Atmospheric Temperature Retrievals

Following the Clive Rodgers text…

As we have shown, the radiance, $L(v)$, leaving the top of the atmosphere at frequency, v, can be written as

$$L(v) = \int_0^\infty B\big(v, T(z)\big) \frac{dX(v,z)}{dz} dz \qquad (1)$$

where X is the atmospheric transmittance. Consider a set of m measurements of radiance, $L(v_i)$, made at m closely spaced frequencies. The frequencies are close enough that the frequency dependence of the Planck function can be ignored. Under these conditions,

$$L_i = L(v_i) = \int_0^\infty B\big(\bar{v}, T(z)\big) K_i(z) dz \qquad (2)$$

where $\bar{v}$ is a representative frequency and $K_i(z) = dX(v_i,z)/dz$.

The equation is linear in B which is the unknown when we are determining the profile of atmospheric temperature. So each radiance L_i is a weighted mean of the Planck function profile with $K_i(z)$ as the weighting function.

Solving or inverting the equation will cause some problems in part because there are only m observations. We therefore write the unknown temperature structure in terms of a set of m basis functions, A.

$$B\big(\bar{v}, T(z)\big) = \sum_{j=1}^{m} w_j A_j(z) \qquad (3)$$

where w_j is a set of coefficients still to be derived from the observations and A_j is a set of functions such as polynomials, z^{j-1}, or sines and cosines, $\sin(\pi jz/Z)$ and $\cos(\pi jz/Z)$, over a finite height range $(0,Z)$ in terms of which the profile is being represented. Subbing (3) into (2) yields

$$L_i = L(v_i) = \sum_{j=1}^{m} w_j \int_0^\infty A_j(z) K_i(z) dz = \sum_{j=1}^{m} C_{ij} w_j \qquad (4)$$

where

$$C_{ij} = \int_0^\infty A_j(z) K_i(z) dz \qquad (5)$$

So C_{ij} is the ith weighting function times the jth vector of the basis set integrated over all altitudes. Each element can be calculated as long as we use a constituent whose mixing ratio is independent of altitude (like O_2 or CO_2). We now have m equations and m unknowns that, in theory, can be solved exactly.

Formal Exact Solution:

From (4), we have the matrix equation, $\boldsymbol{L} = \boldsymbol{C}\,\boldsymbol{w}$, so

