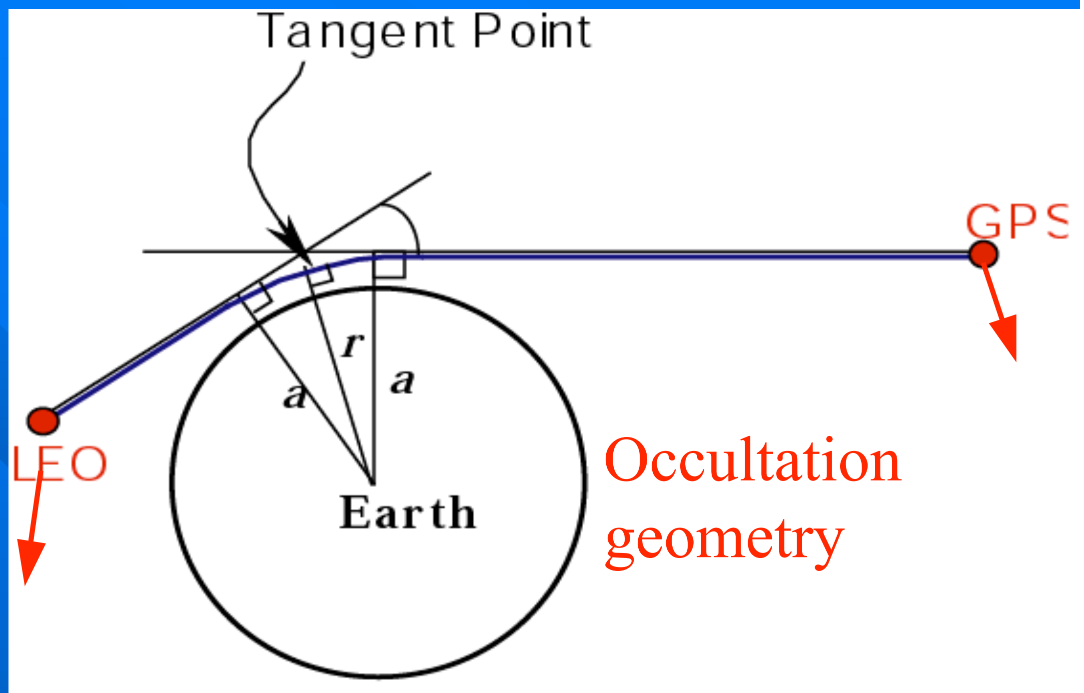
GPS Occultation Introduction and Overview

R. Kursinski

Dept. of Atmospheric Sciences, University of Arizona, Tucson AZ, USA

Occultation Geometry

- An occultation occurs when the orbital motion of a GPS SV and a Low Earth Orbiter (LEO) are such that the LEO 'sees' the GPS rise or set across the limb
- This causes the signal path between the GPS and the LEO to slice through the atmosphere



Occultation geometry

- The basic observable is the change in the delay of the signal path between the GPS SV and LEO during the occultation
- The change in the delay includes the effect of the atmosphere which acts as a planetary scale lens bending the signal path

GPS Occultation Coverage

Two occultations per orbit per LEO-GPS pair

=>14 LEO orbits per day and 24 GPS SV

=>Occultations per day ~ 2 x 14 x 24 ~700

=>500 occultations within $\pm 45^o$ of velocity vector per day per orbiting GPS receiver distributed globally

