# Contents

1	Evaj	poration	1
	1a	Evaporation from Wet Surfaces	1
	1b	Evaporation from Wet Surfaces in the absence of Advection	4
	1c	Bowen Ratio Method	4
	1d	Potential evaporation $(E_p)$	5
	1e	Operational methods for landsurfaces	5
	1f	Evaporation from Dry Vegetation	6

# 1. Evaporation

Evaporation is the phenomenon by which a substance is converted from the liquid state into apour. Evaporation requries energy to supply the latent heat of vapor-ization. From the perspective of the atmospheric scientist, much of the problem relates to determining the surface humidity (air's humidity at the humidity roughness height  $z_q$ . For a wet surface  $q_0$  is the saturated value  $q_0^*$ .

## **1a.** Evaporation from Wet Surfaces

Evaporation from a saturated surface where  $q_0 = q * (T_0)$  is closely related to the concept of potential evaporation  $(E_p)$  which is the maximum possible evaporation from a given surface. Using the above equation for humidity measured at two different levels, we can define the latent heat flux of evaporation as:

$$L_v \tilde{R} = \tilde{Q_E} = \frac{\rho L_v (q_2 - q_1)}{r_{aV}} \tag{1}$$

we can express this equation in terms of vapor pressure since  $q \approx 0.622 e/p$ 

$$\tilde{Q_E} = \frac{\rho L_v 0.622(e_2 - e_1)}{p \, r_{aV}} \tag{2}$$

$$= \frac{\rho c_p (e_2 - e_1)}{\gamma r_{aV}} \tag{3}$$

where  $\gamma = \frac{c_p p}{0.622 L_v}$ 

If your surface is saturated, we can take measurements at only one height because the vapor pressure at the surface is the saturation vapor pressure.

$$\tilde{Q_E} = \frac{\rho c_p(e^*(T_0) - e(T))}{\gamma r_{aV}}$$
(4)

Remember that we have also defined the sensible heat flux as:

$$\tilde{Q_H} = \frac{\rho c_p(\theta_0 - \theta)}{r_{aH}}$$
(5)

And since the total radiation is:  $R_N - G_0 = H_0 + \lambda E_0$ :

$$\tilde{Q_E} = (R_N - G_0) - \frac{\rho c_p(\theta_0 - \theta)}{r_{aH}}$$
(6)

We will now express this relationship in terms of absolute temperature, by assuming that the correction is negligible, that is:  $T(p/p_r)^{-R_d/c_p} - T \approx 0$  where z is the elevation above the reference level, then:

$$\tilde{Q_E} = (R_N - G_0) - \frac{\rho c_p (T_0 - T)}{r_{aH}}$$
(7)

We define:

$$\Delta = \frac{e^*(T_0) - e^*(T)}{T_0 - T}$$
(8)

Then,

$$\tilde{Q_E} = (R_N - G_0) - \frac{\rho c_p(e^*(T_0) - e^*(T))}{\Delta r_{aH}}$$
(9)

Now, let's manipulate this equation:

$$\tilde{Q_E} = (R_N - G_0) - \frac{\rho c_p [(e^*(T_0) - e(T)) + (e(T) - e^*(T))]}{\Delta r_{aH}}$$
(10)

$$\Delta \tilde{Q_E} + \frac{\rho c_p(e^*(T_0) - e(T))}{r_{aH}} = \Delta (R_N - G_0) + \frac{\rho c_p(e^*(T) - e(T))}{r_{aH}}$$
(11)

we can express the second term on the left hand side as a function of  $\tilde{Q_E}$ :

$$\Delta \tilde{Q_E} + \tilde{Q_E} \frac{\gamma r_{aV}}{r_{aH}} = \Delta (R_{N0} - G_0) + \frac{\rho c_p(e^*(T) - e(T))}{r_{aH}}$$
(12)

we finally obtain the expression for latent heat of vaporization:

$$\tilde{Q_E} = \frac{\Delta(R_N - G_0) + \frac{\rho c_p}{r_{aH}}(e^*(T) - e(T))}{\Delta + \frac{\gamma r_{aV}}{r_{aH}}}$$
(13)

This equation has a two-term structure suggesting that the evaporation from a saturated surface has both energy and aerodynamic contributions. This equation has proven successful at modeling evaporation when applied to water surfaces, saturated soil and vegetation with wet foliage. It can be applied at hourly and longer-term.

