RADAR Sensing of Precipitation

Precipitation radars typically operate between 2 and 8 GHz (wavelength: 15 to 4 cm). There are also cloud radars like the one on CloudSat that operate around 94 GHz (wavelength: 3 mm).

The general objective of precipitation radars is to measure the backscattered power and then relate it to the rain rate. Doing so also requires determining the particle size distribution and the fall velocity.

Backscatter crosssection

Precipitation radars generally operate in the Rayleigh regime where the particle size is much smaller than the wavelength ($a << \lambda$). In the Rayleigh regime, the (electromagnetic) backscatter efficiency, Q_b, is given as

$$Q_b = \frac{\sigma_b}{\pi a^2} = 4x^4\left|\frac{m^2-1}{m^2+2}\right|^2 = 4x^4\left|\frac{\varepsilon-1}{\varepsilon+2}\right|^2 = 4\left(\frac{2\pi a}{\lambda}\right)^4\left|\frac{m^2-1}{m^2+2}\right|^2 = 64\pi^4\left(\frac{a}{\lambda}\right)^4|K|^2 \quad (1)$$

where a is the particle radius, x is $2\pi a/\lambda$, $K=(m^2-1)/(m^2+2)$ and m is the complex index of refraction of the scattering particle. The backscatter crosssection is therefore given as

$$\sigma_b = \pi a^2 4x^4\left|\frac{m^2-1}{m^2+2}\right|^2 = \pi a^2 4\left(\frac{2\pi a}{\lambda}\right)^4\left|\frac{m^2-1}{m^2+2}\right|^2 = 64\frac{\pi^5}{\lambda^4}a^6|K|^2 = \frac{\pi^5}{\lambda^4}D^6|K|^2 \quad (2)$$

where D is the particle diameter. The total backscatter is the sum over all backscattering particles

$$\sigma_{b-tot} = \sum_j \sigma_{b_j} = \frac{\pi^5}{\lambda^4}|K|^2\sum_j D_j^6 = \frac{\pi^5}{\lambda^4}|K|^2 Z \quad (3)$$

where Z = $\sum_j D_j^6$ with units of mm^6/mm^3 or in mm^3.

If there is a continuum of particles sizes then the reflectivity, Z, is expressed as an integral over the particle size distribution.

$$Z = \int N(D)D^6 dD \quad (4)$$

where $N(D)$ is the particle size distribution.

Another quantity of interest is the liquid water content expressed as a mass density like g/m^3.

$$M = \int_0^\infty \rho\frac{\pi D^3}{6}N(D)dD = \frac{\pi\rho}{6}\int_0^\infty D^3 N(D)dD \quad (5)$$

Radar equation

(Following *EvZ*…) Consider a pulsed radar that radiates a pulse of peak power, P_t, of duration, τ, through an antenna of area, A, and gain, G. The radiated power density, P_i, at a distance r from the antenna is

$$P_i = \frac{P_t G}{4\pi r^2} = \frac{P_t A}{\lambda^2 r^2} \tag{6}$$

Note that P_t is power whereas P_i is power per unit area (like W/m^2).

The backscattered power, P_s, is the incident power per unit area times the scattering crossectional area. The scattering crossectional area is the scattering crossection per unit volume, σ, times the volume that is scattering the particular radar pulse. The volume that is scattering is equal to propagation distance during the pulse length, τ, times the radar beam area, S, at distance, r, which, if the radar has a square aperture, is $(\lambda/d\ r)^2$ where λ is the radar wavelength and d is the length of one side of the square aperture or more generally $(\lambda r)^2/A$ where A is the area of the radar antenna. So the scattered power is therefore

$$P_s = P_i\ \sigma\ Vol_{scatt} = P_i \sigma \frac{c\tau}{2} S = P_i \sigma \frac{c\tau}{2} S = P_i \sigma \frac{c\tau}{2} \frac{\lambda^2 r^2}{A} \tag{7}$$

The power received by the radar antenna is then

$$P_r = \frac{P_s}{4\pi r^2} A = P_i \sigma \frac{c\tau\lambda^2}{8\pi} = \frac{P_t A}{\lambda^2 r^2} \sigma \frac{c\tau\lambda^2}{8\pi} = \frac{P_t A}{r^2} \frac{c\tau}{8\pi} \sigma = \frac{\pi^4}{8} \frac{P_t A c\tau}{r^2 \lambda^4} |K|^2 Z \tag{8}$$

Note that the actual power received will be reduced by absorption and scattering along the path of length, r.