Distribution of one day of occultations

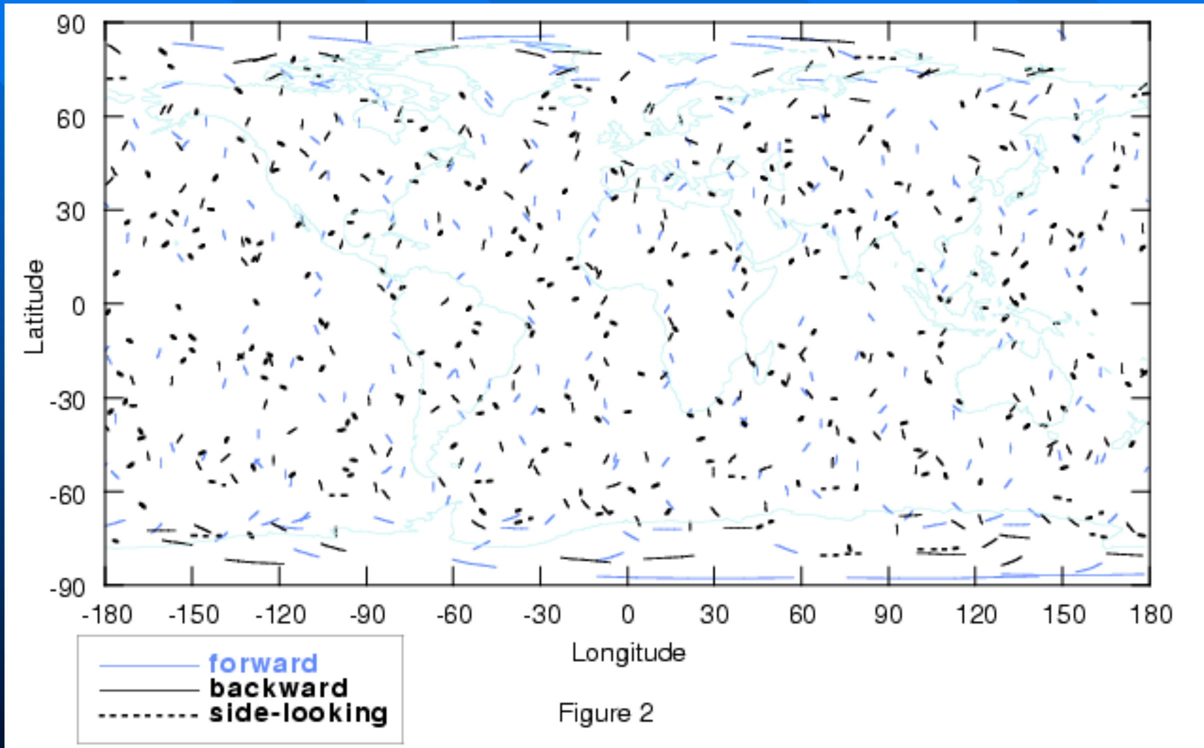
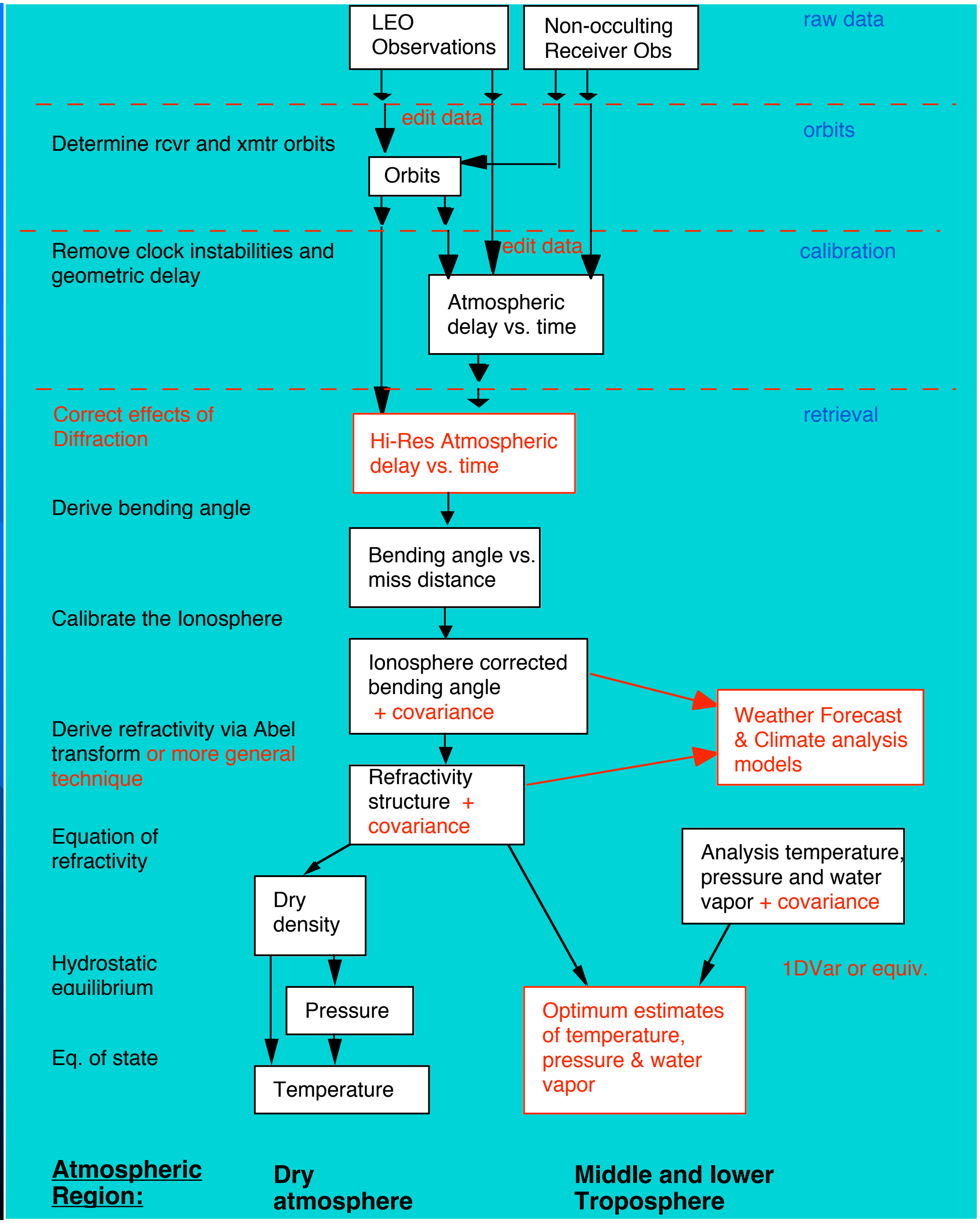


Figure 2

Overview Diagram of Processing

- Figure shows steps in the processing of occultation data



Talk Outline:

Overview of main steps in processing RO signals.

- *Occultation Geometry*
- *Abel transform pair*
- *Calculation of the bending angles from Doppler.*
- *Conversion to atmospheric variables (r , P, T, q)*
- *Vertical and horizontal resolution of RO*
- *Outline of the main difficulties of RO soundings (residual ionospheric noise, upper boundary conditions, multipath, super-refraction)*

Abel Inversion

- Forward problem
  - See also Calc of variations (stationary phase) in Melbourne et al., 1994
  - Snell's law
- Forward integral
  - $dn/dr \Rightarrow \alpha(a)$
- Inverse integral
  - $\alpha(a) \Rightarrow n(r)$