$$\boldsymbol{w} = \boldsymbol{C}^{-1}\boldsymbol{L} \qquad (6)$$

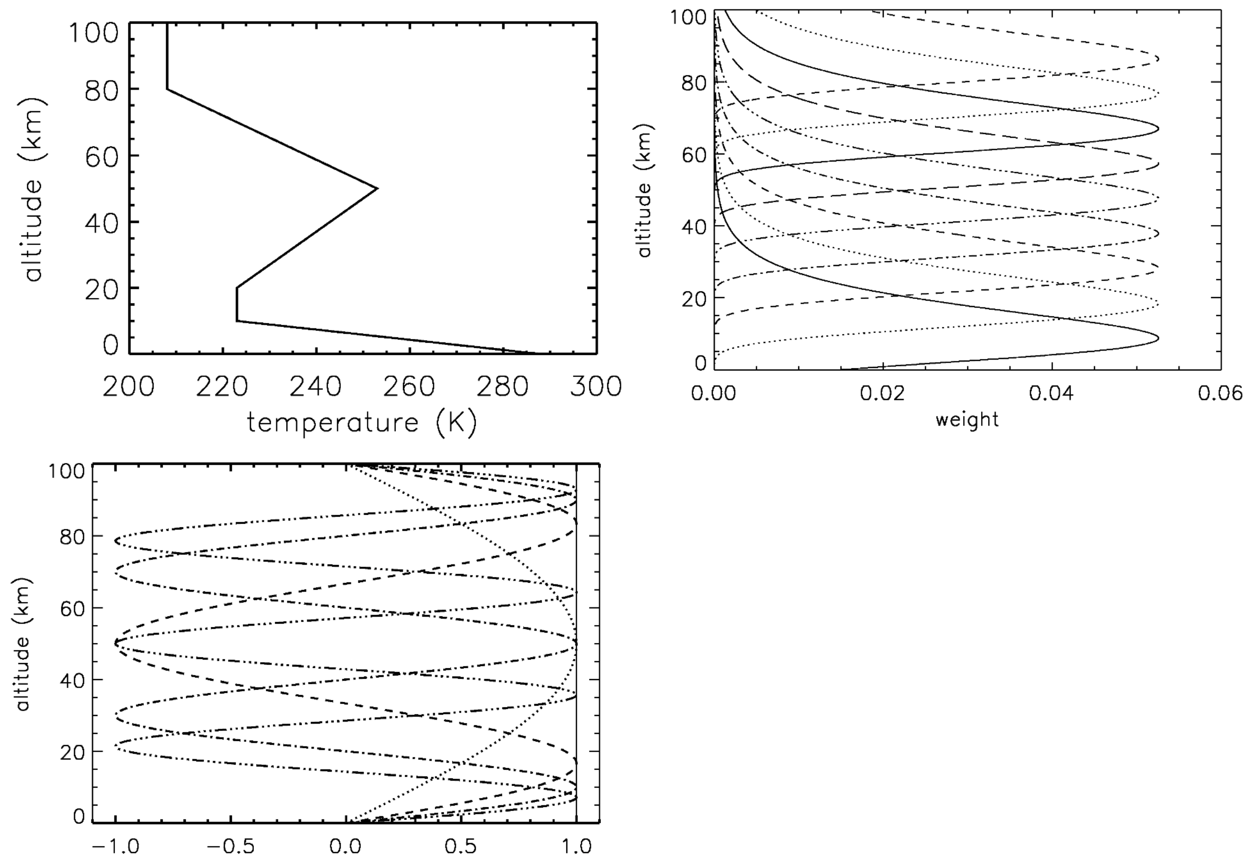
This shows that the set of coefficients that are multiplied times the set of basis functions to describe T are the inverse of the C_{ij} matrix multiplied times the vector of radiance measurements. Plugging this in for the w_j's, we determine the reconstructed temperature profile as follows

$$B(\bar{v},T(z)) = \sum_{j=1}^{m} w_j A_j(z) = \sum_{j=1}^{m} A_j(z) \sum_{i=1}^{m} C_{ji}^{-1} L_i = \sum_{j=1}^{m} G_i(z) L_i \qquad (7)$$

While this looks reasonable, the problem is equation (7) is Ill-conditioned: Any experimental error in the L measurements is greatly amplified in the solution via the inverse C matrix.

EXAMPLE

Consider the following example. Panel A shows the temperature profile. Panel B shows the 9 weighting functions. Panel C is the set of 9 basis functions, A, which in this case are sines and cosines (only the sines are shown)



There are measurements of upward radiance at 9 frequencies. There are 9 basis functions, specifically 4 sines, 4 cosines and one DC or constant term. The reconstruction is shown below in Figure 2. The errors in the reconstruction are shown in Figure 3.