Hydrometeor Fall Rates

To measure rain, we must measure both the amount of condensed water in the atmosphere and its rate of descent through the atmosphere. In theory, these small droplets fall at their terminal velocities because it takes little time for them to accelerate under the force of gravity until an equilibrium is established between the downward gravitational force and the upward directed aerodynamic drag. Terminal velocity, V_t, is the velocity when these two forces sum to zero.

The gravitational force is proportional to the drop mass, m, and therefore D^3, while the frictional force is proportional to the cross-sectional area A of the drop, hence the 2nd power of D. The force balance is as follows:

$$mg = \rho_{h2o} \frac{\pi D^3}{6} g = C_d \rho_{air} V_t^2 A = C_d \rho_{air} V_t^2 \frac{\pi D^2}{4} \tag{9}$$

where g is the gravitational acceleration, ρ is mass density, A is the crossectional area of the droplet, and C_d the drag coefficient. The problem is the coefficient of drag is not a constant. If C_d were constant, then the fall speed would be given as

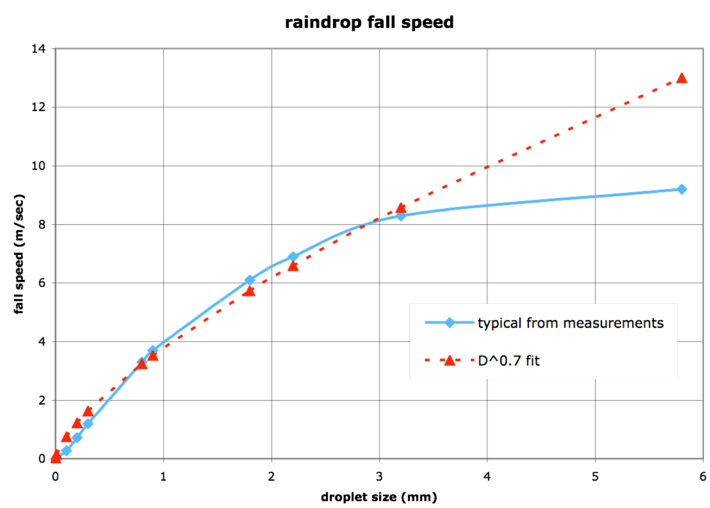
$$V_t = \sqrt{\frac{\rho_{h2o}}{\rho_{air}} \frac{2D}{3C_d} g} \tag{10}$$

such that it would increase as the square root of the particle diameter. However, the variations in C_d with particle size due to eddies as the air flows around the particle and distortions in the particle shape create more complex behavior. Actual behavior must be determined form measurements. Foote and Dutoit (1969) developed the following relationship for raindrop fallspeed, V_t (in m/s) as a function of D (in mm), for 0.1 mm < D < 6 mm (2):

$$V_t = [-0.193 + 4.96\,D - 0.904\,D^2 + 0.0566\,D^3]\exp(z/20) \tag{11}$$

The factor, $\exp(z/20)$, where z is height in km, accounts for the decrease in density (and hence drag) with height in the atmosphere. (The factor of 20 seems a bit large given that density scale heights in the troposphere are more like 10 km). Behavior is approximately linear up to drop diameters of 1 mm above which the increase in V_t with drop size begins to level off. Note that drops larger than 3 mm have a good chance of breaking up into smaller drops. The break-up probability increases rapidly at diameters around 5 mm such that we don't have to worry much about liquid droplets of much larger size. Much of this material is from http://www-das.uwyo.edu/~geerts/cwx/notes/chap09/hydrometeor.html by Bart Geerts at University of Wyoming. The table and figure below show typical behavior

Particle diameter (mm)	Fallspeed (m/s)
0.001	0.0003
0.01	0.03
0.1	0.27
0.2	0.72
0.3	1.2
0.8	3.3
0.9	3.7
1.8	6.1
2.2	6.9
3.2	8.3
5.8	9.2



The rainfall rate scales as rainfall mass times the rainfall descent velocity. The $V_t(D)$ behavior is closer to linear than $D^{1/2}$ at small sizes. The rainfall rate scaling is therefore something like $D^{3+0.75}$. So to measure rainfall with a radar, we want to measure something that depends on the particle size as $\sim D^{3.75}$. This presents a basic remote sensing problem because the radar backscatter which is the most obvious thing to measure depends on D^6. In fact, the D^6 dependence makes the backscattered power very sensitive to the largest particles in the particles size distribution.

