Homework #5
Objective Analysis in the Atmospheric and Related Sciences
ATMO, HWRS, GEOS, GEOG 529: Fall 2013

<u>Data</u>

The following data are needed for Parts II and III:

1) Yearly average winter (DJF) winter surface pressure (mb)

   Period 1950-2013 (64 years) at 2.5º grid spacing for the northern hemisphere. These data are derived from the NCEP-NCAR Global Reanalysis I. The domain spans 0 – 90º N (37 points) and the entire global span of longitude starting at 0º E (145 points). The format of the data is a list of 343,360 numbers (145 X 37 points X 64 years). There are no missing data. Data are written in the following structure in the file:

   First loop: x dimension, 1 to 145
   Second loop: y dimension, 1 to 37
   Third loop: t dimension, 1 to 64

   These data, along with the corresponding GrADS control file, are provided on the course website.

2) Corresponding winter precipitation anomalies from UDEL data and the three winter average climate indices, computed in previous assignments.

<u>Part I</u>

Calculate the EOFs and PCs of the following 4 x 2 matrix by hand as a pencil and paper exercise. Following the convention used in class, the sampling domain lies in **A**'s columns.

$$\mathbf{A} = \begin{bmatrix} -7 & -10 \\ 3 & 8 \\ -6 & 4 \\ 1 & -5 \end{bmatrix}$$

Follow these steps:

1) First calculate $\mathbf{A}^T\mathbf{A}$. Explain why it is more efficient to do eigenanalysis of $\mathbf{A}^T\mathbf{A}$ than $\mathbf{A}\mathbf{A}^T$ in this case.

2) Find the eigenvalues and eigenvectors of $\mathbf{A}^T\mathbf{A}$ (i.e. the EOFs of **A**). Be sure to scale the eigenvectors to length one, as that is how they are displayed in canned routines.

3) Use the relationship $\mathbf{A} = \mathbf{U\Sigma V}^T$ to find the principal component (PC) time series that correspond to the EOFs found in (2). Hint: since $\mathbf{A} = \mathbf{U\Sigma V}^T$, it follows that $\mathbf{AV} = \mathbf{U\Sigma}$. Hence the i^{th} PC can be found by solving $\mathbf{Av}_i = \mathbf{u}_i\boldsymbol{\sigma}_i$. Remember, the singular values ($\boldsymbol{\sigma}$) are the square roots of the eigenvalues of $\mathbf{A}^T\mathbf{A}$.

4) List the orthonormal EOFs and PCs of **A**, and the variance explained by each mode.

5) Demonstrate that $\mathbf{A} = \mathbf{U\Sigma V^T}$ is satisfied.

6) Check your answers using the canned routine for eigenanalysis of the covariance matrix and/or SVD analysis you will be using for the next parts of the assignment. Which canned routine are you using?

Part II

Use EOF analysis to extract the dominant patterns in average surface pressure.

Follow these steps:

1) Prepare data: Compute the surface pressure anomaly map per year by subtracting out the climatological mean surface pressure at each grid point on the map. Do not remove the spatial mean from the data. Since the grid size decreases toward the pole, weight each grid box (i.e. multiply the time series at each grid box by the square root of the cosine of latitude). Weights are based on the square root of the cosine so that the covariance matrix is weighted by the cosine of latitude.

2) EOF/PC analysis: Calculate the first three PCs of the data (corresponding to the three largest eigenvalues of the covariance matrix). Use either SVD or eigenanalysis of the covariance matrix with a canned software routine (e.g. some are provided on the course website). If doing eigenanalysis, it is suggested to eigenanalyze $\mathbf{AA^T}$ when **A** is M X N and M = time dimension, N = spatial dimension.

3) Spatial pattern maps and PC time series: Regress the unweighted surface pressure anomaly field onto the three leading PCs found in (2). You should have one regression map for each PC. Note that the resulting maps will not be identical to the EOFs, since they lie in weighted space.

4) Eigenvalue spectrum: Plot the spectrum of the first 10 eigenvalues (scaled as % variance explained), with error bars. Use the North test to evaluate statistical significance at the 95% level, assuming independence of each year.

Discussion: Compare your findings with those presented in *Monthly Weather Review* paper by Wallace and Gutzler (1981). Are your results the same or different? Why might that be? Are the dominant modes from your EOF analysis explaining physically meaningful variability in cool season climate or are the patterns reflecting just white or red noise? How you answer this last question may depend on which particular mode you are considering.

Part III

Correlate each of the first three PC time series of surface pressure anomalies with 1) the corresponding three climate indices during the antecedent fall (SON average) and 2) Phoenix and Seattle winter SPI from previous assignments. Show results as a table of correlation coefficients. Highlight those correlation coefficients that are statistically significant by a t-test.

Next, considering winter precipitation anomalies from the UDEL data, as already computed in the previous two homeworks, correlate these data with the first three PCs. You should generate a series of three correlation maps. Calculate the statistical significance at the 90% level using a t-test and display the local significance as before. Field significance test is not required.

Discussion: How well do your results using the dominant PCs of winter surface pressure compare with the previous results in Assignment #4? Do any of them have a significant relationship to Phoenix or Seattle winter precipitation? Why or why not? Based on this comparison, how well do these indices capture the dominant modes of the large-scale atmospheric circulation and their associated temperature and precipitation responses in the cool season? How does this bear on potential for seasonal predictability of the climate during the cool season in the United States?

<u>Optional Exercises</u> (for extra credit)

The following optional exercises are progressively more computationally involved and conceptually challenging, so the extra credit is weighted accordingly.

1) <u>Rotated EOFs</u>: Use the varimax rotation algorithm to compute the rotated EOFs of average winter surface pressure. Use only the first 10 original EOFs in the rotation. How does rotation change the spatial patterns of the modes and explained variance? Is this a good thing to do for surface pressure anomalies? Why or why not?

2) <u>EOF analysis with missing data</u>: Perform an EOF analysis on the corresponding winter precipitation anomalies with the data from previous assignments. The analysis procedure is exactly the same as in Part II, except that missing data points are omitted in the analysis. Then correlate the first three PCs of this analysis with the first three PCs of surface pressure anomalies. How well do your results compare with those in Part III? You may want to repeat these steps using rotated EOFs and evaluate the differences in spatial variability in resulting precipitation patterns.

3) <u>MCA or CCA</u>: Use maximum covariance analysis (MCA) and/or canonical correlation analysis (CCA) to find the coupled modes in winter surface pressure anomalies winter precipitation anomalies. Follow the procedures as discussed in class lectures and in the Hartmann notes.

You may want to consider doing one or more of these options if you are using matrix methods for your term project and/or would like to do only part of the next homework assignment on time series analysis. The amount of extra credit will be based on the correct usage of the methods and level of discussion of your results.

Assignment due date: Drop in mailbox no later than November 21. You may hand in optional exercises through the end of the semester.