Atmospheric remote sensing by microwave emission (Ch 9)

Show EvZ Microwave (0-300 GHz) spectra of the atmosphere

Consider a satellite looking down on the atmosphere. We assume no scattering effects which is not too bad at microwave wavelengths. The emission from a height interval with thickness, dz, is

$$\Delta B = \alpha(v,z)\, B(v,T)\, dz \tag{1}$$

This is then attenuated by absorption as it passes through the atmosphere. The radiance from that height interval that leaves the atmosphere is therefore

$$\Delta B_e(v,z) = \alpha(v,z)\, B(v,T)\, e^{-\tau(v,z)}\, dz = \Delta B\, e^{-\tau(v,z)} \tag{2}$$

where τ is the optical thickness **above** altitude of emission, z, where

$$\tau(v,z) = \int_{z}^{\infty} \alpha(v,\zeta)\, d\zeta$$

So the total spectral radiance of the atmosphere is

$$B_a(v) = \int_{0}^{\infty} \alpha(v,z) B[v,T(z)] e^{-\tau(v,z)} dz \tag{3}$$

There is also a contribution from the emission from the surface, B_s. So the total emission measured by the satellite is

$$B_t(v) = B_a(v) + B_s\, e^{-\tau m(v)} . \tag{4}$$

where τ_m is the total optical depth of the atmosphere from the surface to space:

$$\tau_m(v) = \int_{0}^{\infty} \alpha(v,z)\, dz \tag{5}$$

Note: There is a variable, X, called the transmission through the atmosphere which is equal to $e^{-\tau(z)}$. The vertical derivative of X is

$$dX/dz = d\, e^{-\tau(v,z)}/dz = -\, d\tau(v,z)/dz\; e^{-\tau(v,z)} = \alpha(v,z)\, e^{-\tau(v,z)} \tag{6}$$

This allows B_a in (3) to be written somewhat more compactly as

$$B_a(v) = \int_{0}^{\infty} B[v,T(z)] \frac{dX(v,z)}{dz} dz \tag{7}$$

In the Rayleigh-Jeans limit where $hv \ll kT$, the Planck function can be simplified

$$B(v,T) = \frac{2hv^3}{c^2} \frac{1}{e^{hv/kT} - 1}\bigg|_{hv \ll kT} \cong \frac{2hv^3}{c^2} \frac{kT}{hv} = \frac{2v^2}{c^2} kT \tag{8}$$

So, at microwave wavelengths, B is linearly proportional to T, a very nice, simplifying feature of microwave radiation. This allows us to write equations (4) and (3) as

$$T_t(v) = T_a(v) + T_s\, e^{-\tau m(v)} \tag{9}$$

$$T_a(v) = \int_0^\infty T(z)\alpha(v,z)\exp\left[-\int_z^\infty \alpha(v,z)dz\right]dz \tag{10}$$

It is useful to write (10) as

$$T_a(v) = \int_0^\infty T(z)W(v,z)dz \tag{11}$$

where

$$W(v,z) = \alpha(v,z)\exp\left[-\int_z^\infty \alpha(v,z)dz\right] = \frac{dX}{dz} \tag{12}$$

W is known as a weighting function.

In this form we see that the weighting functions, $W(v,z)$, map the atmospheric temperature profile, $T(z)$, to the measured brightness temperatures, $T_a(v)$.

Atmospheric Temperature Sounding

This implies that if we know $W(v,z)$ over some range of v, then we can use satellites to measure the radiances leaving the top of the atmosphere over this range of v and determine the atmospheric temperature profile.

Clearly we need to understand the behavior of $W(v,z)$ a bit better. $W(v,z)$ has a strong vertical dependence as long as the atmospheric constituent absorption line is broadened by collisional broadening. Doppler broadened lines have a weak altitude dependence through the dependence on square root of temperature that is not terribly useful for extracting vertical information from downward looking observations. Collisional broadening has a pressure dependence that varies exponentially with altitude.

Another key point is that the absorption coefficient depends on line strength and line strength depends on the number density of absorbing molecules, n_{abs}, which can be written as $n_{tot}\, n_{abs}/n_{tot}$ where n_{tot} is the total number density of the gas. If we know pressure and temperature, then we know n_{tot} via the ideal gas law or more generally the equation of state. Therefore what we need to know is n_{abs}/n_{tot} which is the mixing ratio of the absorbing species. To make profiling atmospheric temperature relatively simple and straightforward, we want to use gases that are well mixed with constant and well-known mixing ratios that have nice absorption line features. The two best examples in the Earth's atmosphere are O_2 with lines near 60 GHz and a single line near 118 GHz and CO_2 with rotational-vibrational bands in the IR. Note that CO_2 is changing with time so there is an issue here. In the present microwave context, O_2 is the gas solution.

