Homework #5 Key
ATMO 529, Fall 2013
70 points total

Part 1: EOF analysis by hand
One student solution
20 points

Part I

Calculate the EOFs and PCs of the matrix A, where

$$A = \begin{bmatrix} -7 & -10 \\ 3 & 8 \\ -6 & 4 \\ 1 & -5 \end{bmatrix}$$

20

Steps:

1) Calculate $A^T A$.

$$A^T A = \begin{bmatrix} -7 & 3 & -6 & 1 \\ -10 & 8 & 4 & -5 \end{bmatrix} \cdot \begin{bmatrix} -7 & -10 \\ 3 & 8 \\ -6 & 4 \\ 1 & -5 \end{bmatrix} = \begin{bmatrix} 95 & 65 \\ 65 & 205 \end{bmatrix}$$

Since $A^T A$ is 2 by 2 matrix, number of eigenvalues should be 2 and their eigenvector should also be 2 by 2 matrix. Much easier than that of AA^T which is a 4 by 4 matrix. When solve the eigenvalue, it has to solve a quartic equation, much more complicated.

2) Find eigenvalue and eigenvector of $A^T A$.

By definition, the eigenvector and eigenvalue should have the property like following,

$$CE = E\lambda$$

Where $C = A^T A$ in this case, E is the eigenvector, λ is a diagonal matrix with descending order of eigenvalue on its diagonal. In order for such E to exist, $\det(C - \lambda I) = 0$ should have real solutions. That means

$$\begin{vmatrix} 95-\lambda & 65 \\ 65 & 205-\lambda \end{vmatrix} = 0$$

Which $\lambda^2 - 300\lambda + 15250 = 0$, with solution $\begin{cases} \lambda 1 = 235.15 \\ \lambda 2 = 64.86 \end{cases}$. To calculate the eigenvector, take $\lambda 1$ for example:

$$\begin{bmatrix} 95 & 65 \\ 65 & 205 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \cdot 235.15$$

So, we have $\frac{e_1}{e_2} = \frac{65}{140.15}$, after scale the length of eigenvector to 1. We have $E1 = \begin{bmatrix} 0.4207 \\ 0.9072 \end{bmatrix}$

The same process for $\lambda 2$, and the second eigenvector is $E2 = \begin{bmatrix} -0.9072 \\ 0.4207 \end{bmatrix}$.

So the eigenvalue is $\begin{bmatrix} 235.15 & 0 \\ 0 & 64.86 \end{bmatrix}$, the eigenvector is $\begin{bmatrix} 0.4207 & -0.9072 \\ 0.9072 & 0.4207 \end{bmatrix}$.

3) Use the relationship $A = U\Sigma V^T$ to find the principal component corresponding to EOFs from step 2). Knowing that the ith PC can be found by solving $Av_i = u_i\sigma$, where σ is the the square roots of the eigenvalues of $A^T A$.

$$u1 = \begin{bmatrix} -7 & -10 \\ 3 & 8 \\ -6 & 4 \\ 1 & -5 \end{bmatrix} \begin{bmatrix} 0.4207 \\ 0.9072 \end{bmatrix} \frac{1}{\sqrt{235.15}} = \begin{bmatrix} -0.7837 \\ 0.5556 \\ 0.0720 \\ -0.2684 \end{bmatrix}$$

$$u2 = \begin{bmatrix} -7 & -10 \\ 3 & 8 \\ -6 & 4 \\ 1 & -5 \end{bmatrix} \begin{bmatrix} -0.9072 \\ 0.4207 \end{bmatrix} \frac{1}{\sqrt{64.86}} = \begin{bmatrix} 0.2661 \\ 0.0800 \\ 0.8848 \\ -0.3738 \end{bmatrix}$$

4) List the orthonormal EOFs and PCs of A, and variance explained by each mode.

$$\text{EOFs} = \begin{bmatrix} 0.4207 & -0.9072 \\ 0.9072 & 0.4207 \end{bmatrix}$$

$$\text{PCs} = \begin{bmatrix} -0.7836 & 0.2661 \\ 0.5556 & 0.0800 \\ 0.0720 & 0.8848 \\ -0.2684 & -0.3738 \end{bmatrix}$$

good

Variance explained by each mode is 0.7838 and 0.2162 for the first mode and second mode respectively.