Many times, this equation, called the "Penman Equation" is presented as:

$$\tilde{Q_E} = L_v \tilde{R} = \frac{\Delta}{\Delta + \gamma} (R_N - G_0) + \frac{\gamma}{\Delta + \gamma} E_a$$
(14)

where

$$E_a = \frac{0.622\lambda\rho}{Pr_{aH}}(e^*(T) - e(T)) = \frac{\rho}{r_{aH}}(q^*(T) - q(T))$$
(15)

In this way the two-term structure is clearly evident. In the original formulation of Penman (1948) he proposed an empirical equation for  $E_a$ , however, we can express  $E_a$  using similarity theory and the expression for  $r_{aH}$ . Solving this equation also requires an Iterative method. We follow almost exactly the same procedure as before (first assume neutral conditions), calculate the latent heat flux using the Penman equation, the sensible heat can be calculated directly from the net radiation as  $\tilde{Q}_H = R_N - G_0 - \tilde{Q}_E$ . Then, as before, we calculate the Monin-Obukhov and begin iterations.

## **1b.** Evaporation from Wet Surfaces in the absence of Advection

If the air above a surface is completely saturated, the second term in the Penman equation drops. And the evaporation can be expressed as:

$$\tilde{Q}_E = \frac{\Delta}{\Delta + \gamma} (R_N - G_0) \tag{16}$$

Investigations have shown, however, that these conditions are rarely encountered, if ever. However, based on this idea Priestly and Taylor (1972) used this relationship to find an empirical relation for evaporation over wet surfaces in conditions of minimal advection.

$$\tilde{Q}_E = \alpha_e \frac{\Delta}{\Delta + \gamma} (R_N - G_0) \tag{17}$$

Where  $\alpha_e$  is a constant that ranges from about 1.2-1.3 over advection-free water surfaces and moist land surfaces with short vegetation. This relationship has been found to work remarkably well, and furthermore has been modified to include different vegetation stresses to use with remote sensing (Fisher et al. 2008)

# 1c. Bowen Ratio Method

Bowen Ratio is the ratio between sensible heat and latent heat.

$$B_0 = \frac{\tilde{Q}_H}{\tilde{Q}_E} = \frac{\rho c_p(\overline{w'\theta'})_s}{\rho L_v(\overline{w'q'})_s}$$
(18)

We can express these in terms of the bulk transfer coefficients:

$$B_0 = \frac{c_p C_H \overline{U}(\theta_0 - \overline{\theta})}{L_v C_E \overline{U}(q_0 - \overline{q})} = \frac{c_p (\theta_0 - \overline{\theta})}{L_v (q_0 - \overline{q})}$$
(19)

If we know the sensible heat flux, it is quite easy then to determine the latent heat flux. As an aside, if we know the flux of any other scalar (like CO2) which has a similar transfer coefficient we can also use it in the expression.

We can also use the energy equation in combination with the Bowen Ratio to solve for latent heat if we know the net radiation:

$$\tilde{Q}_E = \frac{R_N - G_0}{1 + B_0} \tag{20}$$

So if we know the mean potential temperature and humidity at a certain height and at the surface, and we know the net radiation, we can solve for the latent (and sensible heat). This method has the advantage that we don't need similarity functions for the atmospheric turbulence and the formulation is independent of atmospheric stability.

## **1d.** Potential evaporation $(E_p)$

Generally, calculations of evaporation from wet surfaces are taken to be measures of potential evapotranspiration. We need to clarify some concepts. *Potential evapotranspiration is the maximal rate of evaporation from a large area covered completely and uniformly by an actively growing vegetation with adequate moisture at all times*. It is important to note, however, that that:

- Transpiration, even at the potential rate, involves stomatal impedance to the diffusion of water vapor so it is better to use the term potential evaporation.
- Wet or moist surface is not the same as one that has an adequate moisture supply for the roots of an actively growing vegetation.
- PET is often measured by meteorological data observed under nonpotential conditions, which would not be the same as air under potential conditions. So sometimes we call this the apparent  $E_p$  or  $E_{pa}$ .

NOTE: over short non-wet vegetation with adequate moisture, the evapotranspiration is often similar to the evaporation from open water under the same conditions, this is likely due to the fact that stomatal impedance is compensated by the larger roughness values resulting in larger transfer coefficients.