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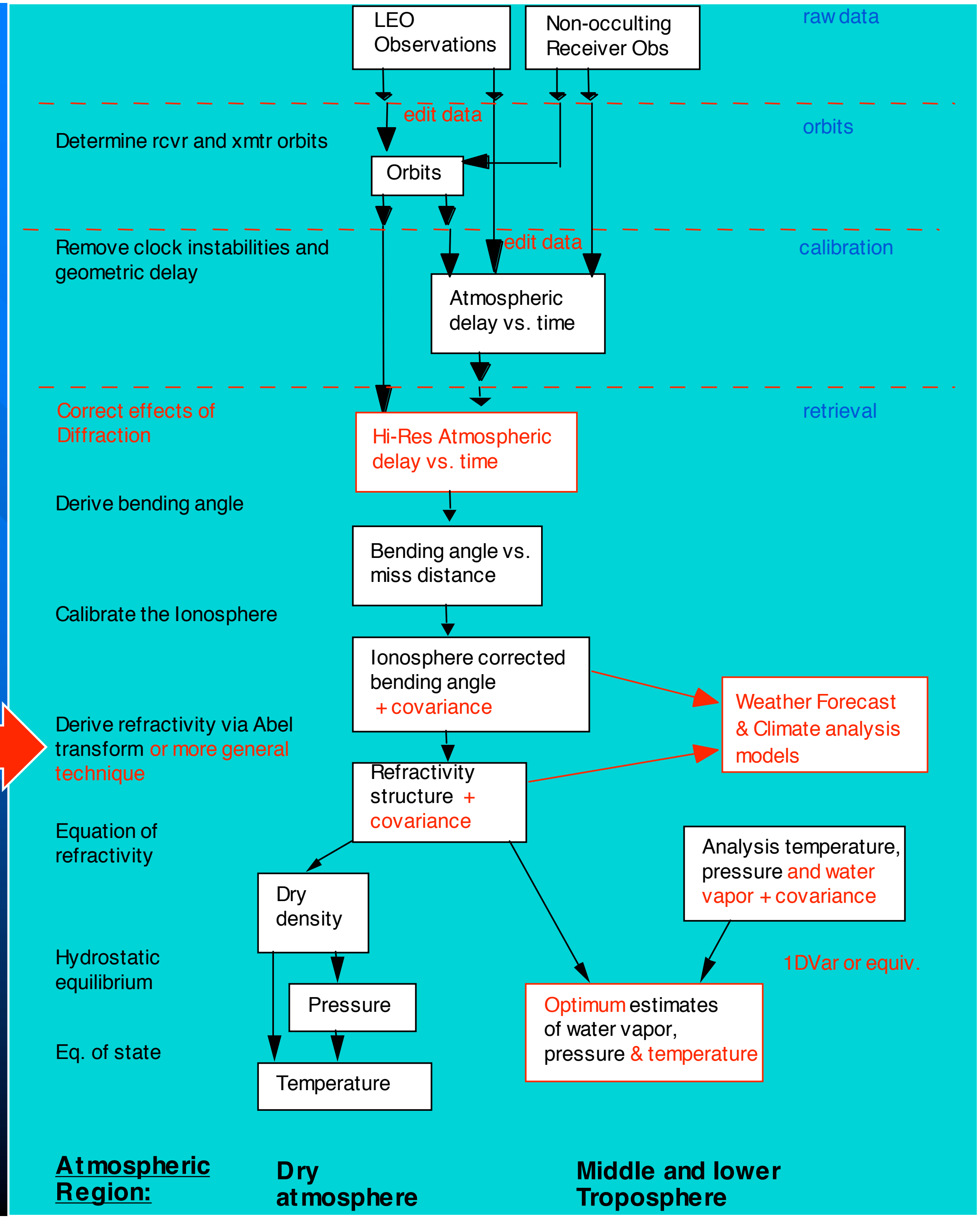
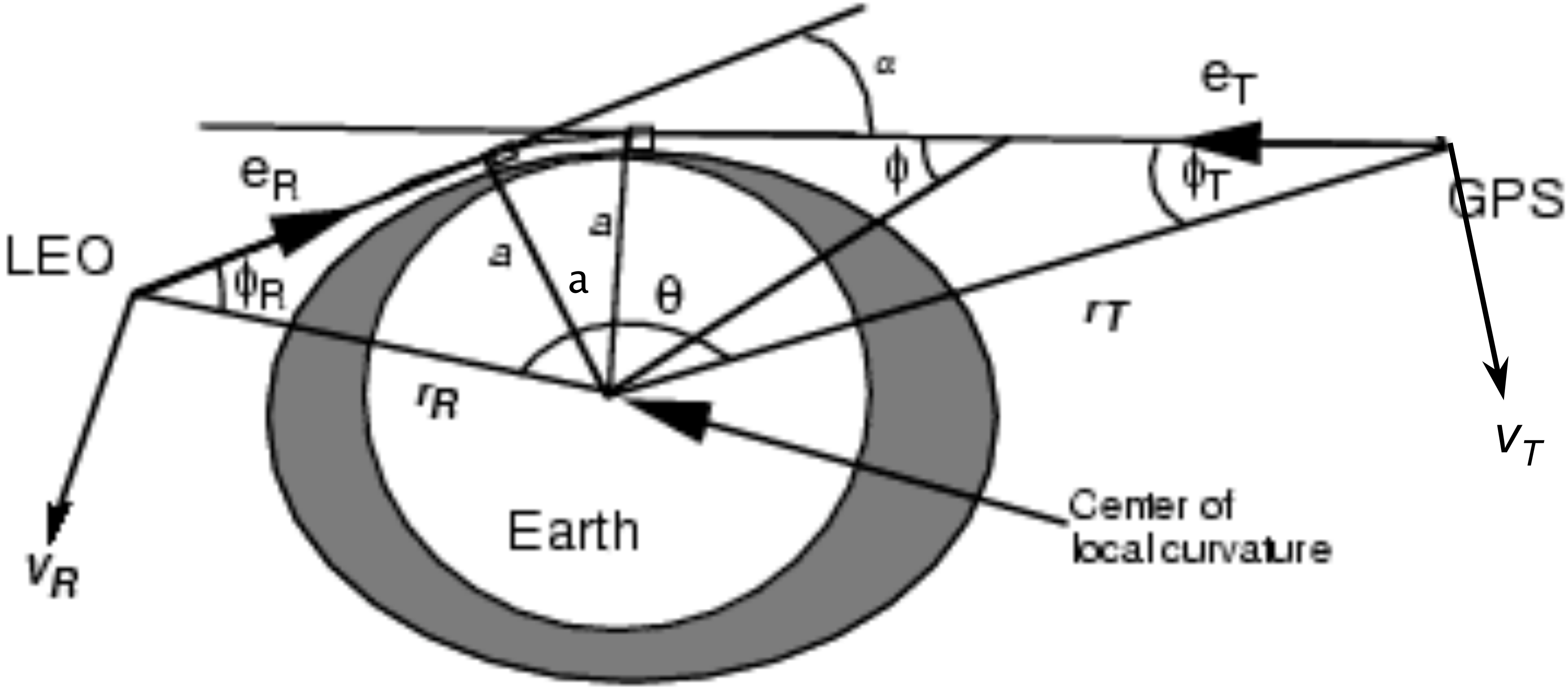


Abel Inversion

- Defining the variables



The Bending Effect

- The differential equation for raypaths can be derived [*Born and Wolf*, 1980] as

$$\frac{d}{ds}\left(n\frac{d\vec{r}}{ds}\right) = \nabla n \qquad (1)$$

where $\vec{r}$ is position along the raypath and *ds* is an incremental length along the raypath such that $d\vec{r} = \hat{s}\,ds$ (2)

where $\hat{s}$ is the unit vector in the direction along the raypath.