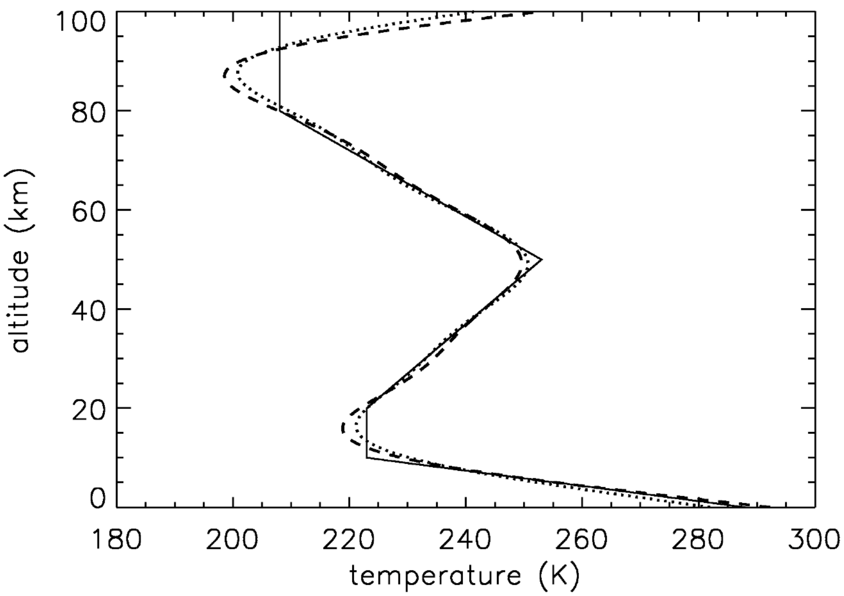


Figure 2. Temperature vs. altitude. The solid line is original temperature. The dotted line is reconstruction based on measurements without error. The dashed line is a reconstruction based on measurements that contain errors with a standard deviation of 0.5 K.

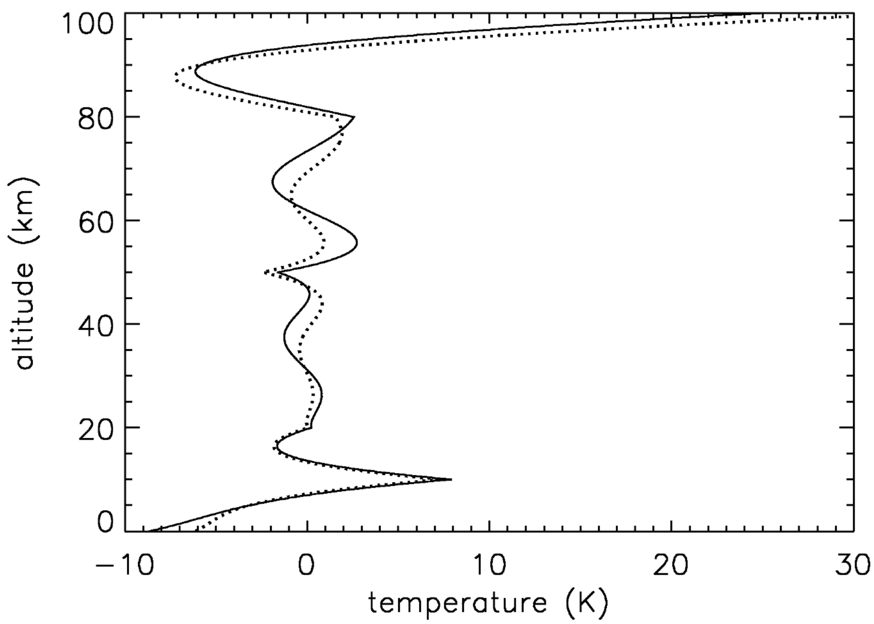


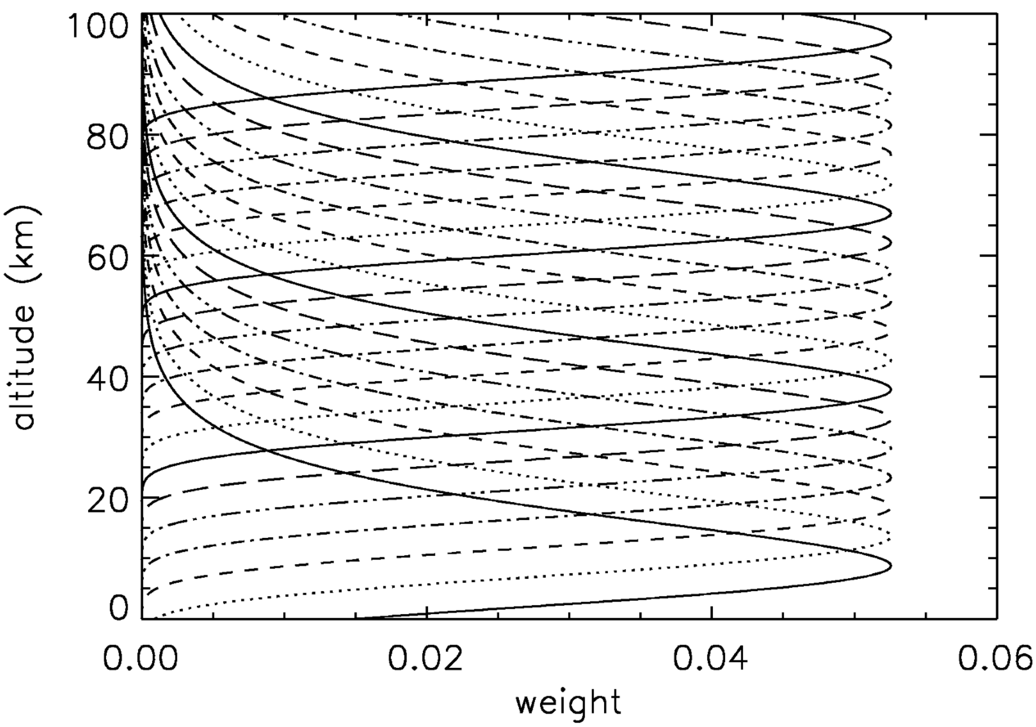
*Figure 3. Temperature reconstruction ERRORS vs. altitude. The solid line represents the errors in the temperature reconstruction based on measurements containing errors with standard deviation of 0.5 K. The dotted line represents the errors in reconstruction based on measurements **without** error.*