5) Demonstrate that $A = U\Sigma V^T$.

$$\begin{bmatrix} -0.7836 & 0.2661 \\ 0.5556 & 0.0800 \\ 0.0720 & 0.8848 \\ -0.2684 & -0.3738 \end{bmatrix} \begin{bmatrix} 15.33 & 0 \\ 0 & 8.05 \end{bmatrix} \begin{bmatrix} 0.4207 & -0.9072 \\ 0.9072 & 0.4207 \end{bmatrix}^T = \begin{bmatrix} -7.1 & -10.7 \\ 2.9 & 8.5 \\ -6.4 & 4.0 \\ 1.1 & -5.2 \end{bmatrix}$$

The result obtained from above is slightly different from A, this is because we applied approximation earlier when calculated eigenvalues, theoretically the matrix obtained above should be the same as A.

6) Use SVD routine from matlab, the obtained $U\ \Sigma, V$ are as followed.

$$U = \begin{bmatrix} 0.7837 & 0.2661 & -0.5572 & 0.0675 \\ -0.5556 & 0.0800 & -0.6872 & 0.4611 \\ -0.0720 & 0.8849 & 0.3565 & 0.2910 \\ 0.2684 & -0.3739 & 0.3001 & 0.8355 \end{bmatrix}$$

$$\Sigma = \begin{bmatrix} 15.3345 & 0 \\ 0 & 8.0531 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

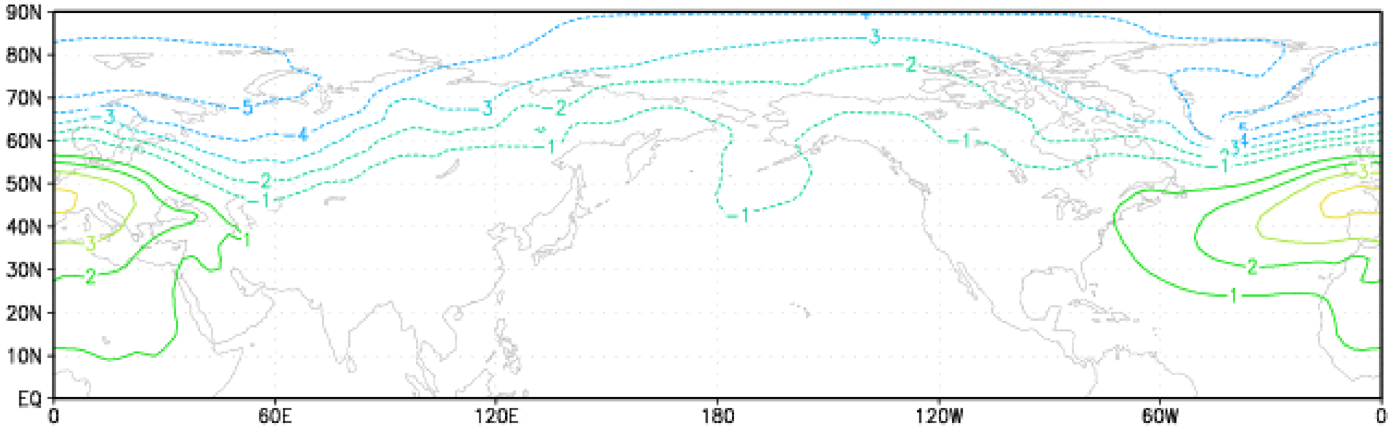
$$V = \begin{bmatrix} -0.4207 & -0.9072 \\ -0.9072 & 0.4207 \end{bmatrix}$$

Which is close to the value obtained from the calculation.