- Consider the change in the quantity, $\vec{r}\times n\hat{s}$ *,* along the raypath given as

$$\frac{d}{ds}\left(\vec{r}\times n\hat{s}\right) = \frac{d\vec{r}}{ds}\times n\hat{s} + \vec{r}\times\frac{d}{ds}\left(n\hat{s}\right) \qquad (3)$$

- From (2), the first term on the right is zero and from (1), (3) becomes

$$\frac{d}{ds}\left(\vec{r}\times n\hat{s}\right) = \vec{r}\times\frac{d}{ds}\nabla n \qquad (4)$$

- (4) shows that only the *non-radial* portion of the gradient of index of refraction contributes to changes in $\vec{r}\times n\hat{s}$.
- So for a spherically symmetric atmosphere, ***a* = *n r* sinϕ = constant** (5)

(Bouguer's rule)

The Bending Effect

- Curved signal path through the atmosphere

The signal path is curved according to Snell's law because of changes in the index of refraction along the path

To first approximation, we assume the refractivity changes only as a function of radius

=> Bouguer's rule applies: $n\, r \sin\phi = a = \text{const}$

So $d(n\, r \sin\phi) = 0 = r \sin\phi\, dn + n \sin\phi\, dr + n\, r \cos\phi\, d\phi$

$$d\phi = -dr\, (r \sin\phi\, dn/dr + n \sin\phi) / (n\, r \cos\phi)$$

- *Straight line*

Notice that the equation for a straight line in polar coordinates is $r \sin\phi_0 = \text{const}$

So for a straight line: $d\phi_0 = -\, dr \sin\phi_0 / (r \cos\phi_0)$

- So the change in direction of the path or the bending along the path (with curving downward defined as positive) is

  $d\alpha = d\phi_0 - d\phi = dr\, (r \sin\phi\, dn/dr) / (n\, r \cos\phi) = dr\, a/n\, dn/dr\, /(nr\, [1-\sin^2\phi]^{1/2})$

$$d\alpha = dr \frac{dn}{n\, dr} \frac{a}{\sqrt{n^2 r^2 - a^2}} \quad (6)$$

Deducing the Index of Refraction from Bending

- The total bending is the integral of dα along the path

$$\alpha = \int d\alpha = 2a\int_{r_t}^{\infty} dr \frac{dn}{n\,dr}\frac{1}{\sqrt{n^2 r^2 - a^2}} \qquad (7)$$

- We measure profiles of $\alpha(a)$ from Doppler shift but what we want are profiles of $n(r)$. But how do we achieve this?
  - The answer is via an Abel integral transform referring to a special class of integral equations deduced by Abel by 1825 which are a class of Volterra integral equations (Tricomi, 1985)
- First we rewrite α in terms of x=nr rather than r:

$$\alpha(a) = 2a\int_{x=a}^{x=\infty} \frac{dn}{ndx}\frac{dx}{\sqrt{x^2 - a^2}} \qquad (8)$$

Deducing the Index of Refraction from Bending

- We multiply each side of (8) by the kernel, $(a^2-a_1^2)^{-1/2}$, and integrate with respect to a from a_1 to infinity (Fjeldbo et al, 1971).

$$\int_{a_1}^{\infty}\frac{\alpha(a)\,da}{\sqrt{a^2-a_1^2}}=\int_{a=a_1}^{a=\infty}\frac{2a}{\sqrt{a^2-a_1^2}}\left[\int_{x=a}^{x=\infty}\frac{dn}{ndx}\frac{dx}{\sqrt{x^2-a^2}}\right]da$$

$$=\int_{x=a_1}^{x=\infty}\frac{dn}{ndx}\left[\int_{a=a_1}^{a=x}\frac{2a\,da}{\sqrt{a^2-a_1^2}\sqrt{x^2-a^2}}\right]dx$$

$$=\int_{x=a_1}^{x=\infty}2\frac{dn}{ndx}\left[\sin^{-1}\sqrt{\frac{a^2-a_1^2}{x^2-a_1^2}}\right]_{a=a_1}^{a=x}dx$$

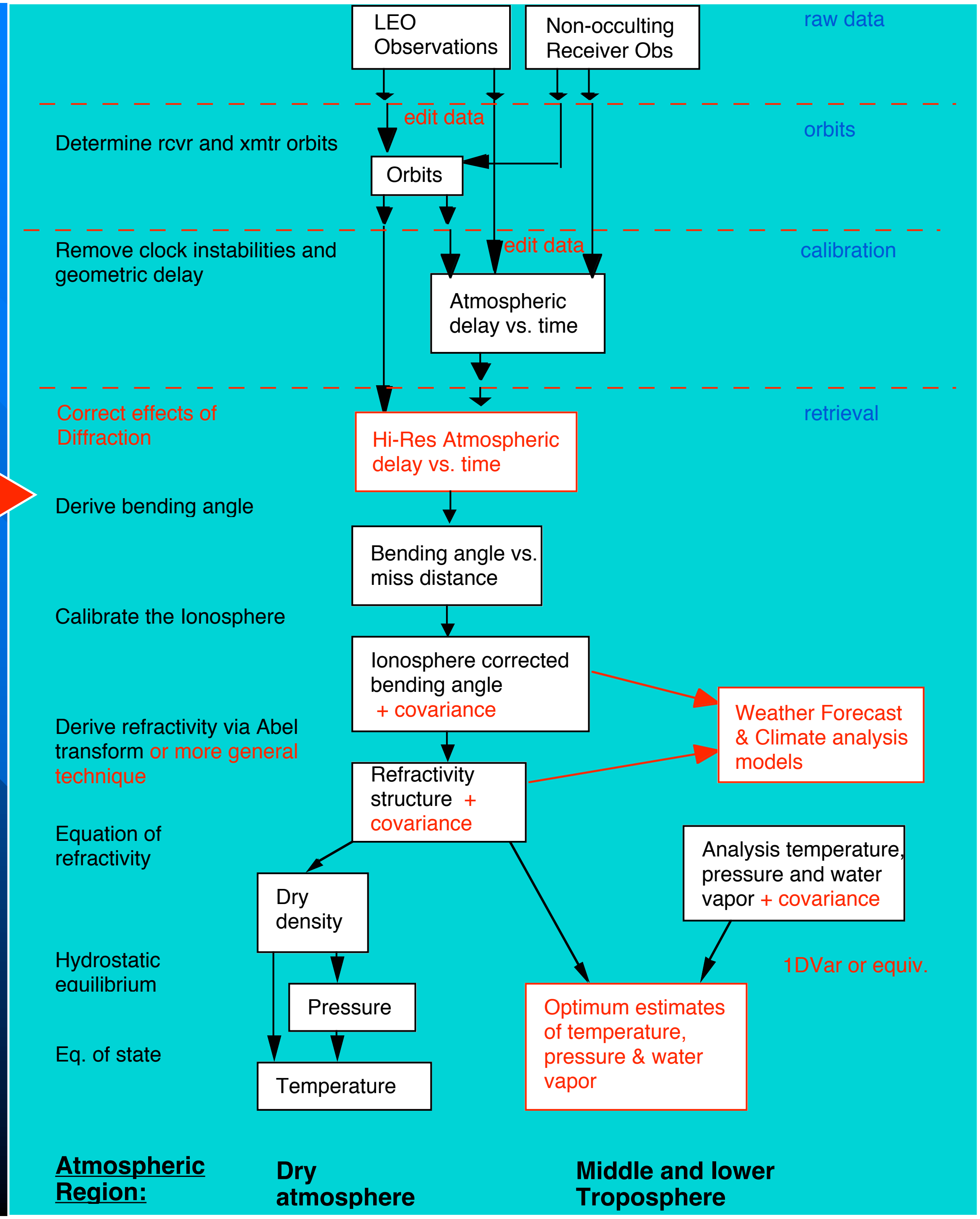
$$=\pi\int_{x=a_1}^{x=\infty}\frac{dn}{ndx}\,dx=-\pi\ln\left[n\left(r_{01}\right)\right]$$

