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## 1. Radiation Fluxes

The net radiation over land  $R_{N0}$  contains a combination of shortwave and long-wave fluxes:

$$R_{N0} = R_{s0}(1 - \alpha_s) + \epsilon_s R_{L0}^d - R_{L0}^u \quad (1)$$

where  $R_{s0}$  is the surface shortwave radiative flux (positive),  $R_{L0}^d$  is the surface downward longwave flux (positive) and  $R_{L0}^u$  is the surface upward longwave flux (negative).  $\alpha_s$  is the shortwave albedo and  $\epsilon_s$  is the surface longwave emissivity (equal to the absorptivity, which is the fraction of the incoming long-wave radiation absorbed by the surface. If the surface behaves as a black body, then  $\epsilon_s = 1$ . The flux of radiation emitted by a black body is given by the Stefan Boltzmann law as  $\sigma T^4$  where  $\sigma = 5.67 \times 10^{-8} W m^{-2} K^{-4}$  is the Stefan-Boltzmann constant. Most natural surfaces are grey rather than black so they emit a longwave radiative flux of magnitude  $\epsilon_s \sigma T^4$  where  $\epsilon_s$  is less than unity. The net radiation can be measured directly, but we can also evaluate the components on the right hand side.

### 1a. Shortwave Fluxes

Radiation emitted by the sun approximates that emitted by a black body with surface temperature close to  $6000K$  and maximum emitted at a wavelength of  $0.48\mu m$  (calculated with Wien's Law). The solar spectrum is confined to .1 to  $4\mu m$  (hence shortwave). At the top of the earth's atmosphere we define the *solar irradiance*  $S_c$  as the flux of solar radiation passing through a plane normal to the solar beam at the top of the atmosphere, with the Earth at its mean annual distance from the Sun,  $S_c = 1367 W m^{-2}$ , but this of course changes as the distance changes.

Scattering, absorption and reflection reduce the solar flux. When radiation is scattered by molecules (Rayleigh scattering) account for diffuse component of shortwave radiation. Total shortwave is the sum of direct and diffuse components. During the night  $R_{s0} = 0$ , during the day ( $\sin(\Psi) > 0$ ), the shortwave radiation can be expressed as:

$$R_{s0} = \tau_a S_c^{app} \sin(\Psi) \quad (2)$$

where  $\Psi$  is the solar elevation angle (angle of the sun above the local horizon),  $S_c^{app}$  is the apparent solar irradiance (that varies with time of year),  $\tau_a$  is the net sky *transmissivity* or the fraction of solar radiation that makes it to the surface. One simple parameterization for the transmissivity is:

$$\tau_a = (0.6 + 0.2\sin\Psi)(1 - 0.4\sigma_{CH})(1 - 0.7\sigma_{CM})(1 - 0.4\sigma_{CL}) \quad (3)$$

Where  $\sigma_C$  is the cloud cover fraction and the subscripts  $H, M, L$  are high, middle and low elevation clouds respectively. If the sun is overhead and there are no clouds,  $\tau_a = 0.8$ , when there are overcast clouds at all levels  $\tau_a = 0.086$ . The determination of the solar elevation angle is a simple geometric exercise:

$$\sin\Psi = \sin\phi\sin\delta_s - \cos\phi\cos\delta_s\cos\left[\left(\frac{\pi t_{UTC}}{12}\right) - \lambda_e\right] \quad (4)$$

where  $\phi$  and  $\lambda_e$  are the latitude (positive north) and longitude (positive west) in radians,  $\delta_s$  is the *solar declination angle* which is the angle of the sun above the equator in radians, and  $t_{UTC}$  is the Coordinated Universal Time in hours. The solar declination angle is:

$$\delta_s = \phi_r \cos\left[\frac{2\pi(d - d_r)}{d_y}\right] \quad (5)$$

where  $\phi_r$  is the latitude of the Tropic of Cancer (0.0409 radians),  $d$  is the number of the day of the year (Julian Day),  $d_r$  (173) and  $d_y$  is the average number of days per year (365.25).

