

Units, Units, Units and Coordinate systems...

See also Wallace&Hobbs, p. 467-468

In general, we will use the *mks* set of units which is shorthand for meters (distance), kilograms (mass) and seconds (as opposed to *cgs* units: centimeters, grams and seconds).

Length: meter (m), also kilometers (km)

Mass: kilograms (kg), also grams (g)

Time: seconds

Temperature: Kelvin, Celsius and Fahrenheit

Check your units: Always check the units of any equation you write. If the units don't make sense, then the equation is incorrect.

Calculations: To avoid unit conversion problems, I strongly suggest that when you do calculations that you do them in *mks* units and then convert the answer to whatever other unit you wish.

Variables related to distance and time...

Position and coordinate systems

Position is the 3D location of a point and is usually written as \vec{x} or \vec{r} where the arrow indicates a vector. Sometimes bold is used to indicate a vector, \mathbf{x} .

- The zonal or east-west direction is indicated by an x with east being positive.
 - The zonal position may also be indicated by longitude in degrees. At the equator, 1° of longitude is approximately 111 km. At higher latitudes, 1° of longitude is approximately $111 \text{ km} * \cos(\text{latitude})$.
- The meridional or north-south direction is indicated by a y with north being positive
 - The meridional position may also be indicated by latitude in degrees. 1° of latitude is approximately 111 km.
- The vertical direction is indicated by a z with up being positive.
 - Pressure may also be used as a vertical coordinate because it simplifies some of the equations of motion. Pressure is indicated with a p . Note that pressure *decreases* monotonically as height *increases*.
 - When pressure is the vertical coordinate, height can still be used but height is a dependent variable indicating the height of a pressure surface like the 500 mb surface which varies with weather conditions.
 - Potential Temperature (θ) may be used as a vertical coordinate when conditions are *stable*, so that potential temperature like in the stratosphere.

Coordinate systems:

Regarding motion, there are two coordinate systems, Eulerian (fixed grid) and Lagrangian (moves with the particle)

Velocity: which is the rate of change in position with time (m/s) written as \vec{v} or \vec{u} or again with bold.

- Wind in the zonal direction is indicated by u with eastward motion being positive. This is also referred to as westerly motion meaning from the west.
- The meridional component of the wind is indicated by v with northward motion being positive. This is also referred to as southerly flow meaning from the south
- The vertical component of the wind is indicated with w with upward motion being positive.
- If the vertical component is pressure then the vertical component of the wind is indicated with ω .

Acceleration: $d\vec{v}/dt$ in m/s^2 .

Mass is a property of matter that refers to its inertial mass and gravitational mass which based on measurements are equivalent.

- Inertial mass is measured as $m = \vec{F}/\vec{a}$, that is the ratio of the force applied to the acceleration of the object.
- Gravitational mass refers to the gravitational acceleration the object creates in attracting other bodies toward it. $\vec{F} = -GMm/r \hat{r}$, where F is the gravitational force between two point objects of mass, M and m , r is the distance between the masses and G is the universal gravitational constant.
- Be careful to distinguish between mass and weight. Weight is the force measured when gravity pulls on a mass. It is not the mass itself.
- Note also that the gravitational acceleration on Earth's surface varies so an object's weight varies with location even when its mass does not.

The mks unit is kilograms = kg. The other standard is grams. Be careful because a mole gives you the mass in grams (NOT kg) equal to the mass of one molecule in amu.

Momentum is the mass of an object times the object's velocity, which is a vector quantity, $m\vec{v}$

Angular momentum

$m\vec{v} \otimes \vec{r}$ (right hand rule)

Force is given by $F = ma$ which says force equals mass times acceleration.

So a unit of force in mks is $kg\ m/s^2$ which is also known as a Newton.

Pressure is force per unit area.