A final point is these lines cannot overlap spectrally with other lines or else interpretation of the radiances would be ambiguous.

So now let's look at the behavior of the collisionally broadened line as a function of the collisional linewidth with depends linearly on pressure which decreases approximately exponentially with altitude.

The Lorenz line shape is given as

$$f(\upsilon) = \frac{1}{\pi} \frac{\upsilon_L}{(\upsilon - \upsilon_0)^2 + \upsilon_L^2} \tag{13}$$

where

$$\upsilon_L = \frac{1}{2\pi t_c} \sim \frac{PA_c}{\sqrt{mk_BT}} \tag{14}$$

To gain some insight into the dependence of $f(v)$ on frequency consider two simple cases.

First, the frequency is the line center frequency. In this case, $v - v_0 = 0$ and

$$f(\upsilon) = \frac{1}{\pi \upsilon_L} \sim \frac{\sqrt{mk_BT}}{\pi PA_c} \tag{15}$$

which scales inversely with pressure.

Second, the frequency, v, is well off line center such that $v - v_0 \gg v_L$ in which case

$$f(\upsilon) \cong \frac{1}{\pi} \frac{\upsilon_L}{(\upsilon - \upsilon_0)^2} \tag{16}$$

and $f(v) \sim P$ (for fixed v) (actually $P/T^{1/2}$).

Now remember, the absorption coefficient, $\alpha = k_{v,v}$ which is

$$k_{v,v} = S_{v,v} f(v - v_0) \tag{17}$$

where

$$k_{v,v} = S_{v,v} f(v - v_0) = n \frac{n_m}{n} \frac{g_i \exp(-E_i/kT)}{Z} \frac{C_{ij}}{c} \left[1 - e^{-hv_{ij}/kT}\right] f(v - v_0) \tag{18}$$

where n is the total number density of the gas $= P/k_BT$ and n_m/n is the volume mixing ratio of constituent m of the gas.

For a well mixed constituent like O_2 where the mixing ratio is constant, and well off line center so the line shape portion of the absorption coefficient, $f(v-v_0)$, scales as $P/T^{1/2}$ then the absorption coefficient scales as $P/T \, P/T^{1/2} = P^2 \, T^{-3/2}$ plus the T scaling in the $exp()$ terms. Since pressure changes approximately exponentially with altitude with a scale height of H_P, P^2 changes exponentially with a scale height of $H_P/2$. This rapid scaling is helpful because it makes the weighting function more sharply defined vertically meaning the observations have more vertical information or resolution.

Consider the variation of the volume absorption coefficient, $k_{v,v} = \alpha$ for O_2 across the troposphere and stratosphere. The pressure dependence causes $k_{v,v}$ (far from line center) to vary by $(1000mb/1mb)^2 = 10^6$. The temperature variation from 200 to 300 K causes variations via the $T^{-3/2}$ term of about $\pm 30\%$. So clearly the pressure effect dominates but the temperature variations must be taken into account for accurate modeling of the weighting function and retrievals.

Now, following EvZ, let's assume $k_{v,v} = \alpha_a$ depends exponentially on altitude which is a reasonable first approximation based on our argument above.

$$\alpha(v,z) = \alpha_0 e^{-z/H} \tag{19}$$

where H here is the scale height of α which is $H_P/2$ if the frequency is well off the absorption line center frequency. Plugging (19) into (12) we get

$$W(v,z) = \alpha_0(v)e^{-z/H} \exp\left[-\int_z^\infty \alpha_0(v)e^{-z/H} dz\right] = \alpha_0(v)\exp\left[-z/H - \alpha_0(v)e^{-z/H}H\right]$$