We define the *albedo* ( $\alpha_s$ ) as the fraction of downwelling radiation at the surface that is reflected.

$$K \uparrow_s = -\alpha_s K \downarrow_s \quad (6)$$

Albedo ranges from 0.95 over fresh snow to 0.4 over light colored soils, 0.2

over grass and agriculture, 0.1 over coniferous forests and 0.05 over dark wet soils.

Table A.8 Garrat.

### 1b. Longwave Fluxes

Results from the emission of the atmospheric gases and the land and water surfaces of the Earth. All materials on Earth have a much lower temperature than the Sun, so that the radiation they emit is at a much longer wavelength. There is almost no overlap, as the radiation from the Earth is at a 4-100  $\mu\text{m}$  wavelength. In what follows I will be speaking about longwave fluxes at the surface - a general discussion on longwave fluxes at any particular height is much more complex.

The upward component of longwave radiation flux is calculated assuming that the ground, canopy or water surface is equivalent to an infinitely deep grey body of uniform temperature and emissivity  $\epsilon_s$

$$R_{L0}^u = \epsilon_s \sigma T_0^4 \quad (7)$$

Where  $T_0$  is the surface temperature, and  $\sigma = 5.67 \times 10^{-8} \text{Wm}^{-2}\text{K}^{-4}$  is the Stefan-Boltzmann constant. Most natural surfaces are almost black with  $\epsilon_s$  greater than 0.9.

The downward longwave flux is results from emissions of IR radiation from whole atmosphere and clouds. Low clouds in particular tend to radiate as black bodies at their cloud-base temperatures. The downward flux is easy to measure, but numerical computation is not straight forward. It is dependent on temperature and humidity vertical distributions, clouds and atmospheric composition. For clear sky conditions:

$$R_{L0clear}^d = \epsilon_{ac} \sigma T_a^4 \quad (8)$$

where  $T_a$  is the air temperature near the ground and  $\epsilon_{ac}$  is the atmospheric emissivity under clear skies. One approximation for the emissivity is  $\epsilon_{ac} = a(e_a/T_a)^b$  where a and b are constants derived to be a=1.24 and b=1/7 when the vapor pressure of the air in in hPa and T in in K.

Because the downward longwave radiation is affected by cloudiness, several empirical methods have found:

$$R_{L0}^d = R_{L0clear}^d (1 + am_c^b) \quad (9)$$

where  $m_c$  is the fractional cloudiness and a and b are constants (for the Great Plains, a=0.0496 and b=2.45).

Problem from Brutsaert: The following data are averages for a typical summer day in a temperate climate: air temperature  $T_a = 17.94^\circ C$ , RH=66% and incoming short wave radiation  $R_s=468 \text{ cal cm}^{-2}d^{-1}$ . Calculate  $R_{N0}$  for a surface covered with short vibrant vegetation. Assume cloudiness doesn't affect the radiation and that  $T_a = T_s$ .

If the wind speed measured at 10m above ground was  $10.4 \text{ km h}^{-1}$ , Assume the ground heat flux G is negligible. a) Find the potential evaporation using Penman's method. b) Potential evaporation using Priestly and Taylor

## 2. Surface Energy Balance

The balance of energy fluxes at the surface constraints the total available energy. The primary forcing of the ABL over land is through absorption of solar radiation.

In the absence of vegetation, conservation of energy at the interface requires that:

$$R_{N0} - G_0 = \tilde{Q}_H + \tilde{Q}_E \quad (10)$$

This is the surface energy balance equation, where  $R_{N0}$  is the net radiation,  $G_0$  is the heat flux into the soil,  $\tilde{Q}_H$  is the sensible heat and  $\tilde{Q}_E$  is the latent heat. The fluxes have units of  $W m^{-2}$ . This equation is exact when horizontal advection is negligible, and vertical fluxes do not vary with height (as is our assumption in the surface layer).