Such that

$$n\left(r_{01}\right)=\exp\left[-\frac{1}{\pi}\int_{a_1}^{\infty}\frac{\alpha(a)\,da}{\sqrt{a^2-a_1^2}}\right] \qquad (9)$$

Note that $r_{01} = a_1/n(a_1)$

Deriving Bending from Doppler



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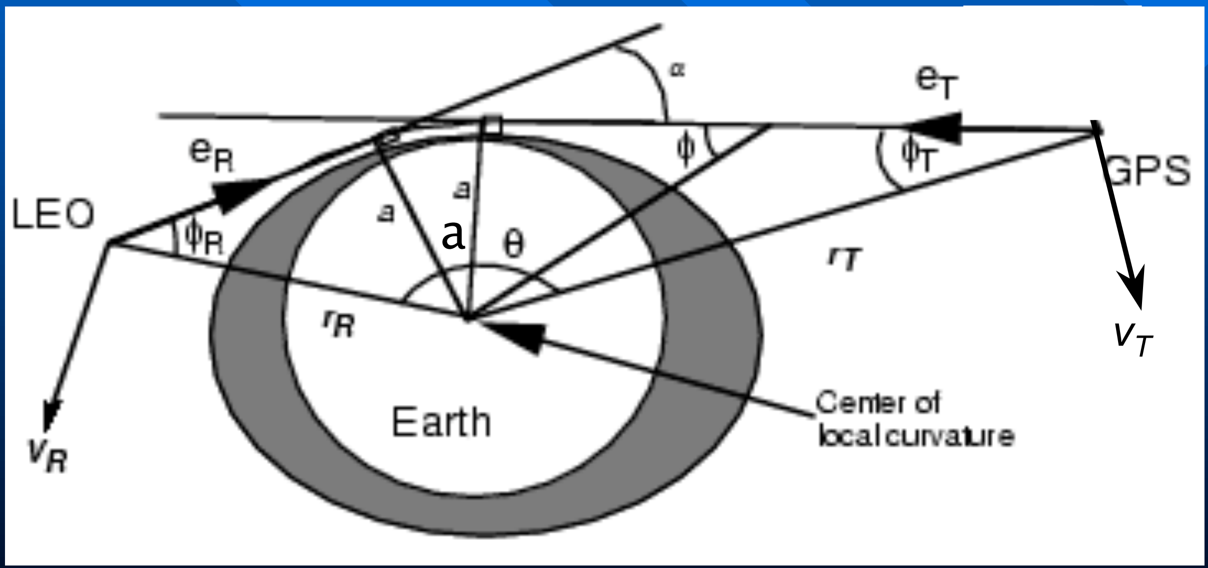
Deriving Bending Angles from Doppler

The projection of satellite orbital motion along signal ray-path produces a Doppler shift at both the transmitter and the receiver

After correction for relativistic effects, the Doppler shift, f_d, of the transmitter frequency, f_T, is given as

$$f_d = \frac{f_T}{c}\left(\vec{V}_T \bullet \hat{e}_T + \vec{V}_R \bullet \hat{e}_R\right) = -\frac{f_T}{c}\left(V_T^r \cos\phi_T + V_T^\theta \sin\phi_T + V_R^r \cos\phi_R - V_R^\theta \sin\phi_R\right) \quad (10)$$

where: c is the speed of light and the other variables are defined in the figure with V_T^r and V_T^θ representing the radial and azimuthal components of the transmitting spacecraft velocity.



Deriving Bending Angles from Doppler

- *Solve for a*

Under spherical symmetry, Snell's law => Bouguer's rule (*Born & Wolf*, 1980).

$$n\, r \sin\phi = \text{constant} = a = n\, r_t$$

where r_t is the radius at the tangent point along the ray path.

So $$r_T \sin\phi_T = r_R \sin\phi_R = a \qquad (11)$$

Given knowledge of the orbital geometry and the center of curvature, we solve nonlinear equations (10) and (11) iteratively to obtain ϕ_T and ϕ_R and a

- *Solve for α*

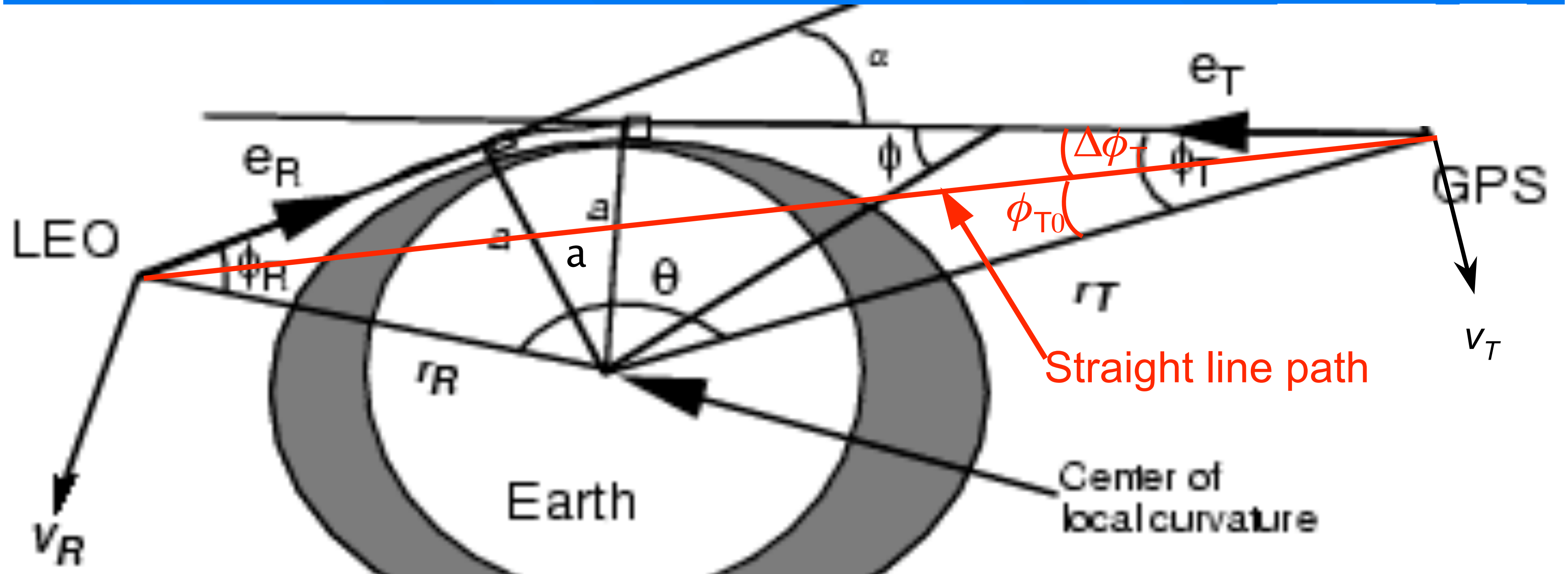
From the geometry of the Figure, $2\pi = \phi_T + \phi_R + \theta + \pi - \alpha$