$$W(v,z) = \alpha_0(v)\exp\left[-z/H - \tau_m e^{-z/H}\right] \tag{20}$$

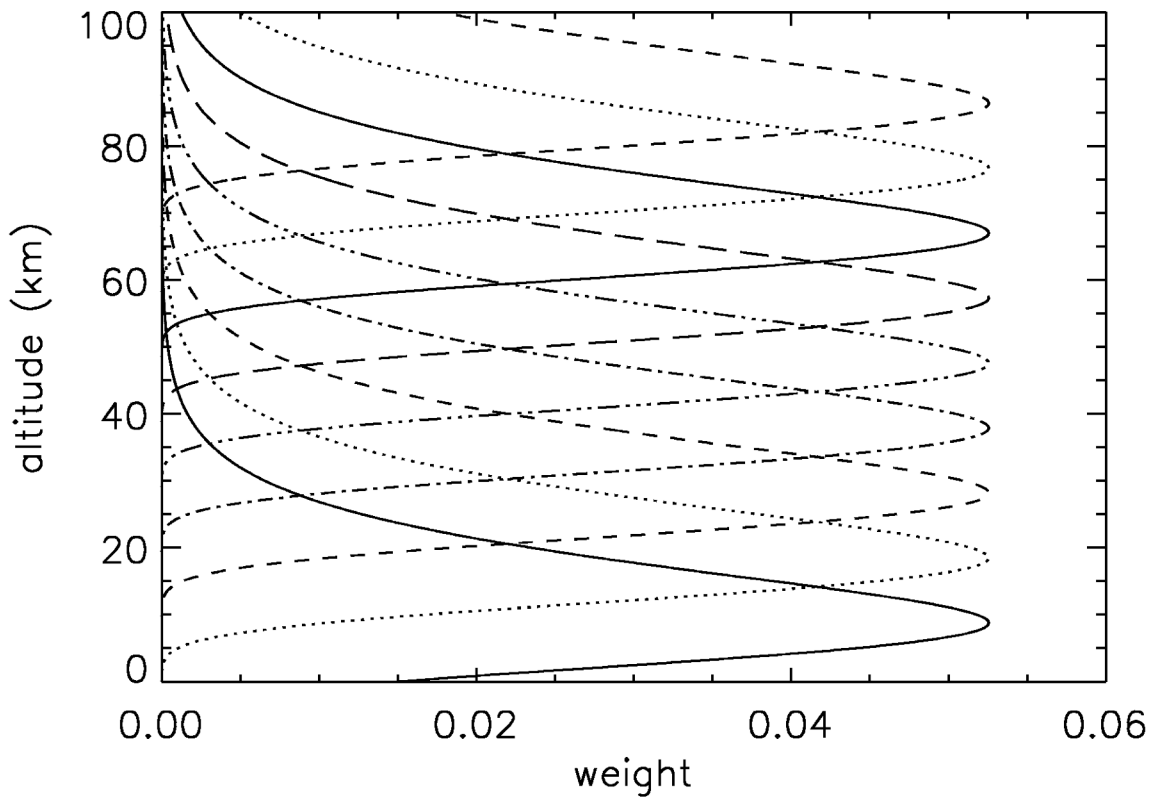where $\tau_m = \alpha_0(v)\,H$ which is the total optical depth across the vertical extent of the atmosphere.

Figure. Weighting functions for H=7 km.

The altitude of the peak of the weighting function occurs where $dW/dz = 0$:

$$\frac{dW(v,z)}{dz} = \alpha_0(v)\exp\left[-z/H - \tau_m e^{-z/H}\right]\left[-\frac{1}{H} + \frac{1}{H}\tau_m e^{-z/H}\right] \tag{21}$$

which occurs where $\tau_m = e^{z/H}$. Therefore

$$z_{\max}(v) = H\log[\tau_m(v)] \tag{22}$$

Changing the frequency, v, closer to line center causes $\tau_m(v)$ to increase, causing the peak of the weighting function to move up in altitude. Moving farther from line center such that $\tau_m(v)$ decreases, causes the peak altitude of the weighting function to decrease. Therefore, measuring

the radiance at the top of the atmosphere at a set of frequencies around a given line provides a range of vertical sampling that allows the vertical (temperature) structure of the atmosphere to be reconstructed (within limits).

There is another important point to be made here in terms of optical depth. As measured from the bottom of the atmosphere, the total optical depth across the atmosphere is $\tau_m(v)$. Consider the optical depth as viewed from above the atmosphere. At the top of the atmosphere the optical depth is 0. As we move down into the atmosphere, the optical depth increases approximately exponentially with decreasing altitude. The optical depth from the top of the atmosphere to the peak of the weighting function is

$$\tau\left(v, z_{max} - \infty\right) = -\int_{z_{max}}^{\infty} \alpha_0(v) e^{-\zeta/H} d\zeta = \alpha_0(v) e^{-z_{max}/H} H = \alpha_0(v) e^{-H \log(\tau_m)/H} H$$

$$= \alpha_0(v) e^{-\log(\tau_m)} H = \frac{\alpha_0(v) H}{\tau_m} = \frac{\alpha_0(v) H}{\alpha_0(v) H} = 1 \qquad (23)$$

So the altitude of the peak emission is the altitude where the optical depth as measured from the top of the atmosphere is 1. This is a very important point in developing an understanding about optical depth. If you are at any point in the atmosphere and you want to know where the peak of the emission you are observing is coming from, it is coming from the points with an optical depth of ~1 relative to where you are (assuming optical depths over larger distances are larger than 1). This is a general property of optical depths and radiative transfer.