### **1e.** Operational methods for landsurfaces

To predict evaporation operationally, generally we use the potential evaporation in conjunction with a procedure to derive the actual evaporation from it. Probably the oldest method is the bucket method.

$$E = \beta_e E_p \tag{21}$$

where  $E_p$  is the potential evaporation rate and  $\beta_e$  is a reduction factor reflecting the moisture availability. In practice we use  $E_pa$ . The reduction factor is a function of soil water content. A common assumption is:

 $\begin{array}{l} \beta_e = 1 \text{ for } w > w_0 \\ \beta_e = (w-w_c)/(w_0-w_c) \text{ for } w \leq w_0 \end{array}$ 

where  $w_0$  is a critical soil water content above which  $E = E_p$  and  $w_c$  is a lower cut-off value below which E = 0. One difficulty in applying this formulation is that as the surface dries out  $\beta_e$  approaches zero while  $E_p = E_{pa}$  tends to become large so they move in the opposite direction and may lead to an unstable product of a large and a small quantity.

#### **1f.** Evaporation from Dry Vegetation

Evaporation is reduced below the potential rate when the surface humidity is less than the saturation value. In the leaf surface, there exists a surface resistance to water vapor transfer through foliage stomata whether the vegetation is under negligible or sever water stress. We need to introduce a **surface resistance** denoted by  $r_s$  that applies to unsaturated surfaces and most usually to vegetation (when  $q_0 < q^*(T_0)$ ). In the case of vegetation this represents an effective stomatal resistance to the transfer of water vapor from the internal water sites in the plant to the leaf exterior. Despite the fact that this is a gross simplification, it does provide the best available simple description of the surface control on transpiration. Because evaporation also takes place from the soil suface, the basic idea is usually extended to include this transport as well. The pores are providing a resistance to the diffusion of water vapor.

In this case, the latent heat flux is now a function of both the stomatal plus a boundary layer resistance:

$$\tilde{Q}_E = \frac{\rho c_p(e^*(T_0) - e(T))}{\gamma r_{aV} + r_s}$$
(22)

And the equation for the evapotranspiration becomes:

$$\tilde{Q}_E = \frac{\Delta(R_N - G_0) + \frac{\rho c_p}{r_{aH}}(e^*(T_0) - e(T))}{\Delta + \frac{\gamma(r_{aV} + r_s)}{r_{aH}}}$$
(23)

We can assume that  $r_{aV} = r_{aH} = r_a$ ,

we finally obtain the expression for latent heat of vaporization called the **Penman-Monteith Equation**:

$$\tilde{Q}_E = \frac{\Delta(R_N - G_0) + \frac{\rho c_p}{r_a} (e^*(T_0) - e(T))}{\Delta + \gamma \left(1 + \frac{r_s}{r_a}\right)}$$
(24)

- $r_s$  stomatal resistance of the individual leaf
- $r_a$  boundary layer resistance
- $R_N G_0$  flux of net radiation into leaf
- $\tilde{Q}_E$  flux of latent heat from leaf
- $\rho$  density of air
- $c_p$  specific heat or air

The importance of this equation is that it gives evaporation in terms of measureable meteorological variables  $(R_N, T, D)$  and known resistances. Although these resistances are really difficult to estimate in reality. While the physical nature of  $r_a$  is well understood based on turbulence theory, the conceptual significance of the resistance concept remains problematic.

Equations for whole-canopy exchanges are identical to single leaf, except that the energy balance is for the canopy as a whole. Single layer canopy models are usually used in numerical atmospheric models where we are interested in scales that are larger than the canopy scale. Multilayer models are appropriate when resolving details within the canopy, so for forest stands. We will deal with single layer models.

The Penman-Monteith Equation for *whole canopy* is:

$$\tilde{Q}_E = \frac{\Delta (R_N - G_0) + \frac{\rho c_p}{r_a} D}{\Delta + \gamma \left(1 + \frac{r_{sT}}{r_a}\right)}$$
(25)

Where  $r_{sT}$  is the bulk stomatal resistance of the whole canopy. If  $r_{si}$  is the resistance for a single leaf, then  $r_{sT}$  is the parallel sum of these leaf stomatal resistances with:

$$\frac{1}{r_{sT}} = \frac{1}{A} \sum \frac{L_{A,i}}{r_s i} \tag{26}$$

where  $L_{A,i}$  is the area of the  $i^{th}$  leaf, A is the ground area LAI the leaf area index is  $LAI = \sum L_{A,i}/A$ , and the approximation generally used is:

$$r_{sT} \approx r_{si}/LAI \tag{27}$$

The values for several canopy types is listed in Garrat Table 5.1.

While this equation works with a dense canopy, for an open canopy where LAI < 1, the underlying single source assumption (only transpiration) doesn't work. Total evapotranspiration becomes a combination of transpiration from different plant types and evaporation from open areas. The equation must be applied separately to vegetated and bare areas.