Lecture 3 E&M

This set of notes is not meant to be comprehensive but simply supplement EvZ Chapter 2.
.

    Electromagnetic spectrum
    Maxwell's equations (use EvZ)
    Wave equation
    EM wave Polarization
       Stokes parameters
    (Temporal) Coherency
    Phase and group velocity
    Doppler
    Detection:
       Instrument: Aperture diffraction-limited resolution
    Definition of radiation quantities
       Irradiance = radiant energy flux. Radiance vs emittance (Elachi p. 289)
    Sources: Creation of EM waves: manmade and natural BB
    Interaction with matter

Units: wavenumber cm^{-1}

Wave equation

    In homogeneous, isotropic and nonmagnetic media with no electric charge and no conductors, Maxwell's equations can be combined to give the wave equation

$$\nabla^2 \vec{E} - \mu_0 \varepsilon_0 \mu_r \varepsilon_r \frac{\partial^2 \vec{E}}{\partial^2 t} = 0$$

where μ_0 is the permeability of vacuum and ε_0 is the permittivity of vacuum and μ_r and ε_r are the relative permeability and permittivity of the medium through which the light is passing. For a sinusoidal solution, the wave equation can be written as

$$\nabla^2 \vec{E} + \frac{\omega^2}{c_r^2} \vec{E} = 0$$

where the speed of light in the medium, c_r, is

$$c_r = \frac{1}{\sqrt{\mu_0 \varepsilon_0 \mu_r \varepsilon_r}} = \frac{c}{\sqrt{\mu_r \varepsilon_r}}$$

and c is the speed of light in a vacuum. The index of refraction is

$$n = \frac{c}{c_r} = \sqrt{\mu_r \varepsilon_r}$$

Kursinski 01/28/09

In most media, $\mu_r = 1$ (for nonmagnetic media). ε_r ranges from 1 to 80 and depends on frequency and temperature. The high end, near 80, is for liquid water at microwave frequencies.

Sinusoidal solution

The simplest solution to the wave equation is a transverse plane wave written here as propagating in the +z direction,

$$\vec{E} = \vec{A}e^{i(kz-\omega t+\phi)}$$

where the electric field vector is in the x-y plane.

Remote sensing of atmospheric gas concentrations and temperature structure probes the frequency dependent absorption/emission spectra of the gases.

Polarization refers to the orientation of the electric field vector and can be used to infer properties of the atmospheric particulates like rainfall rate. Polarization can be used to infer ocean surface roughness to infer near-surface wind speed from passive emission measurements and near-surface wind speed and direction from scatterometers (a form of radar).

Radars and lidars measure the Doppler shift of signals backscattered from atmospheric particulates and inhomogeneities in the index of refraction to determine velocities of the objects.

GPS radio occultations utilize the signal phase to very precisely measure the time rate of change of the signal path length as the signal path descends through the atmosphere during an occultation to infer atmospheric bending angle and profile the atmospheric index of refraction.

GRACE measures changes in signal phase to determine very subtle changes in the distance between the two GRACE spacecraft associated with gravity pulling on the spacecraft as it flies over a mass structure like the Greenland ice sheet.

Altimeters measure group delay, the time between the time a modulated signal is transmitted and the time the reflected signal is received, to determine the varying height of the ocean surface.

Temporal coherency

Note importance of coherent vs incoherent summation discussed in EvZ p. 31

Phase and Group Velocity

The phase velocity is the velocity of the phase of a sine wave. Based on the $e^{i(kr-\omega t)}$ term, the phase is $kr-\omega t$. The speed at which surface of constant phase or phase front propagates is

$$\frac{d(kr-\omega t)}{dt} = 0 = k\frac{dr}{dt} - \omega$$

such that the phase velocity is

$$v_p = w/k \qquad\qquad (3.1)$$

   The group velocity, v_g, is the speed of any modulation on a sine wave and the speed of energy propagation. The distinction between these two can be important for radar propagating through the ionosphere (and atmospheric and ocean waves).

   The standard approach for explaining the difference between phase and group velocity of electromagnetic waves is to consider the sum of two equal amplitude, monochromatic signals with similar but not identical frequencies.

$$A(t) = Ae^{i([k+\Delta k]r - [\omega + \Delta \omega]t)} + Ae^{i([k - \Delta k]r - [\omega - \Delta \omega]t)} \tag{3.2}$$

We make use of $e^{i\phi} = \cos(\phi) + i\sin(\phi)$ and the trigonometric relations:

sin C + sin D = 2 sin 1⁄2(C + D) cos 1⁄2(C - D)
sin A - sin B = 2 cos 1⁄2(A + B) sin 1⁄2(A - B)
cos C + cos D = 2 cos 1⁄2(C + D) cos 1⁄2(C - D)
cos A - cos B = - 2 sin 1⁄2(A + B) sin 1⁄2(A - B)