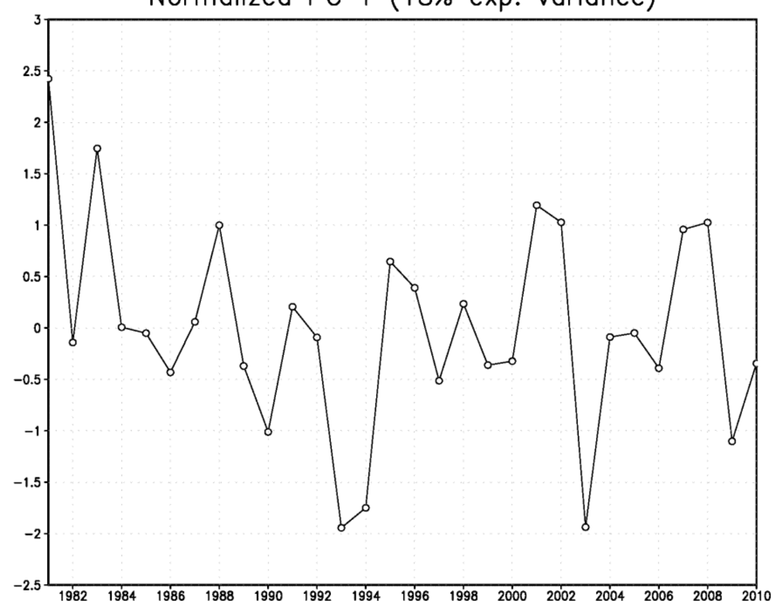
Part 2: EOF analysis of 500-mb height anomalies

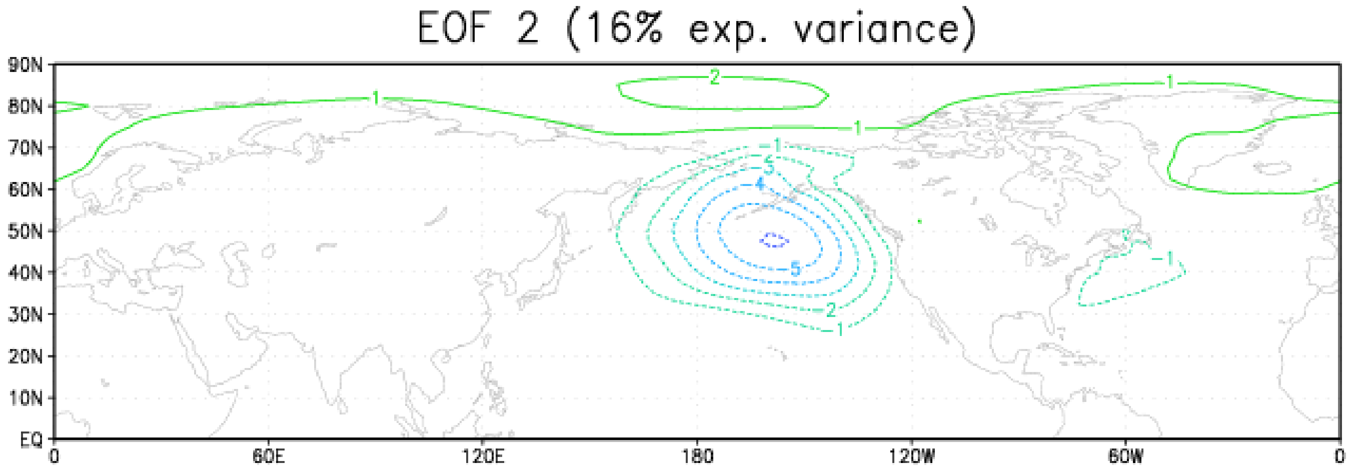
30 points

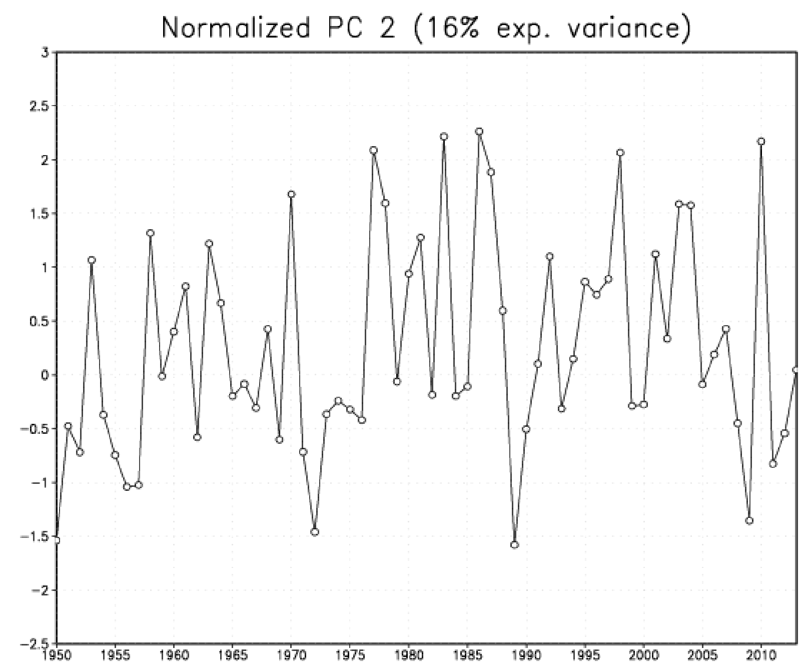
EOF 1 (33% exp. variance)

