One of the standard units of the extinction coefficient is inverse length. This can be thought of as crosssectional area (or crosssection) per unit volume.

A word on Crosssections

The concept of a molecule or particle having a crosssection is a useful concept for developing physical insight into extinction and how it works. Crosssection is the effective area of the particle in this case that interacts with light, either by absorbing or scattering the light. It is important to note that the electromagnetic crosssection may be similar to or quite different from the *geometric* area crosssection of the molecule. The two crosssections are quite different particularly when the wavelength of the light is much larger than the size of the particle.

The crossections of individual molecules can then be added to get the collective crossection of many molecules per unit volume.

Extinction coefficient in a cloud

Clouds offer a very useful example for which we can apply this crosssection concept. I will focus here on liquid water clouds. For spherical droplets, we can use Mie scattering theory developed by Mie in 1908 which provides an exact solution for the scattering and absorption of light by spherical particles.

Extinction = scattering + absorption

There are two parts to extinction: scattering and absorption. Scattering removes energy from the incident beam by scattering it into another direction. Absorption removes energy from the incident beam when the particle absorbs some of the energy from the beam, heating up the particle in the process. So the extinction coefficient, k_{ext}, is the sum of the absorption coefficient, k_{abs}, and the scattering coefficient, k_{scat}.

$$k_{ext} = k_{abs} + k_{scat} \tag{1}$$

These coefficients depend on a number of variables: droplet composition, size and shape and the wavelength of the light. Scattering depends predominantly on the speed of the light in the droplet relative to that in air (real part of the index of refraction). Absorption depends on the imaginary part of the index of refraction which defines how much the signal is attenuated per distance as the light propogates. If the complex index of refraction does not differ much from that of air, the droplet will have very little effect on the light.

In terms of water, water absorbs very little at visible wavelengths. So k_{abs}(visible) is very small and the dominant effect of water droplets on visible light is via scattering. At IR wavelengths, water absorbs quite well and both k_{abs} and k_{scat} are generally significant at IR.

IR absorption and emission by clouds

In this present context, we will focus on the absorption coefficient at IR wavelengths.

Clouds are often defined in terms of their liquid water content (*LWC*). A range of cloud *LWC* and other properties is given at http://www-das.uwyo.edu/~geerts/cwx/notes/chap08/moist_cloud.html. Note that *LWC* is normally written in units of grams per cubic meter (rather than mks units of kilograms per cubic meter). The *LWC* is related to the droplet size and number density as follows

$$LWC = \sum_{drops} V_{drop}\rho_{drop} = n_{drop}\int_{r_{\min}}^{r_{\max}} V_{drop}(r_{drop})\,\rho_{drop}(r_{drop})\,f_{drop}(r_{drop})\,dr_{drop} \tag{2}$$

where f_{drop} is the probability distribution of particles and N_{drop} is the total number of particles per m^3. If all of the drops are the same size, then we can write this more simply as

$$LWC = V_{drop}\,\rho_{drop}\,n_{drop} \tag{3}$$

where V_{drop} is the droplet volume, ρ_{drop} is the density of the condensed water and n_{drop} is the number density of the droplets. Under these conditions, the droplet number density

$$n_{drop} = \frac{LWC}{V_{drop}\rho_{drop}} \tag{4}$$

With the further simplification of spherical droplets,

$$V_{drop} = 4/3\,\pi\,r^3 \tag{5}$$

For liquid water droplets, ρ_{drop} = 1 g/cc = 10^3 kg/m^3 = 10^6 g/m^3.

The extinction coefficient is the electromagnetic crosssection per unit volume which is the electromagnetic crosssection per particle, $\sigma_{\mathrm{ext}}(\lambda)$, times the number of droplets per unit volume

$$k_{ext} = \sigma_{\mathrm{ext}}(r,\lambda)\,n_{\mathrm{drop}} \tag{6}$$

For spherical particles, the electromagnetic crosssection, $\sigma_{\mathrm{ext}}(r,\lambda)$, is a function of the droplet radius and the wavelength of the electromagnetic radiation. The relation between the electromagnetic crosssection, $\sigma_{\mathrm{ext}}(r,\lambda)$, and the geometric crosssection, $\sigma_{\mathrm{geom}}(r)$, is written as

$$\sigma_{\mathrm{ext}}(r,\lambda) = Q(r,\lambda)\,\sigma_{\mathrm{geom}} \tag{7}$$

where $Q(r,\lambda)$ is known as the efficiency. For spherical particles, $Q(r,\lambda)$ can be calculated via Mie scattering. For spherical particles,

$$\sigma_{\mathrm{geom}} = \pi r^2 \tag{8}$$

Combining these equations yields

$$k_{ext} = \sigma_{ext}n_{drop} = Q_{ext}(r,\lambda)\sigma_{geom}\frac{LWC}{V_{drop}\rho_{drop}} \tag{9}$$