Rain drop Size distributions

There have been many attempts to find a size distribution that is both realistic and mathematically tractable.

Mono-distribution

If all the particles are the same size,

$$Z = N\,D^6 \text{ and } M = \pi\rho/6\,N\,D^3 \tag{12}$$

and $Z \sim M^2$ or $M \sim Z^{0.5}$. (Unrealistic but a conceptually useful starting point.)

Exponential distribution

An exponential distribution has been used because of its simplicity and its qualitative consistency with large particles occurring less frequently than smaller particles.

$$N(D) = N_0 e^{-D/\overline{D}} \quad (13)$$

Plugging this distribution into the equations for Z and M we get

$$Z = 720 N_0 \overline{D}^7 \quad (14)$$

$$M = \pi \rho N_0 \overline{D}^4 \quad (15)$$

so $Z \sim M^{1.75}$ or $M \sim Z^{4/7}$. The problem is real distributions show fewer large drops as well as small drops than the exponential distribution.

<u>*Gamma distribution*</u>

A more complicated but more flexible and realistic distribution is the gamma distribution.

$$n(D) = N_0 D^{\mu} \exp(-\Lambda D) \; (0 < D < D_{max}) \quad (16)$$

N_0 has units of $m^{-(\mu+3)}$. Three examples of the gamma distribution are shown below *from Ulbrich, (1983) J. Clim & Applied Met.*. Note that the exponential distribution is a special case of the Gamma distribution with $\mu = 0$.

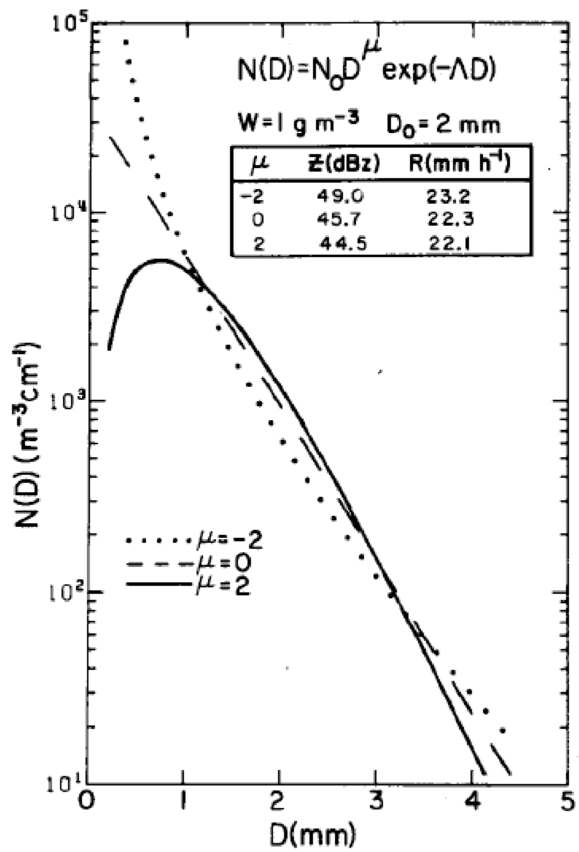


FIG. 2. Examples of the gamma raindrop size distribution for $\mu = -2$, 0 and 2 and with liquid water content $W = 1$ g m^{-3} and median volume diameter $D_0 = 2$ mm. The inset table shows the corresponding values of radar reflectivity factor (Rayleigh approximation) and rainfall rate.