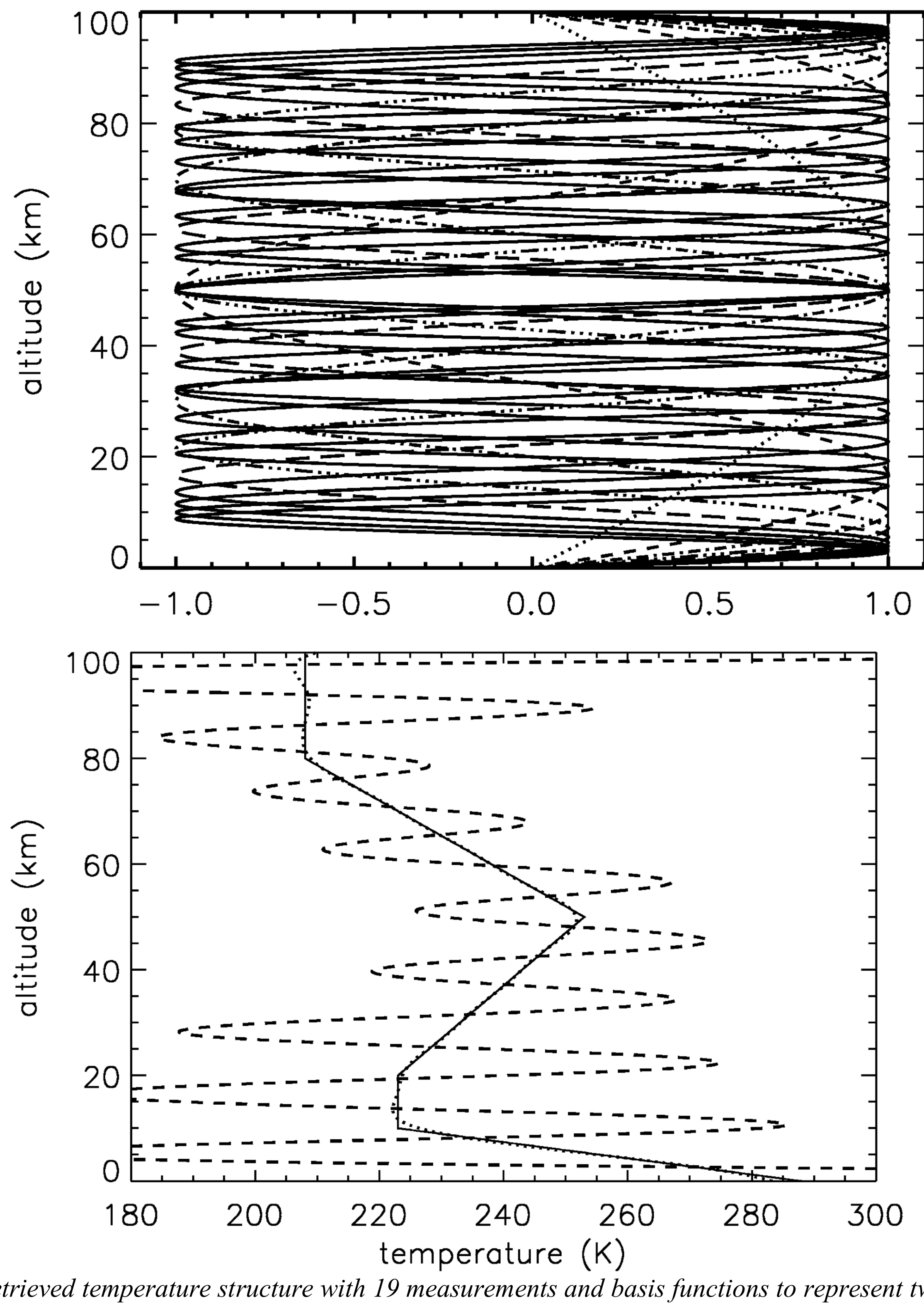
The standard deviation of the errors in the temperature reconstruction done using measurements ***without*** any errors is 4.2K whereas the errors in the temperature reconstruction based on measurements with errors of 0.5K is 5.4K.