For spherical particles, (9) becomes

$$k_{ext} = Q_{ext}(r,\lambda)\pi r^2\frac{LWC}{\frac{4}{3}\pi r^3\rho_{drop}} = Q_{ext}(r,\lambda)\frac{3\,LWC}{4r\rho_{drop}} \tag{10}$$

So for spherical particles, if Q_{ext} is constant, then k_{ext} is inversely proportional to r. Thus, as r decreases, the extinction coefficient increases because there is a larger crossectional area per LWC when the droplets are small. Q_{ext} can in fact be approximately constant near 1 to 2 when $2\pi\, r$ is comparable to or greater than the wavelength of light, λ. Mie scattering is in fact expressed in terms of a dimensionless variable, x, defined as

$$x = \frac{2\pi r}{\lambda} \qquad (11)$$

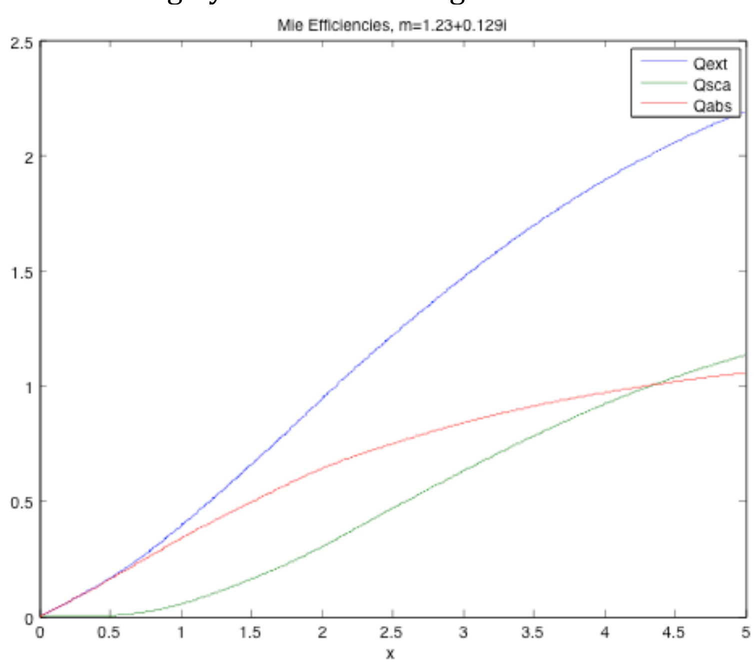
So at IR wavelengths, if x ~1 or larger, Q tends to be in the range 1 to 2.

Example: Liquid Water Cloud

From the website above, we see that a representative cloud LWC is 0.3 g/m^3. We can estimate the number of droplets per unit volume if we know the typical cloud droplet size, which the website indicates is a radius of ~5 microns. The droplet volume is therefore 4/3 $\pi\, r^3$ = 5E-16 m^3. Since the mass density of liquid water is 1e6 g/m^3, the mass per droplet is volume * liquid density = 5e-16 m^3 * 1e6 g/m^3 = 5e-10 g. Therefore the number of droplets per cubic meter is LWC / mass per droplet = 0.3 g/ m^3 / 5e-10 g ~ 6e8 m^{-3}.

As we have shown previously in the Planck function and blackbody discussion, thermal IR wavelengths in Earth's atmosphere are centered around 10 microns. The next question is how large is the electromagnetic crossection of cloud water droplets at these wavelengths.

For spherical droplets, we can use Mie scattering theory. The ratio of the electromagnetic crosssection to the geometric crosssection is Q. The figure below shows Q for 5 micron radius droplets for wavelengths around 6 microns. Since Q_{abs} ~ 1, the absorption crosssection is roughly the same as the geometric crosssection.



5 micron cloud droplet: Mie efficiencies at IR wavelengths. $x=2\pi r/\lambda = 3$: $\lambda=10$ microns, The index of refraction varies somewhat over this frequency range so this efficiency spectrum should be viewed as representative.

The geometric cross-sectional area of a 5 micron radius droplet is πr^2 = 0.8E-10 m^2. Since $Q_{abs} \sim 1$ at 10 micron wavelength, the electromagnetic crossection of these droplets near 10 micron wavelengths is roughly equal to the geometric crosssection.

The absorption extinction coefficient, k_{abs}, is the crosssection per unit volume which is the crosssection per droplet times the number of droplets per cubic meter. For the *LWC* of 0.3 g/m3 and droplets of 5 microns in radius, the absorption coefficient is

$$k_{\text{abs}} = \sigma_{\text{abs}}\, n = 0.8\text{E-10 m}^2\ \ 6\text{e}8\ \text{m}^{-3} = 5\text{e-2 m}^{-1} \tag{12}$$

So for every meter of propagation through the cloud, the IR signal intensity decreases by 5%.