The *n*th moment of a Gamma distribution is

$$\langle D^n \rangle = N_0\, \Lambda^{-(\mu+n+1)}\, \Gamma(\mu+n+1) \tag{17}$$

where Γ is the gamma function, which is the factorial function generalized for non-integer numbers. Therefore

$$\overline{D} = \int N(D) D^1 dD = \int_0^{D_{max}} N_0 D^{\mu+1} e^{-\Lambda D} dD = N_0 \frac{\Gamma(\mu+2)}{\Lambda^{\mu+2}} \tag{18}$$

$$M = \frac{\pi\rho}{6}\int_0^{\infty} D^3 N(D) dD = \frac{\pi\rho}{6} N_0 \frac{\Gamma(\mu+4)}{\Lambda^{\mu+4}} = \frac{\pi\rho}{6}\frac{\overline{D}}{\Lambda^2}\frac{\Gamma(\mu+4)}{\Gamma(\mu+2)} = \pi\rho\frac{\overline{D}}{\Lambda^2} \tag{19}$$

$$Z = \int N(D) D^6 dD = \int_0^{D_{max}} N_0 D^{\mu+6} e^{-\Lambda D} dD = N_0 \frac{\Gamma(\mu+7)}{\Lambda^{\mu+7}} = \frac{720}{\Lambda^5}\overline{D} \tag{20}$$

Note that for a given water amount, M, N_0 can be determined given μ and Λ.

$$N_0 = M\frac{6}{\pi\rho}\frac{\Lambda^{\mu+4}}{\Gamma(\mu+4)} \tag{21}$$

So one can't really write $Z \sim M^{\alpha}$ for the gamma distribution but $Z/M = 720/(\pi\rho\, \Lambda^3)$. The bottom line is Z tends to scale as the particle size to a power about +3 higher than the mean mass, M, and ~ +2 higher than the rain rate dependence on particle size. This makes the measured backscatter more sensitive to the larger particles than one would like for determining rain rate. This makes an understanding of the particles size distribution very important for interpreting the radar reflectivity.

$$Z = B\, R^{b} \tag{22}$$

where R is the rain rate and $1.6 < b < 2$ and B is a constant

$B \sim 200$ for rain and $B \sim 2000$ for snow

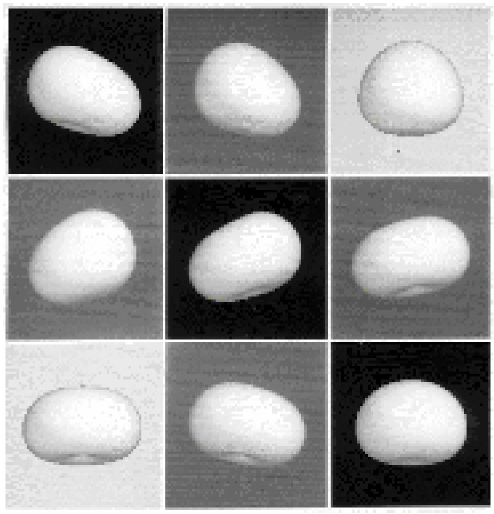
for R in mm/hr.

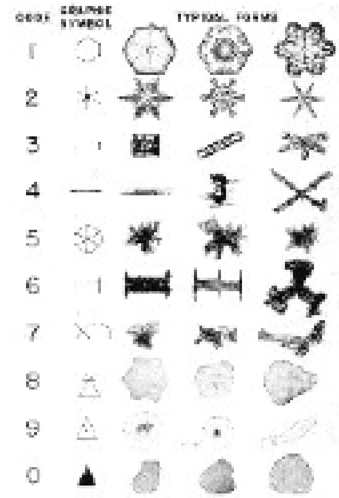
The values for b and B actually vary with the type of rain. The ambiguity is because Z scales as D^6 whereas R scales as $D^{3.5} - D^4$. This ambiguity is fundamental to backscatter measurements because backscatter does not depend on the particle size in the same way that rain rate does. Therefore, to determine a unique relation between Z and R, one must know the particle size distribution which in fact one almost nevers knows.

An improvement in this situation can be achieved using polarized radar measurements. (However I don't think this works well for orbiting radars because of their downward looking geometry).