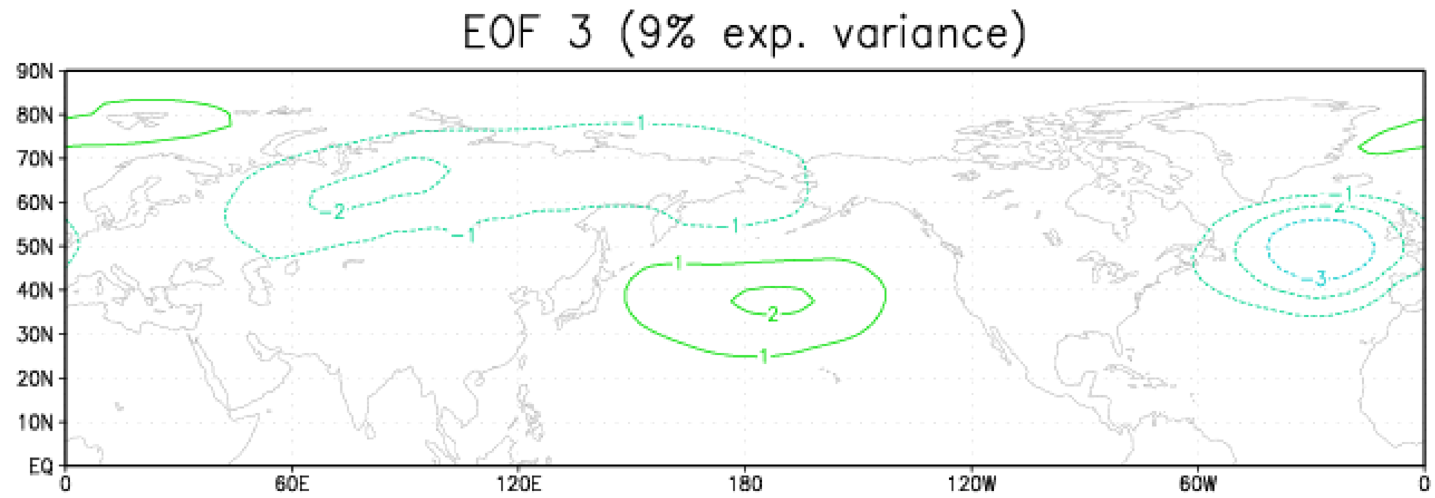


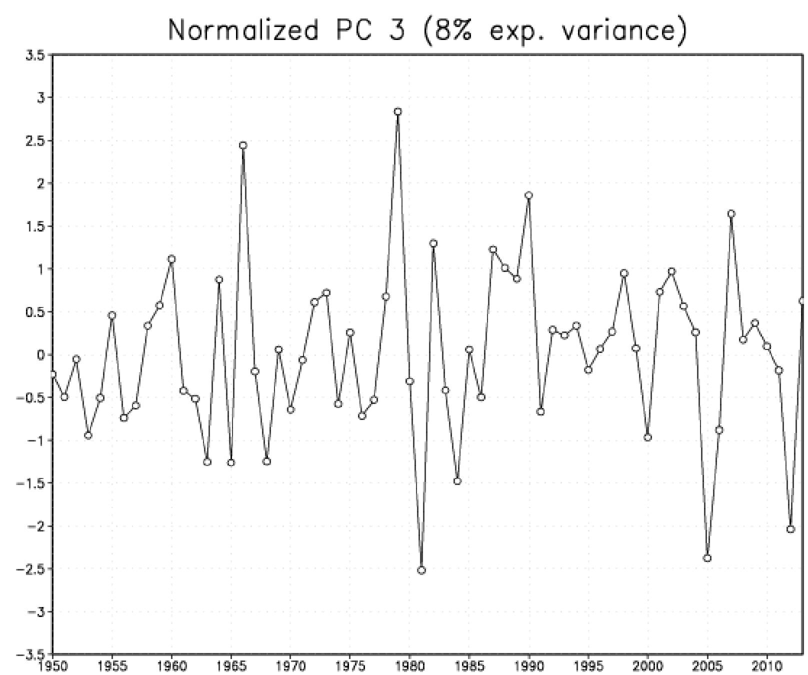
Normalized PC 1 (18% exp. variance)