So $$\alpha = \phi_T + \phi_R + \theta - \pi \qquad (12)$$

Knowing the geometry which provides θ, the angle between the transmitter and the receiver position vectors, we combine ϕ_T and ϕ_R in (12) to solve for α.

Linearized Relation Between α, a and f_d

- Consider the difference between bent and straight line paths to isolate and gain a better understanding of the atmospheric effect



- The Doppler equation for the straight line path is

$$f_{d0} = -\frac{f_T}{c}\left(V_T^r \cos\phi_{T0} + V_T^\theta \sin\phi_{T0} + V_R^r \cos\phi_{R0} - V_R^\theta \sin\phi_{R0}\right) \quad (13)$$

Linearized Relation Between a, a and f_d

We have the following relations (using the Taylor expansion):

$\Delta\phi_T = \phi_T - \phi_{T0}$

$\sin(\phi_T) = \sin(\phi_T - \phi_{T0}) \sim \sin(\phi_{T0}) + \cos(\phi_{T0})\, \Delta\phi_T$

$\cos(\phi_T) = \cos(\phi_T - \phi_{T0}) \sim \cos(\phi_{T0}) - \sin(\phi_{T0})\, \Delta\phi_T$

The difference in the Doppler frequencies along the bent and straight paths is the atmospheric Doppler contribution, f_{atm}, which is given as

$$f_{atm} = f_d - f_{d0} = \frac{f_T}{c}\left[\left(V_T^r \sin\phi_{T0} - V_T^\theta \cos\phi_{T0}\right)\Delta\phi_T + \left(V_R^r \sin\phi_{R0} + V_R^\theta \cos\phi_{R0}\right)\Delta\phi_R\right] \quad (14)$$

Notice that the velocity components in (14) are *perpendicular* to the straight line path

So the relevant velocity responsible for the atmospheric Doppler shift is the descent or ascent velocity (orthogonal to the limb) of the straight line path

Now we also know from the Taylor expansion of Bouguer's rule that $r_T \cos(\phi_{T0})\, \Delta\phi_T = r_R \cos(\phi_{R0})\, \Delta\phi_R$

such that $\Delta\phi_R / \Delta\phi_T = r_T \cos(\phi_{T0}) / r_R \cos(\phi_{R0})$

We also know from geometry that $\Delta\phi_T + \Delta\phi_R = \alpha$

Linearized Relation Between α, a and f_d

- Now we also know from the Taylor expansion of Bouguer's rule that $r_T \cos(\phi_{T0})\,\Delta\phi_T = r_R \cos(\phi_{R0})\,\Delta\phi_R$ such that

$$\Delta\phi_R / \Delta\phi_T = r_T \cos(\phi_{T0}) / r_R \cos(\phi_{R0})$$

- We also know from geometry that $\Delta\phi_T + \Delta\phi_R = \alpha$
- For the GPS-LEO occultation geometry $r_T \cos(\phi_{T0}) / r_R \cos(\phi_{R0}) \sim 9$
  Therefore $\Delta\phi_R / \Delta\phi_T \sim 9$ and $1.1\,\Delta\phi_R \sim \alpha$ or $\Delta\phi_R \sim \alpha$
- Therefore we can write

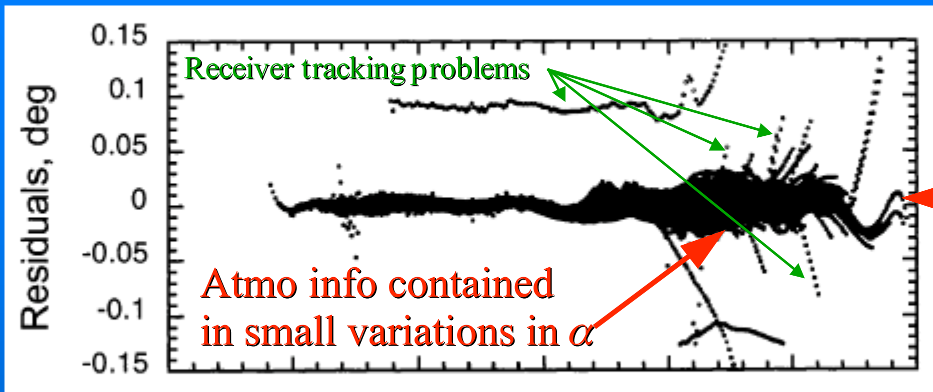
$$f_{atm} \cong \frac{f_T}{c} V_\perp \alpha \qquad (15)$$