Polarization radar

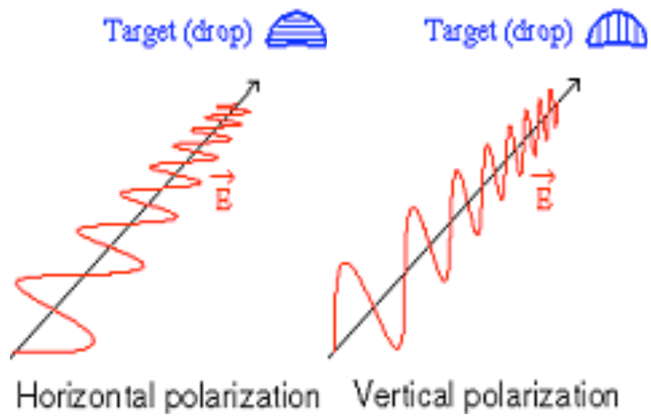
The shape of atmospheric water particles is not actually a sphere except perhaps for vary small liquid droplets. This can be used to help determine particle size and type. But more information is needed to do so. In this regard, using polarized radar signals can be quite helpful.





Ken Beard, University of Illinois, model of oscillating 5mm rain drop

Liquid droplets flatten as they fall. To maximize reflection, single polarization radars are therefore linear horizontal polarized. The reflectivity is written as Z_{HH} where the first H subscript refers to transmitting horizontally polarized light and the second H refers to receiving horizontally polarized light. When the droplets are flattened, the reflectivity for the vertical polarization, Z_{VV}, is smaller than Z_{HH} . This difference in reflectivity can be used to constrain the particle size distribution because larger droplets flatten more than smaller droplets.



Typical amounts of flattening are shown in the following figure.