Eqn (3.2) becomes

$$A(t) = A\Big[\cos([k + \Delta k]r - [\omega + \Delta \omega]t) + i\sin([k + \Delta k]r - [\omega + \Delta \omega]t)\Big]$$
$$+ A\Big[\cos([k - \Delta k]r - [\omega - \Delta \omega]t) + i\sin([k - \Delta k]r - [\omega - \Delta \omega]t)\Big] \tag{3.3}$$

Setting C = $(k+\Delta k)$r $-(\omega+\Delta\omega)t$ and D = $(k-\Delta k)$r $-(\omega-\Delta\omega)t$ then $(C+D)/2 = (kr - \omega t)$ and $(C-D)/2 = \Delta k\, r - \Delta\omega\, t$. Plugging these relations into the eqn (3.3)

$$A(t) = A\Big[\cos(C) + i\sin(C) + \cos(D) + i\sin(D)\Big] = 2A\left[\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right) + i\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)\right]$$

$$A(t) = 2Ae^{i\left(\frac{C+D}{2}\right)}\cos\left(\frac{C-D}{2}\right) = 2Ae^{i(kr-\omega t)}\cos(\Delta kr - \Delta\omega t)$$

   So the sum of the two signals is a signal at the mean frequency and wavenumber of the two signals with twice the amplitude of the individual signals multiplied by a modulation term whose wavenumber and frequency are the difference between those of the two signals.

   The propagation speed of the $e^{i(kr-wt)}$ term is ω/k which is the phase velocity (3.1). The propagation speed of the modulation term is $\Delta\omega/\Delta k$ which can be generalized to

$$v_g = \frac{\partial \omega}{\partial k} \tag{3.4}$$

which is called the group velocity. If the medium is nondispersive, then $\omega = ck$ and the two velocities are equal to c. If the medium is dispersive so ω is a nonlinear function of k then the two velocities are different. In the ionosphere, the phase velocity is greater than the speed of light and the group velocity which is the speed that information and energy can be transferred, is less than the speed of light.

Kursinski 01/28/09

Doppler shift

Motion causes a signal's frequency to shift. This is useful for measuring motion and determines the linewidth of absorption lines at high altitudes in the atmosphere.
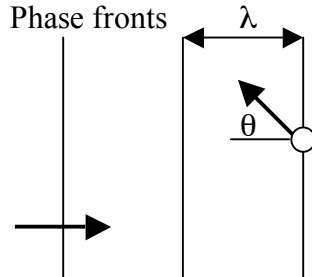
At time t=0, a phase front passes the observer. The time the next phase front passes the observer is T' such that

$$\lambda' = cT' = \lambda - v\cos\theta\ T'.$$

where v is the speed of the observer. The frequency measured by the observer, f', is $1/T'$. So
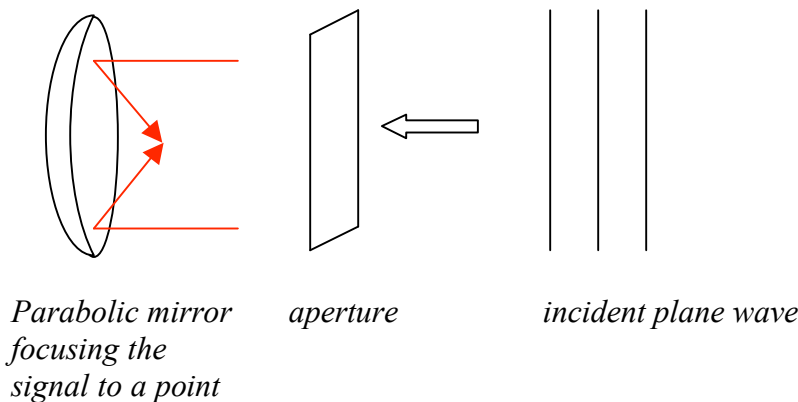
$$c/f' = \lambda - v\cos\theta/f'.$$

$$f' = c/\lambda + v\cos\theta/\lambda = f + v\cos\theta\ f/c = f(1 + v\cos\theta/c) = f + f_d.$$

where the Doppler shift in the signal frequency is $f_d = f\,v\cos\theta/c$. The Doppler shift is positive if the observer is moving towards the EM signal source.

**Instrument: Resolution limit set by Aperture Diffraction**

A key feature of any instrument is the resolution it can produce, that is it ability to resolve features in whatever object is being remotely sensed. A key part of resolution is the resolution limit associated with aperture diffraction.

The combination of the size of the aperture and the wavelength, λ, of an instrument determine its aperture diffraction. This can be understood as follows. Suppose we have a simple square aperture of size D x D where D is the length of a side of the square aperture.

Parabolic mirror   *aperture*   *incident plane wave*
focusing the
signal to a point

Kursinski 01/28/09

Now consider a plane wave incident on the aperture. The optics behind the aperture collects all of the light that passes through the aperture at a single point, focus. The aperture and following optics are designed such that the distance from any point on a plane wave to the focus is identical for all light passing through the aperture in the direction orthogonal to the aperture plane. The focusing optics could be a parabolic reflector.