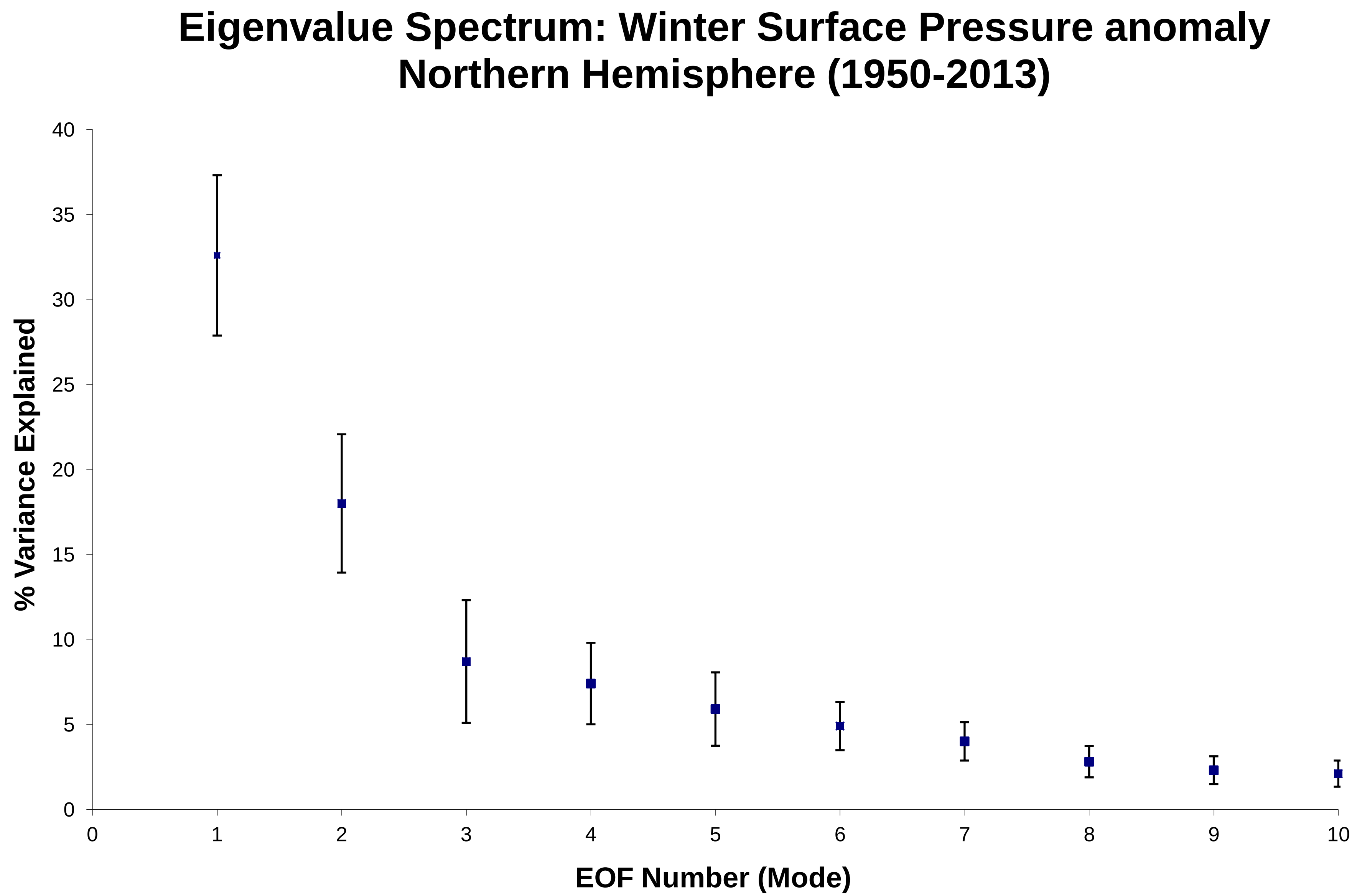



EOF 2 (16% exp. variance)
90N
80N
70N
60N
50N
40N
30N
20N
10N
EQ
0
60E
120E
180
120W
60W
0