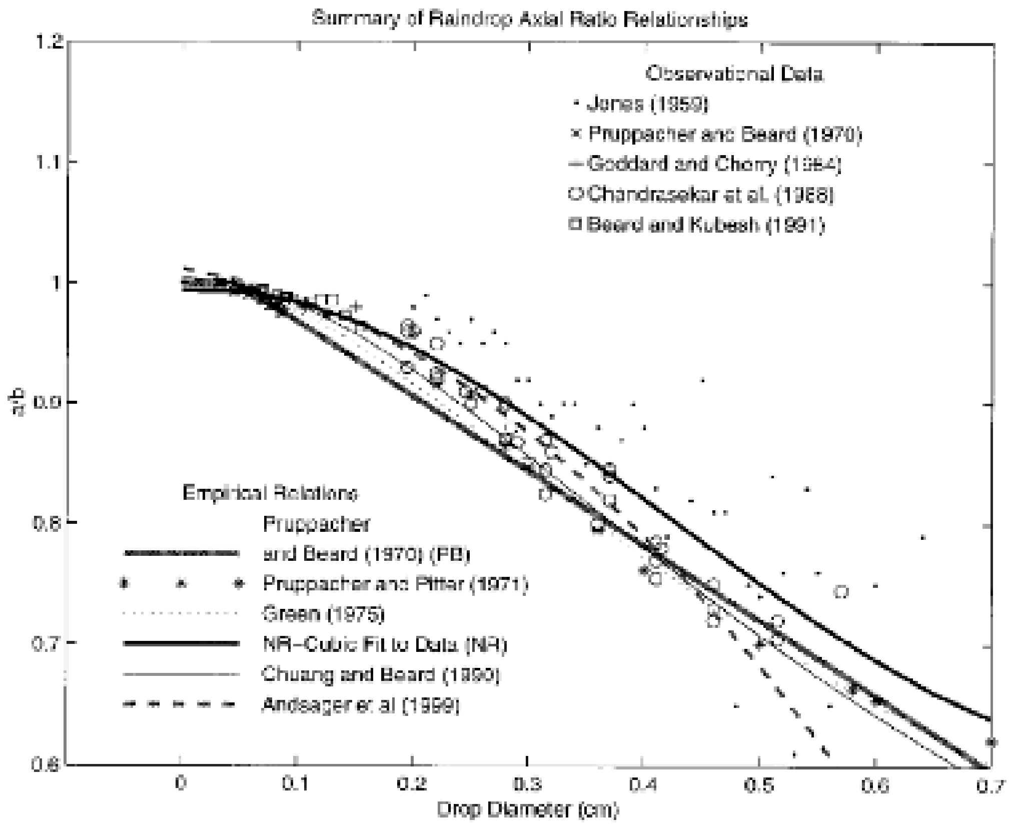


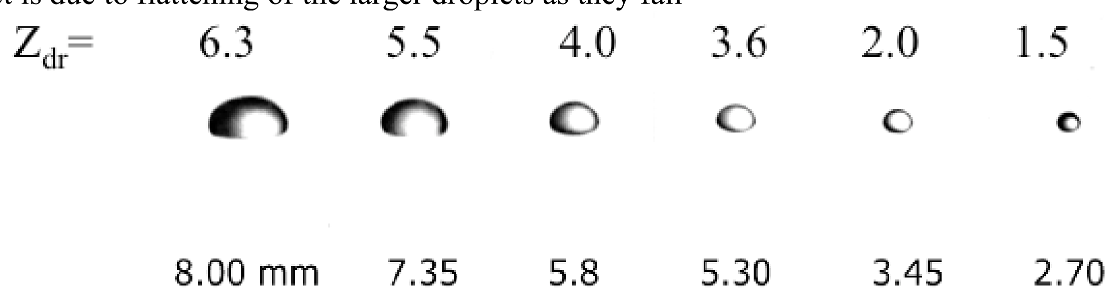
FIG. 1. Summary of experimental data and empirical relations showing raindrop axis ratio a/b as a function of D_e (cm).

The raindrops become more flattened as they become larger. I believe D_e is the diameter of the spherical droplet of equivalent volume.

The differential reflectivity, Z_{DR}, is the ratio of the reflectivity at the 2 wavelengths, defined in the case used in the following table as

