^2 \sin\varphi}\frac{\partial}{\partial \varphi}\left(\sin\varphi \frac{\partial f}{\partial \varphi}\right) + \frac{1}{r^2 \sin^2\varphi}\frac{\partial^2 f}{\partial \theta^2}. \tag{4}$$

(where θ represents the azimuthal angle and φ the polar angle).

For a homogeneous, isotropic and nonmagnetic media with no electric charge and no conductors, the wave equation becomes

$$\nabla^2 \vec{E} + \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial^2 t} = 0 \tag{5}$$

or it can be written as

$$\nabla^2 \vec{E} + \mu_0 \varepsilon_0 \mu_r \varepsilon_r \frac{\partial^2 \vec{E}}{\partial^2 t} = 0 \tag{6}$$

where μ is the permeability of the medium and ε is the permittivity of the medium and μ_r and ε_r are the relative permeability and permittivity of the medium through which the light is passing defined as

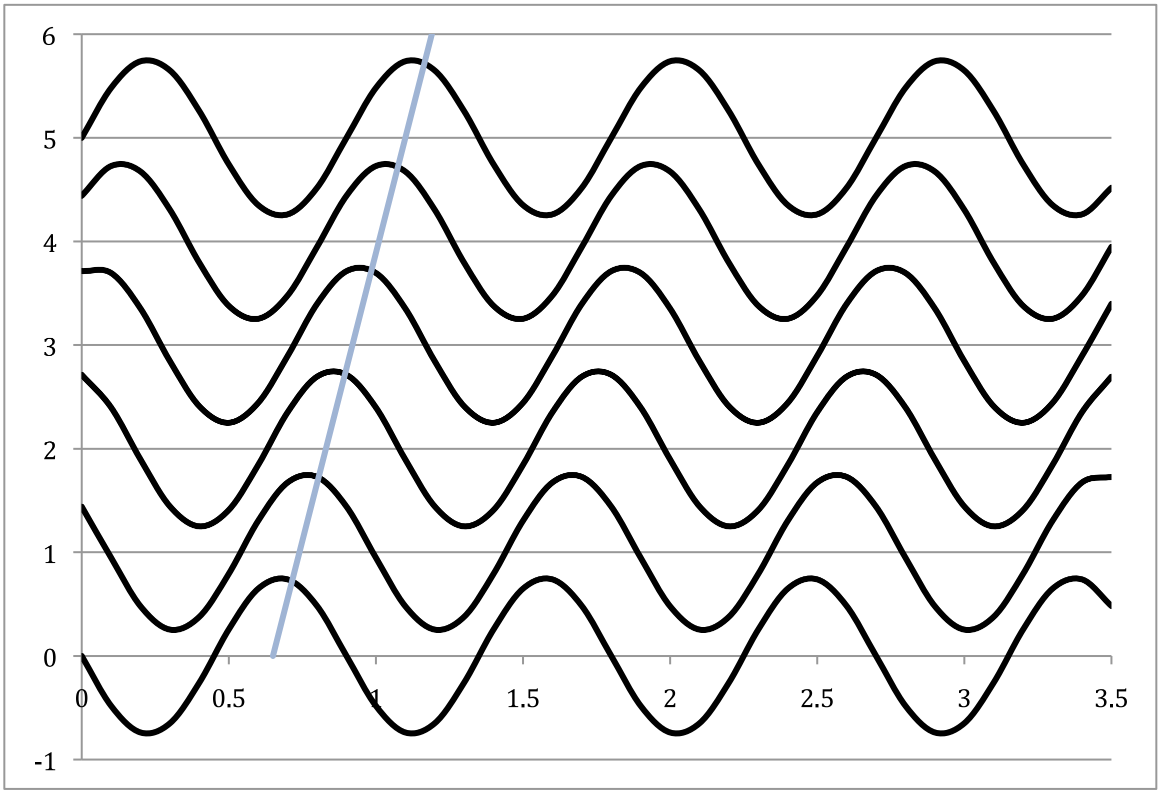
$$\mu_r = \frac{\mu}{\mu_0} \quad \text{and} \quad \varepsilon_r = \frac{\varepsilon}{\varepsilon_0} \tag{7}$$