Normalized PC 2 (16% exp. variance)
3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
1950
1955
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005
2010


EOF 3 (9% exp. variance)
90N
80N
70N
60N
50N
40N
30N
20N
10N
EQ
0
60E
120E
180
120W
60W
0


Normalized PC 3 (8% exp. variance)
1950
1955
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005
2010


Eigenvalue Spectrum: Winter Surface Pressure anomaly
Northern Hemisphere (1950-2013)
% Variance Explained
EOF Number (Mode)
0
5
10
15
20
25
30
35
40
0
1
2
3
4
5
6
7
8
9
10

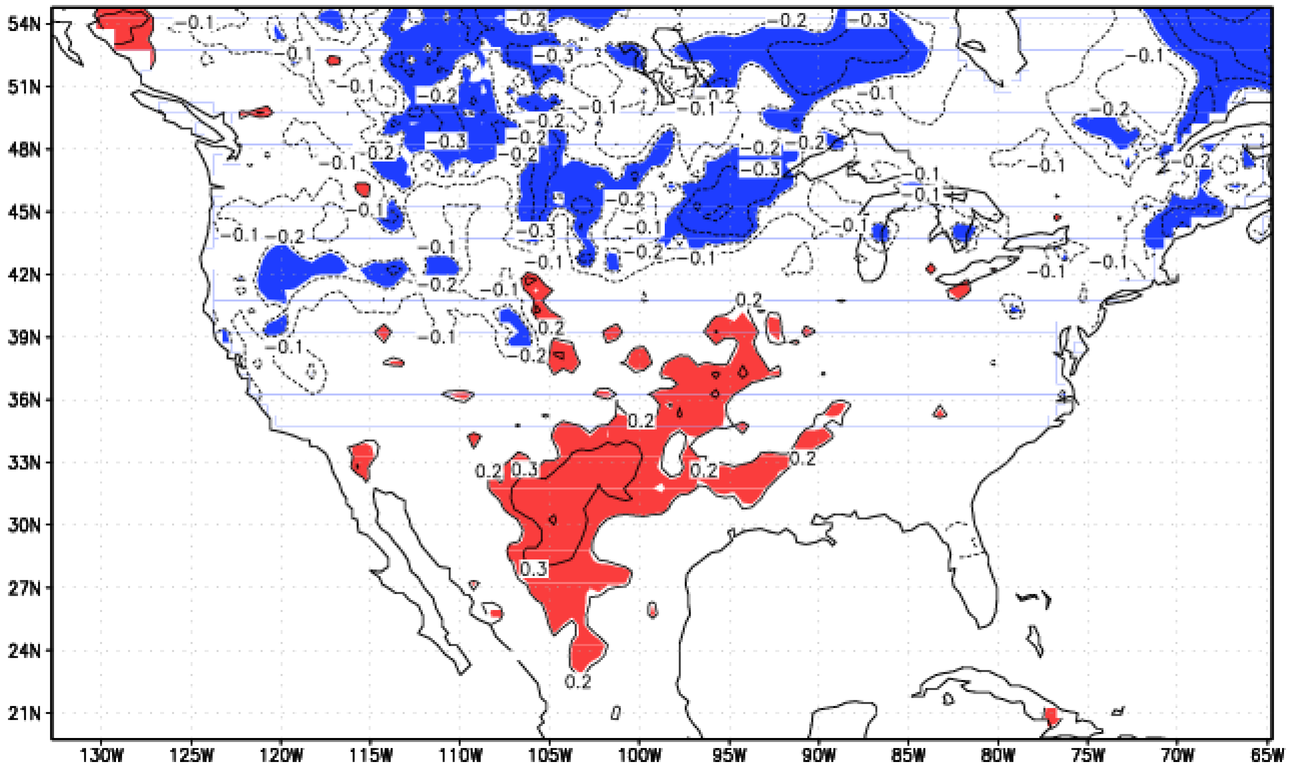
Correlation Analyses of PCs and antecedent fall climate indices, Seattle and Phoenix winter SPI