As long as the plane wave is parallel to the aperture plane and propagating perpendicular to it, the distance of any point along the phase front to the detector is identical. Therefore all of the light that passes through the aperture will add coherently and the detector at the focus will see the full expected signal power.
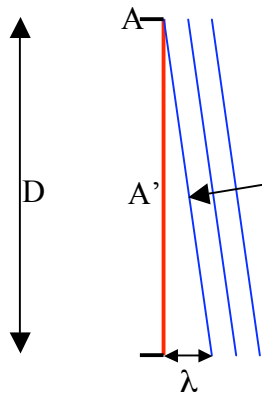
Now consider the case where the plane wave propagation direction forms a slight angle, α, with the orientation of one of the sides of the aperture. Now the distance from any point along a phase front to the detector is no longer identical.

Now consider a specific angle, $\alpha = \lambda/D$. Now divide the aperture into two equal halves. Consider the contribution of the plane wave passing through the 2 halves. Consider the light at points A and A' on the aperture. A is at the aperture edge and A' is at the midpoint of the aperture. Note that the phase of the light at A' is one half wavelength behind that of the light at point A. That means that when these two contributions of light pass through the optics and reach the detector they will be exactly a half cycle out of phase and will cancel.

$$A \cos (\phi) + A \cos (\phi + \pi) = A \cos (\phi) - A \cos (\phi) = 0 \qquad (1)$$

When $\alpha = \lambda/D$, the contributions of light passing through two points, x and x' along the aperture where x' = x + D/2, will cancel.

So for a square aperture, when the incident plane wave is tilted by an angle $\alpha = \lambda/D$ relative to the boresight direction of the antenna aperture, no net light reaches the detector. This 0 angle is called the first null of the antenna and sets the approximate aperture diffraction resolution of the antenna.

LEO Example

Consider a satellite orbiting the Earth at 800 km altitude (= z). The downward looking antenna aperture is 1 m in diameter and the instrument wavelength is 1 cm. What is the approximate spot size (defined by the aperture diffraction) that the instrument can resolve on the Earth's surface looking straight down.

The spot size is approximately $\lambda/D * z = 0.01m/1m * 800$ km = 8 km.

Geosynchronous Examples

The spot size for the same instrument in geostationary orbit would be 0.01m/1m * 36,000 km = 360 km. This very large spot size is a key reason why there are as yet no microwave instruments in geosynchronous orbit.

Now suppose the wavelength were 1 micron (near-IR) and the aperture were 1 cm in diameter, then the aperture diffraction limited spot size would be 10^{-6} m/0.01m * 36,000 km = 3.6 km. This is a useful resolution.

The GOES atmospheric sounder instrument observes the Earth in an IR window channel at about 10 μ wavelength. "Window" means there are no significant gas absorption lines in the channel. Its resolution on the surface is about 4 km. Therefore the diameter of the aperture is about λ z / spot = 10^{-5} m * 3.6×10^7 m/4×10^3 m = 0.1 m = 10 cm = 4 inches.

Notice that the angle in the last case was 10^{-4} radians or 6 millidegrees. This is quite small and creates challenges for knowing where exactly your instrument is pointed.

Consider a microwave sounder in geosynchronous orbit profiling temperature via the 118 GHz band of O_2. The wavelength is about 2.5 mm. If you want a 4 km diameter footprint, then the diameter of the aperture is about λ z / spot = 2.5×10^{-3} m * 3.6×10^7 m/4×10^3 m = 23 m or one quarter the width of a football field. Nontrivial to put such a large aperture in space and point it accurately!

Signal Detection

Frequency and Wavelength Units

The obvious units for wavelength spanning IR, visible and UV is micrometers or microns. Wavelengths over these bands span about 100 μ to less than 0.03 μ. Visible wavelengths run from 0.4 to 0.7 μ. Microwave wavelengths tend to be given in cm or mm.

In terms of frequency, microwave signal frequencies tend to be given in MHz (megahertz) = 10^6 Hz or GHz (gigahertz) = 10^9 Hz. Today, instruments are being built between microwave and IR with sub-mm wavelengths at frequencies of THz (terahertz) = 10^{12} Hz. This is an interesting band full of absorption lines that are stronger than at lower frequencies.

At IR and shorter wavelengths, frequencies are extremely high and a different unit is used, called wavenumber, $\tilde{\upsilon}$, in cm^{-1}. This unit is the inverse of the wavelength expressed in cm. This is proportional to frequency because $1/\lambda = \nu/c$. As an example, consider a wavelength of 10μ, right in the middle of the Earth's BB emission spectrum. The wavelength is 10 μ = 10^{-3} cm. So the wavenumber, the frequency is 1000 cm^{-1}. A wavelength of 1 mm has a wavenumber of 10 cm^{-1} and a frequency of 300 GHz.

Be careful about the use of wavenumber. In the propagating electric field equation, wavenumber means $2\pi/\lambda$. But in this spectroscopic case it is used to mean $1/\lambda$.