Improving the vertical resolution

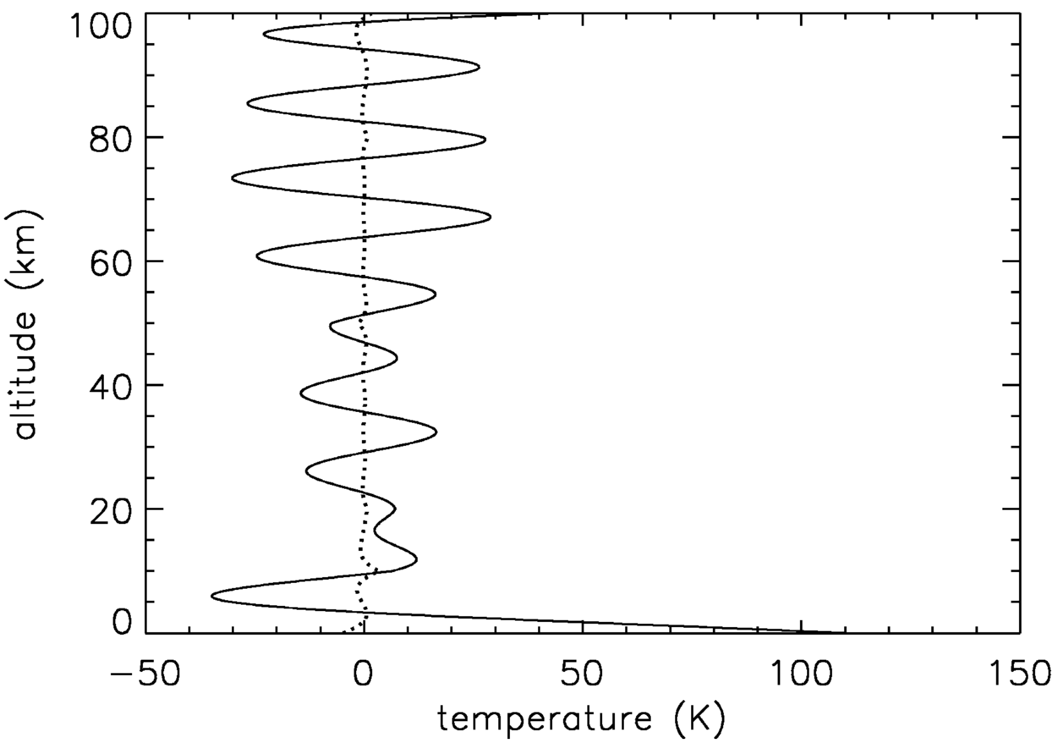
We'd like to improve the vertical resolution. The obvious way to accomplish this is to add additional measurements of radiances at more frequencies. So let's double the number of radiance measurements. This example addresses a question raised by Prof. Zeng in a Atmospheric Science lecture I gave in 2008. I stated that the nadir viewing, microwave weighting functions were vertically too coarse to see the planetary boundary layer very well. He suggested that one can add more measurements at more frequencies to gain vertical resolution. My response was that the problem is noise amplification. We shall see…

So we increase the number of radiance measurements and as the number of basis functions to 19 and do the temperature retrieval again…





Retrieved temperature structure with 19 measurements and basis functions to represent twice as many levels in the atmosphere



Error in retrieved temperature structure with 19 measurements and vertical levels in the atmosphere for no noise added to the measurements (dotted line) and 0.5 K noise added to the measurements.

