Chapter 3. Uniform Particle Motion

This Chapter covers <u>uniform</u> motion therefore no acceleration (although we bend this rule when deriving an expression for V_{TS}).
Here we consider just two (opposing) forces (gravity and drag). We will add more forces later.

Drag Force

Two approaches yield the same result

- Newton's Resistance Law, originally for high Reynolds numbers:

  $F_D = C_D \pi \rho_{gas} D_p^2 V^2/8$
  Since C_D is proportional to σ $1D_pV$
  Then F_D is proportional to D_pV

- Stokes's Law, derived for certain specific conditions including low Reynolds numbers (derivation assumes that inertial terms are insignificant. See § 3.10 Appendix)
  Solving Navier – Stokes equation and the continuity equation yields:

  $F_D = 3 \pi \eta V D_p$
- Drag Coefficient

  Equating Newton and Stokes results in expression for drag coefficient, C_D

  $C_D = 24/R_e$

  i.e., Stokes is a special case of Newton's Resistance Law (Fig. 3.1)

Terminal Velocity or Settling Velocity

See class notes for general derivation starting with a particle at rest, and then allowing it to accelerate to its terminal velocity.

$$V_{TS} = \frac{\rho_p D_p{}^2 g}{18\eta}$$

Note that V_{TS} is proportional to D_p^2

Relaxation Time

Note that the characteristic relaxation time, τ, is very short (μs) for most natural aerosols (see class notes; and, looking ahead, §5.1, 5.2).

Mobility (mechanical)

$$B = \frac{V}{F_D} = \frac{\tau}{m_p}$$

i.e., mobility is velocity per unit force or it can also be viewed as relaxation time per unit mass.

Corrections to Drag Force Equations

- Cunningham slip correction factor C_c accounts for gas molecules that slip past aerosol particles without inducing drag (Fig. 3.2)

  $C_c \approx 1$ when $D_p > \sim 10\mu m$
- Drag coefficient, C_D, is included in Newton's Friction Law, although Newton himself used a constant of proportionality, K, - a subtle distinction (compare eq. 3.3, 3.4).
- Shape Factor, χ, empirically obtained and included in Stokes's Law for non-spherical particles (Table 3.2). Values range from 1.0 to nearly 2, depending on the particle.

Various Diameters Defined

Read § 3.6: aerodynamic, vs. Stokes vs. equivalent sphere diameters.

Stirred Settling

A very common situation, e.g., rooms, containers
Particle number concentration decays exponentially (eq. 3.36).

$$N = N_0 \exp(-V_{TS}t/H)$$

Instrumentation and Equipment

Based on V_{TS} (and therefore D_P) and used to separate/classify particles, e.g., horizontal elutriator (Fig. 3.6, 3.7) and aerosol cyclone (eq. 3.14)