	PC 1	**PC 2**	**PC 3**
AO	-0.09	0.11	**0.26**
MEI	-0.06	**0.62**	**0.28**
PDO	0.06	**0.27**	**0.35**
Seattle SPI	-0.16	-0.06	-0.01
Phoenix SPI	-0.11	-0.01	0.08

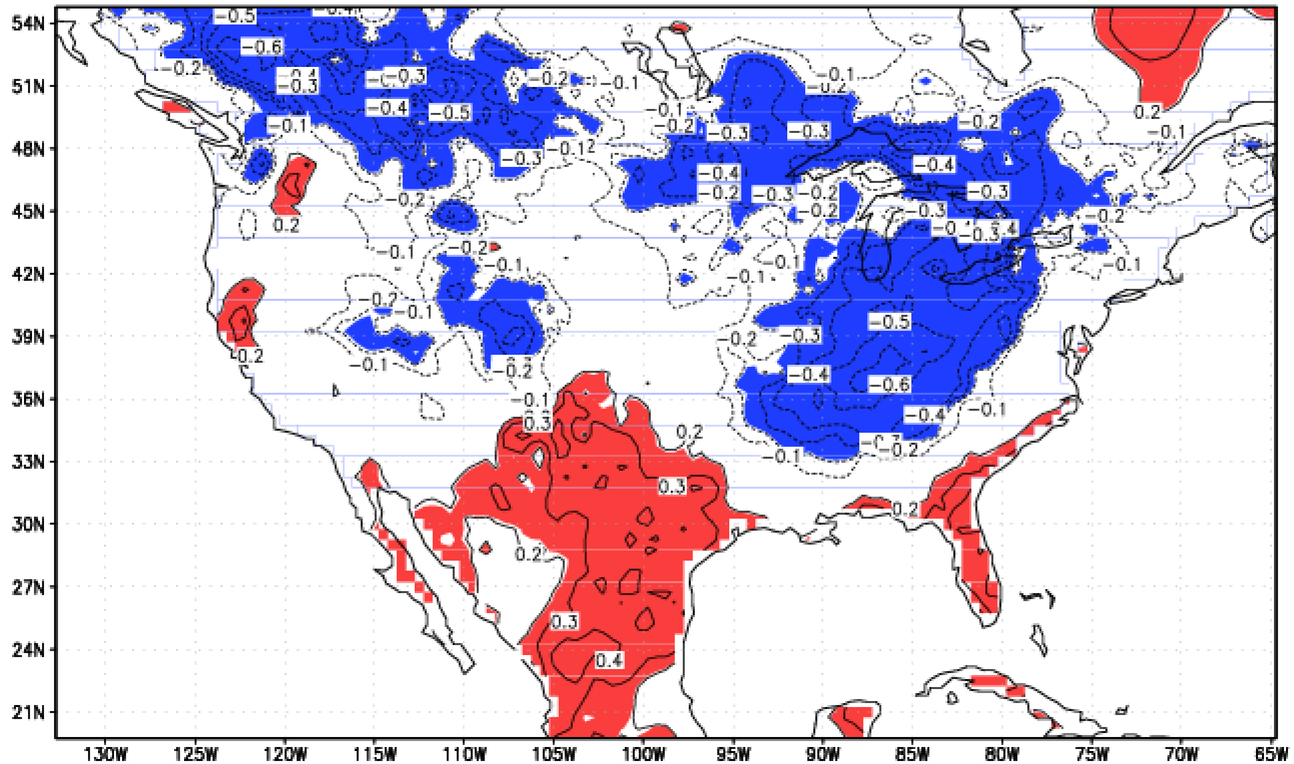
Part 3: Correlation analysis of PCs with coincident winter precipitation with statistical significance shaded

20 points

PC 1: Corr. Coef. with Winter Precipitation Anomalies



PC 2: Corr. Coef. with Winter Precipitation Anomalies



PC 3: Corr. Coef. with Winter Precipitation Anomalies