We see a couple of things in these temperature error plots.

(1) the standard deviation of the error in the temperature reconstruction derived from ***error-free*** measurements has ***improved*** to 0.67K from 4.2K in the previous case with half as many measurements and basis functions.

(2) However, we also see a strong problem where the temperature reconstruction has become much more sensitive to the noise in the measurements (which is still 0.5 K standard deviation) that has resulted in an error standard deviation of 45K! This level of error indicates the temperature reconstruction at this resolution is not useful.

Where is this noise amplification coming from? It is coming from the C_{ji}^{-1} matrix which is the inverse of the C_{ij} matrix:

$$C_{ij} = \int_0^\infty A_j(z) K_i(z) dz \qquad (5)$$

where A_j is the set of basis functions for representing the temperature profile and K is the set of weighting functions, one for each radiance measurement. The two figures below show the rows of the C_{ji}^{-1} matrix. Each column multiplies the radiances to obtain the jth w coefficient to multiply the jth basis function for representing $T(z)$.

$$B(\bar{v}, T(z)) = \sum_{j=1}^{m} w_j A_j(z) = \sum_{j=1}^{m} A_j(z) C_{ji}^{-1} L_i = \sum_{j=1}^{m} G_i(z) L_i \qquad (7)$$

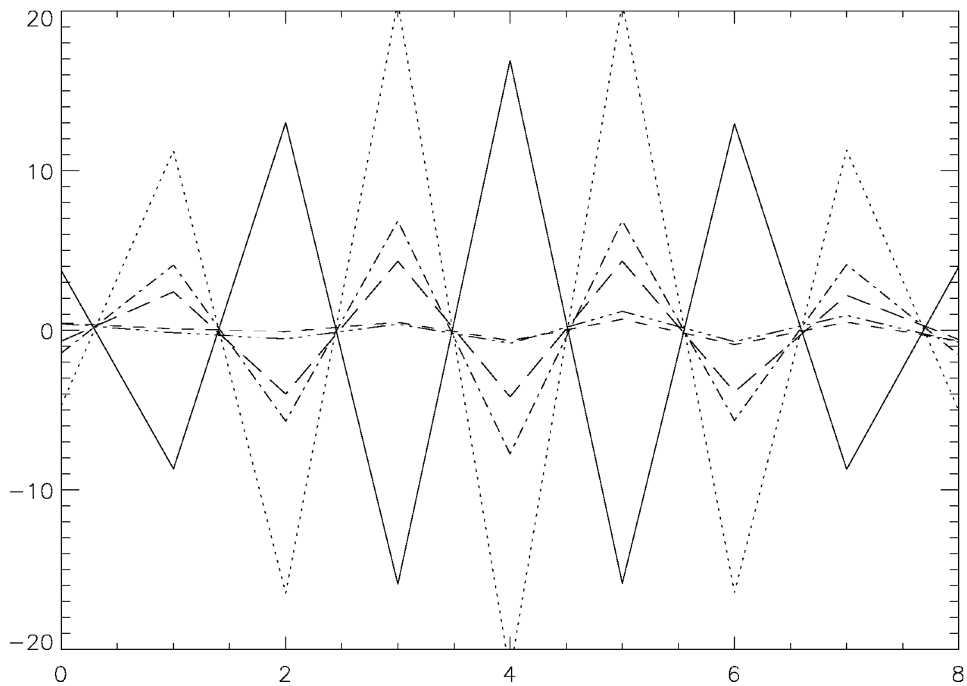


Figure showing the C_{ji}^{-1} for 9 radiances and 9 basis functions. Solid line shows the 9 components from the row 0 of C_{ji}^{-1} to estimate the coefficient of basis function 0. Dotted line shows row 1. Dashed line shows row 2. (Done in IDL using "linestyle")

