Mesoscale Meteorological Modeling: Spring 2008
Homework #3

In this assignment, you will be using linearization techniques to explore different types of waves which exist in the atmosphere. In addition to the class texts, I would suggest the Holton and Gill texts as helpful references for this assignment.

The terms in each of the four sets of equations are as follows:

Variable	Meaning
x	Zonal direction
y	Meridional direction
z	Zonal direction
u	Zonal velocity
v	Meridional velocity
w	Vertical velocity
p	Pressure
ρ	Density
θ	Potential temperature
g	Gravitational constant
f	Coriolis parameter

For each of the equation sets:

1. Identify the simplifications, averaging assumptions, and scaling arguments which have been used to derive the system, in comparison to the full form primitive equations (2-38) to (2-42) in the Pielke text.

2. Linearize about a basic state. In your linearization procedure, remember to assume the products of perturbation quantities are negligible. If you make any additional simplifications to the equations in this process, state what these are.

3. Derive an atmospheric wave equation, wave dispersion relation, and phase and group velocities for the wave (in each direction). Identify the resulting atmospheric wave type. Are these types of waves dispersive or non-dispersive? Why is this important?

4. Describe the physical characteristics of the atmospheric waves. This discussion should include the typical phase speed of the waves, their dominant spatial scale (i.e. microscale to global scale), and where they occur in relation to atmospheric phenomena. Given that an atmospheric model can only resolve features which are four times the grid spacing,

what would be the minimum model grid spacing necessary to resolve the waves? Which waves are more important on the mesoscale?

5. Where applicable, discuss how these waves might be affected by the presence of a (parameterized) diabatic source or sink term (S_θ) in the thermodynamic equation. (Hint: think about what key atmospheric parameter the vertical variation of temperature is related to)

EQUATION SET #1

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial x} = 0$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) = 0$$

$$\frac{\partial \theta}{\partial t} + u\frac{\partial \theta}{\partial x} = 0$$

EQUATION SET #2

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} + \frac{1}{\rho}\frac{\partial p}{\partial x} = 0$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z} + \frac{1}{\rho}\frac{\partial p}{\partial z} + g = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0$$

$$\frac{\partial \theta}{\partial t} + u\frac{\partial \theta}{\partial x} + w\frac{\partial \theta}{\partial z} = 0$$

EQUATION SET #3

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} - fv + \frac{1}{\rho}\frac{\partial p}{\partial x} = 0$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + fu + \frac{1}{\rho}\frac{\partial p}{\partial y} = 0$$

$$\frac{1}{\rho}\frac{\partial p}{\partial z} + g = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\frac{\partial \theta}{\partial t} + u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} + w\frac{\partial \theta}{\partial z} = 0$$

EQUATION SET #4

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} - fv + \frac{1}{\rho}\frac{\partial p}{\partial x} = 0$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + fu + \frac{1}{\rho}\frac{\partial p}{\partial y} = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$