$$Z_{DR} = 10\log\frac{Z_{HH}}{Z_{VV}} \quad \text{(in dB)} \tag{23}$$

Effect is due to flattening of the larger droplets as they fall



From Beard and Chuang, 1987

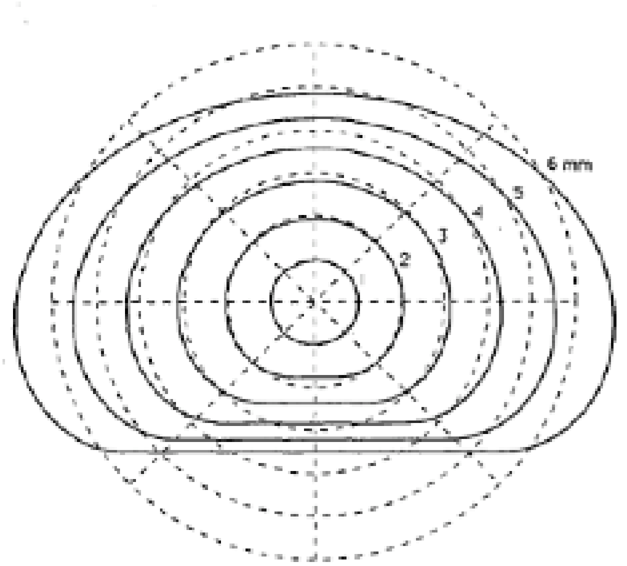
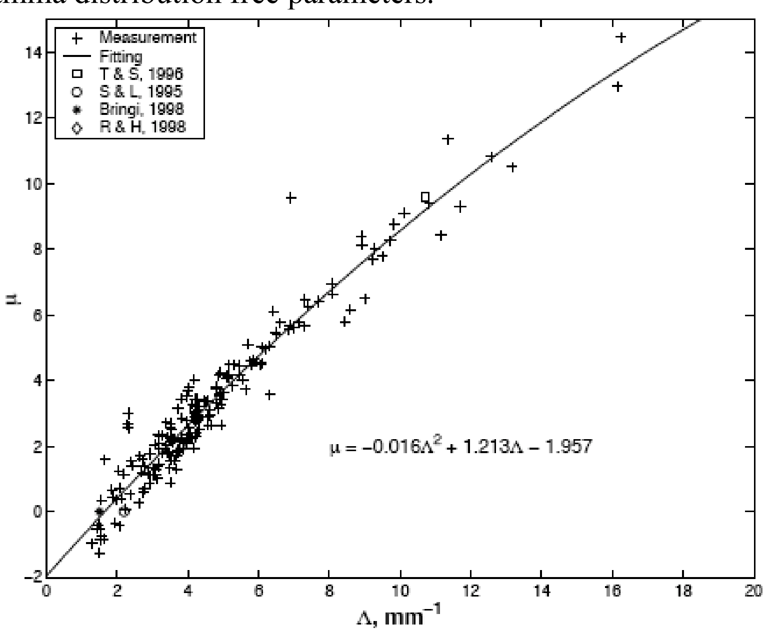
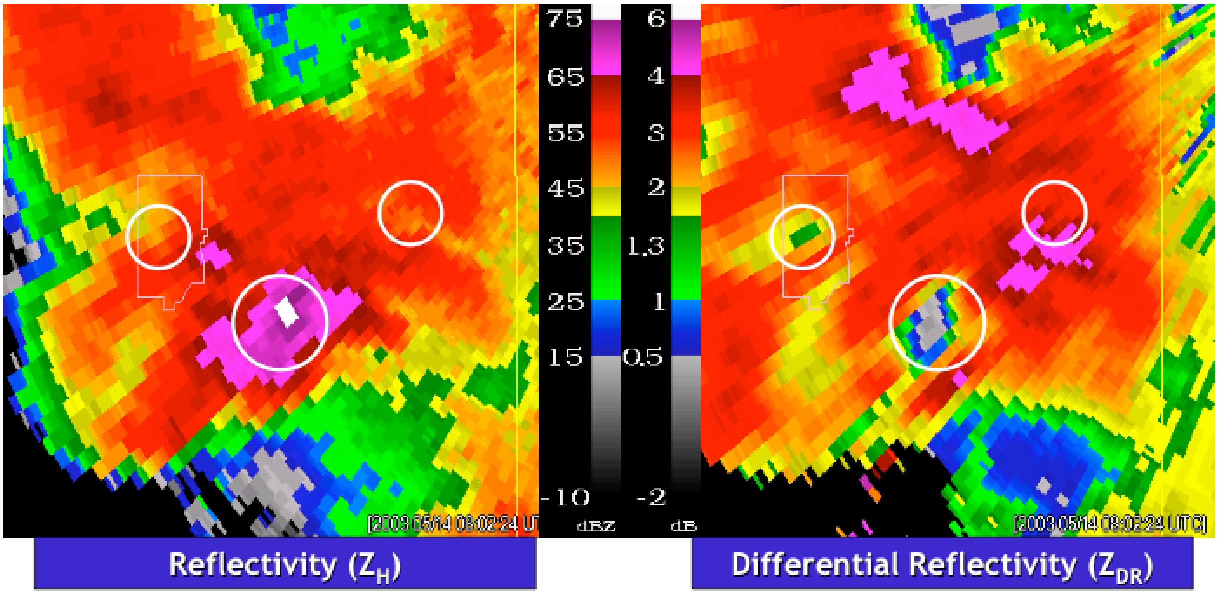


FIG. 2. Sessile drop profiles for d = 1, 2, 3, 4, 5 and 6 mm. Shown for comparison are dashed circles of diameter d divided into 45 degree sectors.

3 parameters are needed to describe the gamma particle size distribution. A video disdrometer can measure the size distribution. Such measurements at least at high rain rates indicate that μ and Λ are quite correlated as shown in the Figure below. Under these conditions there are only 2 unknowns, N_0 and either μ or Λ and measurements of Z_{DR} and Z_{HH} provide enough information to solve for the gamma distribution free parameters.





Differential Phase:

Note that there is also differential phase information to be used from polarized radar signals. The HH signal travels more slowly than the ZZ signal and yields a useful differential phase delay.

One must distinguish between ice and liquid water particles or water coated ice particles which will substantially change the dielectric properties of the particles and therefore the scattering and absorbing backscatter crossections.

References

Guifu Zhang, J. Vivekanandan, and Edward Brandes, A Method For Estimating Rain Rate And Drop Size Distribution From Polarimetric Radar Measurements, IEEE, IGARSS, 2000