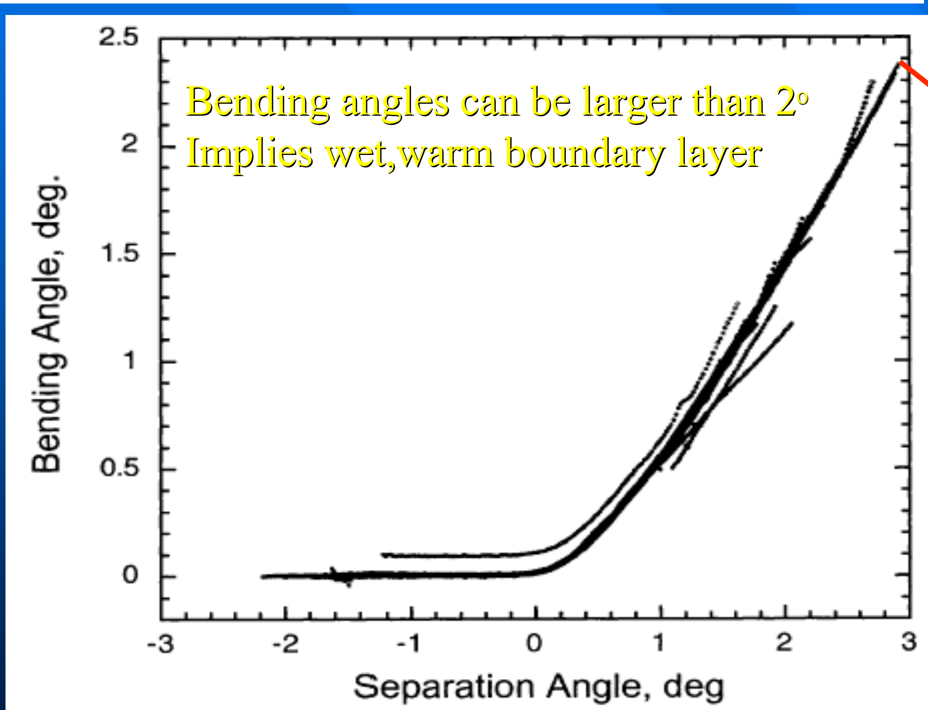
- So atmospheric Doppler is ~ linearly proportional to
  - bending angle and the straight line descent velocity (typically 2 to 3 km/sec).
  - $1^o \sim 250$ Hz at GPS freq.
- $a \sim r_R \sin(\phi_{R0}) + r_R \cos(\phi_{R0})\,\alpha$

$$a \cong \underbrace{r_R \sin\phi_{R0}}_{\text{Distance from center to straight line tangent point}} + \underbrace{r_R \cos\phi_{R0}}_{\text{Distance from LEO to limb}} \frac{c\, f_{atm}}{V_\perp f_T} \qquad (16)$$

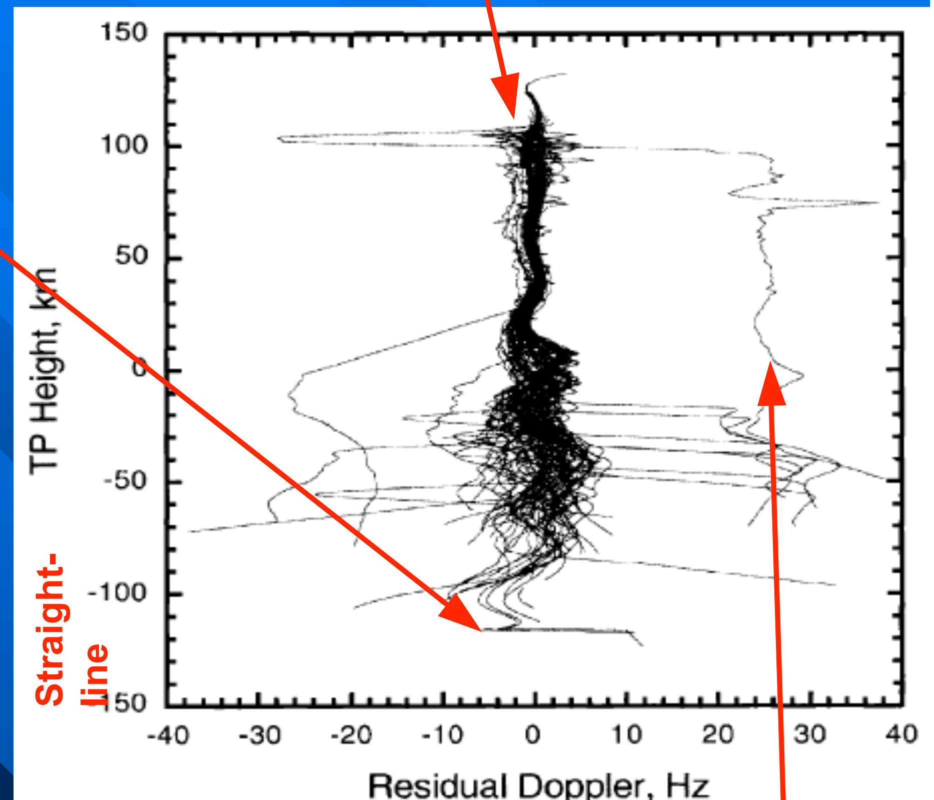
Results from July 1995 from Hajj et al. 2002