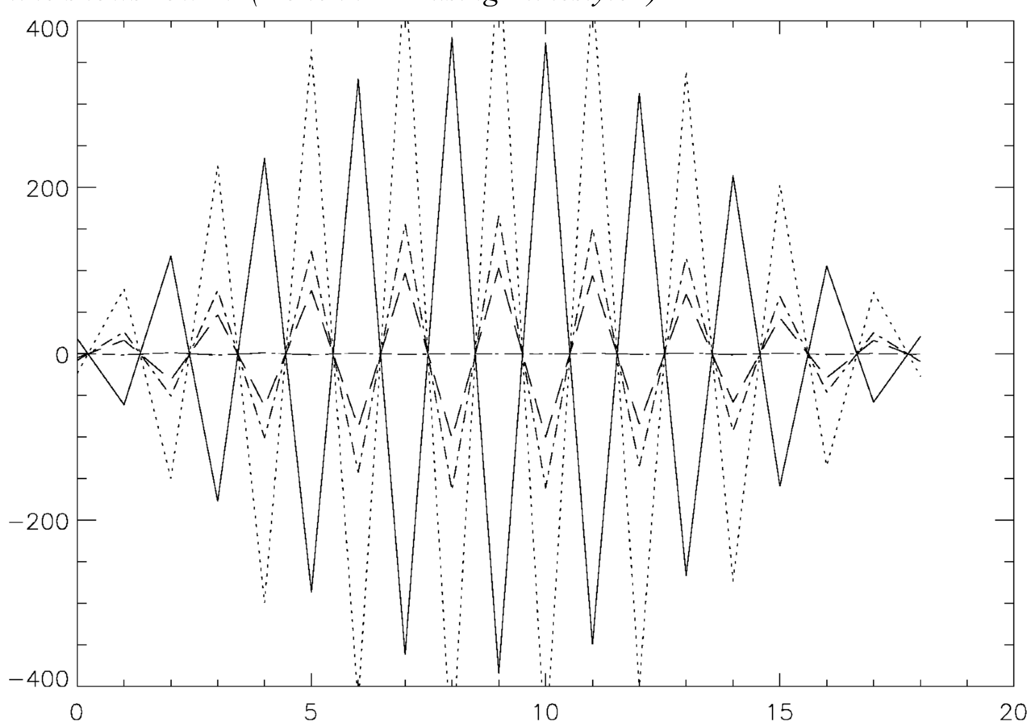


Figure showing the C_{ji}^{-1} for 19 radiances and 19 basis functions over the same vertical interval as the previous figure. Solid line shows the 9 components from the row 0 of C_{ji}^{-1} to estimate the coefficient of basis function 0. Dotted line shows row 1. Dashed line shows row 2. (Done in IDL using "linestyle")

Notice that the C_{ji}^{-1} terms span from about -20 to +20 in the case of 9 radiances whereas the span has grown to -400 to +400 when the number of radiances and vertical levels in the reconstruction were increased by approximately a factor of 2 to 19. The dramatic increase in the C_{ji}^{-1} entries is responsible for the dramatic growth in the errors between the two cases. The amplification grows greatly when the resolution becomes finer than the width of the weighting function because at that point you are deconvolving the weighting function which involves differencing consecutive weighting functions to create finer vertical structure.

So trying to extract higher vertical resolution from these nadir viewing measurements can be done but only at a cost of dramatically higher amplification of measurement noise. The reason is that the relatively coarse vertical resolution of the individual weighting functions simply don't provide the information needed to derive much finer resolution, ***UNLESS***, the measurements come with *very* high signal to noise ratio (SNR). Note the SNR of such measurements is limited by the temperature of the thermal source which determines how much energy the source is radiating, the equivalent noise temperature of the instrument and the integration time of the measurement. Integration times for observations from polar orbiting satellites tend to be short because they are moving at ~7 km/sec.

So this is why I answered Prof. Zeng comment saying you can't just add more radiance measurements to increase the vertical resolution because it will dramatically amplify the measurement noise. Note that a geosynchronous satellite would have the advantage of potentially being able to integrate longer depending on the horizontal resolution and update time and might therefore achieve somewhat better vertical resolution through increased integration time and SNR.

The reconstruction is exactly fitting the noise which is bad unless the noise is very small.

The key point is there is insufficient information in the nadir viewing radiance observations to determine a unique atmospheric profile unless it is quite smoothed vertically.

So we have to develop another approach: On to Bayesian retrievals…