- Its natural units in mks are Newtons per square meter = $kg\ m/s^2 / m^2 = kg/s^2/m$ which is known as a Pascal (Pa).
- 1 millibar (mb) = 1 hectopascal (hPa) = 100 Pa
- 1 bar = 1000 mb
- 1 atmosphere ~ seal level pressure = 1013.25 hPa

The following pressure measurements are not absolute because they depend on gravity and, in the case of mercury, temperature.

- Inches of mercury. It is defined as the pressure exerted by a column of mercury of 1 inch in height at (0°C) at the standard acceleration of gravity.

- 1 inHg = 3,386.389 Pa at 0 °C.
- 1 Torr = 760 millimeters of mercury = 1 atm
- Pounds per square inch (psi). 1 atm = 14.696 psi.

Energy.

I find the easiest way to remember the units of energy is via kinetic energy which is $\frac{1}{2} m v^2$. So the standard units of energy in *mks* units must be $\text{kg m}^2/\text{s}^2$ which is known as a Joule (J).

Power

Power is the rate of change of energy or dE/dt . Its *mks* units are Joules per second which is known as a watt (W).

Density

Mass density is mass per unit volume, ρ or ρ_m in kg/m^3 .

Number density is the number density, typically in number of objects per unit volume or in the case of molecules, moles per unit volume.

Energy density is energy per unit volume, ρ_E in J/m^3 which is also Pa. Note that pressure and energy density have the same units and are related but are not the same.

Flux

Flux is defined as a density times a velocity and is a vector quantity.

For example, energy flux is $\vec{F}_E = \rho_E \vec{v}$ which has units of $\text{J}/\text{m}^2/\text{s} = \text{W}/\text{m}^2$ which is power per unit area.

Flux divergence

The flux divergence is defined as $\nabla \cdot F = \frac{\partial F}{\partial x} + \frac{\partial F}{\partial y} + \frac{\partial F}{\partial z}$ which indicates how much the flux is

spreading out spatially. A negative flux divergence means convergence, that is the flux, F , is converging or accumulating into a certain region or volume.

- For instance, when it is raining, atmospheric water vapor must be converging into the region where it is raining in order to supply the water that is condensing out and falling to the ground. So the water vapor flux divergence would be negative in this region while it is raining.

Temperature

- Absolute temperature in Kelvin (K) is directly proportional to internal energy
- Celsius (C) = T(K) - 273.15
- Fahrenheit (F) = T(C) * 9/5 + 32

Frequency

Temporal frequency is the rate of cycles of oscillation of something per second.

- The natural unit is 1 cycle per second = 1 Hertz (Hz).
- mHz (millihertz): 0.001 Hz, kHz (kilohertz): 1000 Hz, MHz (Megahertz): $1e6$ Hz
- GHz (gigahertz): $1e9$ Hz, THz (terahertz): $1e12$ Hz

Another initially odd-looking unit: For electromagnetic radiation, the speed of light and frequency, f , and wavelength, λ , are related $f = c/\lambda$ where the speed of light in a vacuum (c) is 3×10^8 m/s.

- These frequency numbers get very large when written in Hz. So another unit was created, the wavenumber in cm^{-1} which is $1/\lambda = f/c$.
- A 10 micron (μ) wavelength (0.001 cm) has a wavenumber of $1000 \text{ cm}^{-1} = 30 \text{ THz}$
- A 0.5 μ wavelength (0.00005 cm) has a wavenumber of $20,000 \text{ cm}^{-1} = 600 \text{ THz}$

Atmospheric composition:

Bulk atmosphere

- Number density in moles/m^3 or analogous units
- Mass density in kg/m^3 or analogous units. Note the density of air at the surface is about 1 kg/m^3 .

Trace species

- Often done in volume mixing ratio: ppmv = parts per million volume or pptv = parts per trillion
 - Other than water vapor, this is done in terms of number of molecules of a particular molecule per unit volume divided by the total number of molecules per unit volume
- Can be done in mass mixing ratio. For water vapor this is called specific humidity and written in units of g/kg meaning grams of water vapor per kilogram of air.
-