A key question is what path length through the cloud is required to get an optical depth of unity. The answer is $k_{\text{abs}}\, z = \tau = 1$. So

$$z(\tau=1) = 1/k_{\text{abs}} = 1/\ \sigma\, \text{n}. \tag{13}$$

(13) should look familiar. Recall the concept of a mean free path, *mfp*, that we discussed in introducing the concept of diffusion. The *mfp* concept is also relevant here in radiative transfer. The mean free path of a photon is the path it takes between collisions with particles or molecules. Recall that the mean free path was given as the *mfp* σ = volume per particle = $1/n$. Therefore

$$mfp = (n\ \sigma)^{-1} = \text{x}(\tau=1) \tag{14}$$

So the *mfp* of a photon in a medium defines the path length associated with optical depth = 1.

Why do we care?

The *mfp* is approximately the minimum thickness that a cloud must have to be optically thick (in terms of *absorption* as opposed to *scattering*) at IR wavelengths.

In the case of a cloud with LWC = 0.3 g/m^3, the thickness that the cloud needs to be to be optically thick at IR wavelengths is at least $1/k_{\text{abs}}$ = 1/5e-2 m^{-1} ~ 20 m. This is quite small. So a typical liquid water cloud needs to only be 20 m thick to be optically thick at IR. For cumulonimbus clouds, with a LWC of 1 g/m^3 and 10 micron radius droplets, the cloud only needs to be about 7 m thick to be a good IR absorber and emitter.

Such optically thick clouds are both good absorbers and emitters. They also tend to be so at all thermal IR wavelengths and can therefore be treated as blackbody absorbers and emitters. This then allows one to apply the Stephan Boltzmann law to clouds: σT^4. Note that this σ is the constant 5.67e-8 W/K^4, not the crosssection!

Need spectral examples

A quick look at Cirrus Clouds

Cirrus clouds are also very effective IR absorbers and emitters. According to the web page noted above, the effective optical radius of cirrus clouds

Temp (C)	effective optical radius (microns)	number of droplets per cc	Number of droplets per m^3	Water content (g/m^3)	k_{abs} (m^{-1})	tau=1 thick (m)
-25	92	0.11	1.1e5	0.03	3e-3	330
-50	57	0.02	2e4	0.002	2e-4	5000

Schwartz and Mace (SM)

Another source of information is from Chris Schwartz and Jay Mace ASR talk. "Analysis of Cirrus Cloud Particle Size Distributions Measured During Sparticus". Aircraft measurements were used from 58 flight legs. More than 10,000 fits were done of the particles size distribution to a Poisson distribution type form:

$$n_D = N_0 (D/D_0)^{\alpha} \exp(-D/D_0)$$

D_0 is apparently in cm.
N_0 is apparently in #/cc.

Cirrus clouds in the upper troposphere (UT) are supposed to be a big issue for IR cooling of the atmosphere. A small increase in cirrus in the UT would decrease the IR to space causing Earth to warm. Let's try to check the optical thickness of cirrus clouds in the UT.

From reading SM, at -60°C, $D_0 \sim 5$ microns, $N_0 \sim 100$ to 1000/cc. The small size of these ice crystals is important because a cloud composed of relatively lots of small particles will be more effective at interacting with the IR radiation than a cloud composed of fewer big particles, as long as the particles are big enough that $x_{IR} > 1 \Rightarrow r > \lambda_{IR}/2\pi \sim 1.5$ microns. That criterion seems to be met as the particles at -60C seem to be more like 5 microns.

For snowflake type particles with a radius of 5 microns and a thickness of 1 micron, and 100 per cc, k_{abs} is 8e-3 m^{-1}. So a cloud 125 m thick would have an optical depth of 1. This seems high but may be about right. The facet of ice at these temperatures may not be snowflakes. If we assume these are ice cubes, 5 microns on a side, then k_{abs} is 2.5e-3 m^{-1} and a cloud 400 m thick would have an optical depth of 1. This is not too different (a factor of 3) suggesting that we are in the right ball park of solution.

Such a cloud would dramatically reduce the outgoing long wave (IR) radiation (OLR) if it were over an otherwise clear region. A surface at 288 K would radiate 390 W/m^2 whereas a cloud at 213 K (= -60C) would only radiate 117 W/m^2 to space. The reduction in OLR over the area of the cloud would be 273 W/m^2. To put this into perspective, doubling CO_2 is supposed to reduce the OLR by 4 W/m^2.

Remote Sensing via IR

This sensitivity of OLR to cloud top temperature is what allows us to determine the temperature and altitude of cloud tops using IR satellites observations. It also means that IR cannot penetrate much below cloud top.