The electric field of a plane wave solution to the wave equation is

$$E(z,t) = E_0 \exp(-i[\omega t - kz]) \tag{8}$$

where k is the wavenumber, $= 2\pi/\lambda$, where λ is the wavelength, z is the direction of propagation and ω is the angular frequency in radians per second which is $2\pi f$ where f is frequency in cycles per second. Consider when the phase of the sine is constant

$$\omega t - kz = \text{constant} \tag{9}$$



A propagating sine wave showing the motion of the phase with time

The straight line in the figure shows how a point of constant phase moves as the wave propagates. From this we can determine the speed of propagation of the phase in the medium which is

$$\frac{dz}{dt} = \frac{\omega}{k} = f\lambda = v \tag{10}$$

In a vacuum, the speed of light, *c*, is 3e8 m/s. In the atmosphere it is very slightly slower by an amount that depends on the wavelength of the light.

So, for a sinusoidal solution, we can write the wave equation as

$$\nabla^2 \vec{E} + \frac{\omega^2}{c_r^2}\vec{E} = 0 \tag{11}$$

where the speed of light in the medium, c_r, is

$$c_r = \frac{1}{\sqrt{\mu_0 \varepsilon_0 \mu_r \varepsilon_r}} = \frac{c}{\sqrt{\mu_r \varepsilon_r}} \tag{12}$$

and *c* is the speed of light in a vacuum. The index of refraction is

$$n = \frac{c}{c_r} = \sqrt{\mu_r \varepsilon_r} \tag{13}$$

In most media, $\mu_r = 1$ (for nonmagnetic media). ε_r ranges from 1 to 80 and depends on frequency and temperature. The high end, near 80, is for liquid water at microwave frequencies.

The index of refraction that you have probably seen before is $m_r = c/v$. This is actually the real part of the index of refraction which is in general complex. The fact that the light generally travels slower in the medium than in a vacuum as defined by m_r but the frequency of the light remains the same as it would in a vacuum, the wavelength of the light in the medium therefore differs from the wavelength of that light in a vacuum. Defining k' as the wavenumber ***in the medium*** yields

$$\omega = k'v = kc \qquad \text{so} \qquad k' = c/v\ k = m\ k \tag{14}$$

Plugging this into the plane wave solution in the medium yields

$$E(z,t) = E_0 \exp(-i[\omega t - k'z]) \tag{15}$$

m is complex. Examining the spatial part, $\exp(ik'z)$, yields

$$\exp(ik'z) = \exp(imkz) = \exp(i[m_r + im_i]kz) = \exp(im_r kz)\exp(-m_i kz) \tag{16}$$

We see two differences relative to propagation in a vacuum. First, the wavenumber has been modified from k to $m_r k$ and, for typical non-vacuum mediums where $m > 1$, $k' = m_r k$ is therefore a bit larger than in a vacuum because the wavelength has become ***shorter*** relative to a vacuum because the light is traveling slower than it would in a vacuum.

The second change is there is now an attenuation term associated with the imaginary part of the index of refraction. Therefore light traveling through a medium with an imaginary component of the index of refraction is attenuated.

We will now apply this complex index of refraction to understand scattering.

Extinction and cross-sections

We return to Beer's Law where

$$dI = -I\ \alpha\ dz = -I\ d\tau \tag{17}$$

where I is radiance, α is the extinction coefficient, z is path length and τ is known as optical depth. α has units of inverse length representing how much attenuation there is per unit length. There is a useful concept called the cross-section which is the effective cross-sectional area of a particle. From an electromagnetic standpoint, we can think of the cross-section removing or more generally altering a portion of the propagating light.

If there are n_p particles per unit volume and each particle has a cross-section, σ_p, then the cross-sectional area per unit volume is $n_p\ \sigma_p$ which has units of $m^2/m^3 = m^{-1}$ and is equal to α. So

$$\alpha\ dz = n_p \sigma_p\ dz = d\tau \tag{18}$$

Extinction is the total attenuation of the incident radiation due to scattering and absorption.