Let's focus on  $G_0$ . Heat flux to the soil is related to the soil temperature using the requirement for heat conservation:

$$\rho_s c_s \frac{\partial T_s}{\partial t} = - \frac{\partial G}{\partial z'} \quad (11)$$

where  $C_s = \rho_s c_s$  is the volumetric heat capacity (units of  $Jm^{-3}K^{-1}$ ),  $\rho_s$  is soil density and  $c_s$  is the soil specific heat, and  $z'$  is the vertical coordinate in the soil (positive down). Remember that the heat capacity characterizes the amount of heat required to change a body's temperature by a given amount.

The change in temperature for a slab of finite depth can be expressed as:

$$\rho_s c_s \Delta z' \frac{\partial T_s}{\partial t} = -(G_1 - G_0) \quad (12)$$

Where  $G_1$  is the heat flux at a depth  $\Delta z'$  into the soil. Substituting equation 12 into 10:

$$R_{N0} - G_1 = \tilde{Q}_H + \tilde{Q}_E + \frac{\partial W_s}{\partial t} \quad (13)$$

where  $W_s = \rho_s c_s \Delta z' T_s$  is the energy storage. For a canopy or layer of vegetation of depth  $h_c$ , the balance equation at the canopy top can be written in an analogous form:

$$R_{N0} - G_1 = \tilde{Q}_H + \tilde{Q}_E + \frac{\partial W_c}{\partial t} + D_h \quad (14)$$

where  $W_c = \rho_c c_c h_c T_c$  is the energy storage and  $D_h$  is a horizontal flux of energy due to advection (generally neglected).

The values of soil heat flux at the surface or at a shallow depth depend on solar radiation, soil type and soil moisture content. The subsurface heat flux  $G$  at any level  $z'$  can be described by Fourier's law for heat conduction in a homogeneous body:

$$G = -k_s \frac{\partial T_s}{\partial z'} \quad (15)$$

where  $k_s = \rho_s c_s \kappa_s$  is the thermal conductivity.  $k_s$  depends on soil wetness because water is a good conductor of heat, while air isn't.  $\kappa_s$  is the thermal diffusivity. See Table A7. By replacing equation 15 into 16, we obtain the heat conduction equation:

$$\frac{\partial T_s}{\partial t} = \frac{k_s}{\rho_s c_s} \frac{\partial^2 T_s}{\partial z'^2} = \kappa_s \frac{\partial^2 T_s}{\partial z'^2} \quad (16)$$

In this equation  $T_s$  is a function of ( $z'$  and  $t$ ), and we can solve for  $T_s$  and hence for  $G$  using either an analytical solution for simple sinusoidal forcing or numerical

methods for real world conditions. If we use a pure sinusoidal forcing function we can get some idea of how diurnal and annual temperature waves penetrate into the ground. If we assume that the temperature at  $z' = 0$  (ground surface)  $T_s(0, t) = \bar{T} + A_0 \sin(\omega t)$ .

For the diurnal cycle,  $2A_0$  is the diurnal temperature amplitude,  $\bar{T}$  is the daily average temperature and  $\omega$  is the angular velocity of the Earth's rotation ( $7.292 \times 10^{-5} \text{ rad s}^{-1}$ ). For the annual cycle,  $2A_0$  is the annual temperature amplitude,  $\bar{T}$  is the annual average temperature and  $\omega$  is the velocity of the earth's cycle around the sun ( $.0172 \text{ rad day}^{-1}$ ). Using  $\bar{T}$  as the bottom boundary condition, the solution of the equation is:

$$T_s(z', t) = \bar{T} + A_0 \exp\left(\frac{-z'}{D}\right) \sin\left(\frac{\omega t - z'}{D}\right) \quad (17)$$

where  $D = (2\kappa_s/\omega)^{1/2}$  is the *damping depth*, the depth at which the temperature amplitude has decreased to  $1/e$  of the surface amplitude. For the diurnal wave and typical soils,  $D \approx 0.1m$ .

- As we go deeper into the soil, the amplitude of the oscillation dampens, and there is a lag that grows larger as we go deeper.
- Notice that  $D$  depends on  $\omega$ , the frequency of heating. This means that a longer cycle (like the annual) will have a greater damping depth, which means that the wave will penetrate deeper into the soil.
- Notice that  $D$  depends on the diffusivity. Dry soils will have a smaller diffusivity than wet soils which will have smaller diffusivity than, say, rock. Again, larger diffusivities mean deeper penetration of the wave.
- Notice that the lag means that there is a delay in the effect of the surface over the soil. Larger frequency  $\omega$  means a larger lag.

In reality, however, the temperature is not a sinusoid but is best represented as a Fourier series. In numerical models the equation is solved using finite difference forms.

Notice that there is also a phase shift of  $z'/D$ . The soil heat flux can now be determined using equation 15. At  $z' = 0$  this gives:

$$G_0 = \rho_s c_s (\kappa_s \omega)^{1/2} A_0 \sin(\omega t + \pi/4) \quad (18)$$

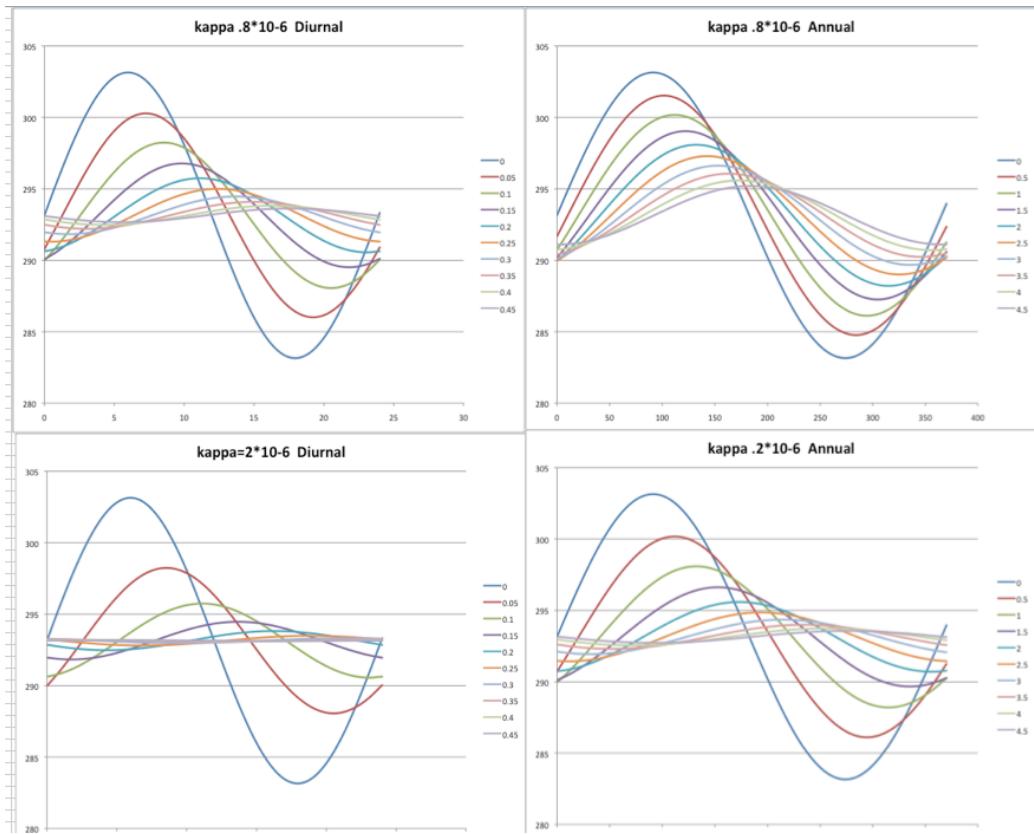


Figure 1: Soil Temperatures as a function of time for different diffusivities and for the diurnal and annual cycle

So that the heat flux is  $\pi/4$  out of phase with the temperature. In practice, several heat flux plates are buried in the soil. These plates measure temperature and the properties of the soil must be known.