- 85 GPS/MET occultations
- Note scaling between bending & Doppler residuals





Angle between transmitter and receiver
0 deg. is when tangent height is at 30 km

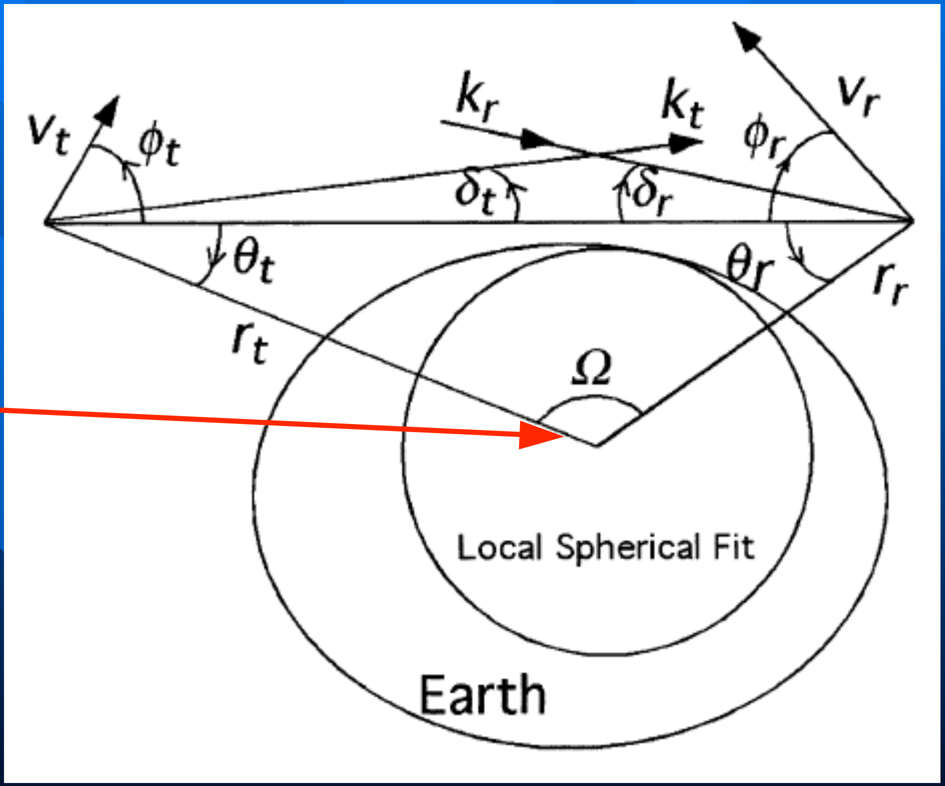


$$f_{atm} \sim f_T\, V_{perp}/c\ \alpha$$
$$= 1.6\text{x}10^9\ \text{x}10^{-5}\ 0.1^o{*}0.017\ \text{rad}/^o = 27\ \text{Hz}$$

Center of curvature

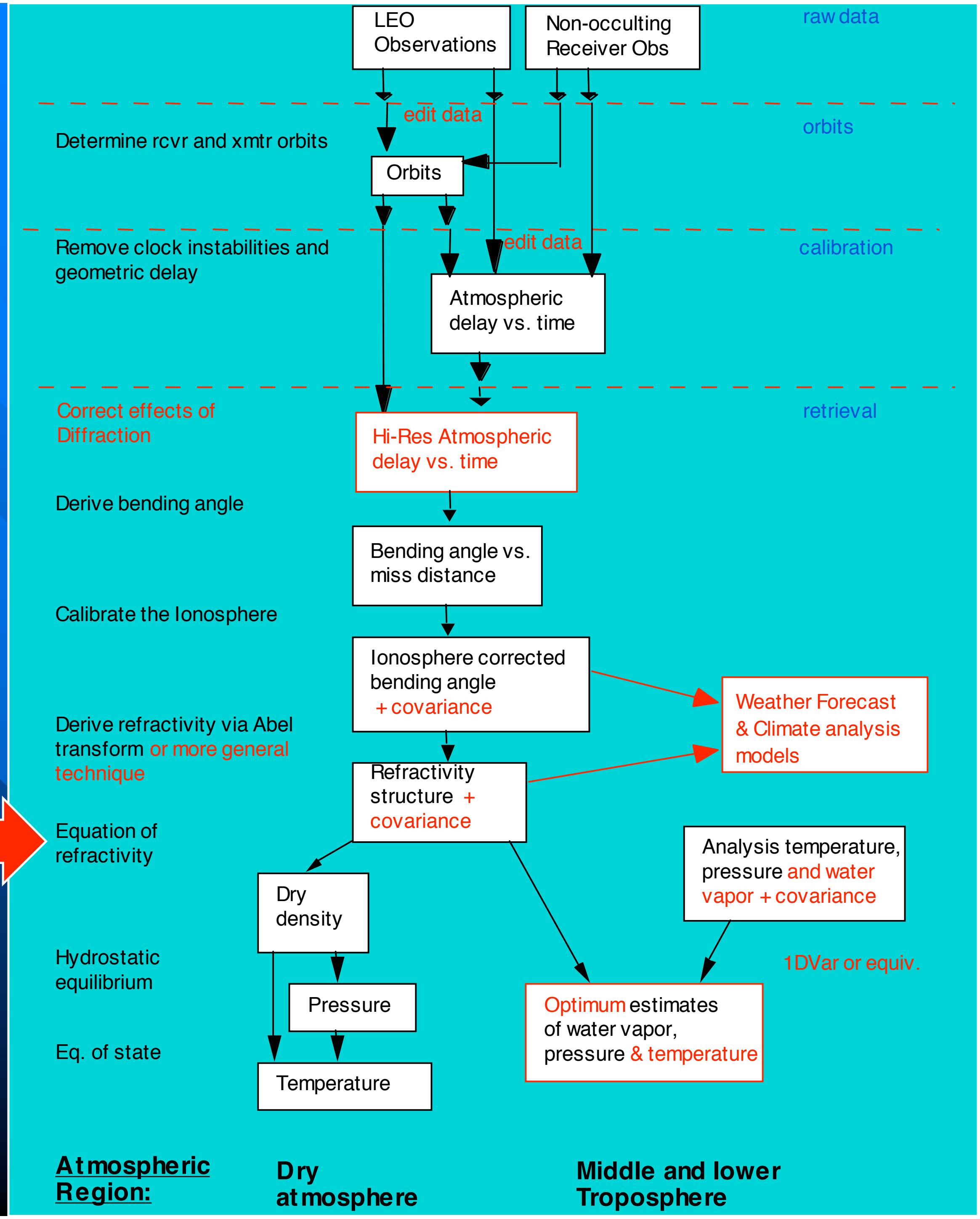
- The conversion of Doppler to bending angle and asymptotic miss distance depends on knowledge of the satellite positions and the reference coordinate center (center of curvature)
- The Earth is slightly elliptical such that the center of curvature does not match the center of the Earth in general
- The center of curvature varies with position on the Earth and the orientation of the occultation plane

The center of curvature is taken as the center of the circle in the occultation plane that best fits the geoid near the tangent point
See Hajj et al. (2002)



Conversion to Atmospheric Variables (r, P, T, q)

- N equation
- Deriving *P* and *T* in dry conditions
- Deriving water vapor



Conversion to Atmospheric Variables (r , P, T, q)

Refractivity equation: $N = (n\text{-}1)*10^6 = c_1\, n_d + c_2\, n_w + c_3\, n_e + c_4\, n_p$ (12)

n : index of refraction = c/v N : refractivity

n_d , n_w , n_e , n_p : number density of "dry" molecules, water vapor molecules, free electrons and particles respectively

Polarizability: ability of incident electric field to induce an electric dipole moment in the molecule (see Atkins 1983, p. 356)

Dry term: a polarizability term reflecting the weighted effects of N_2, O_2, A and CO_2

Wet term: Combined polarizability and permanent dipole terms with permanent term >> polarizability term:

$$c_2\, n_w = (c_{w1} + c_{w2} /T)\, n_w$$

1st term is polarizability, 2nd term is permanent dipole

Ionosphere term: Due first order to plasma frequency, proportional to $1/f^2$.

Particle term: Due to water in liquid and/or ice form. Depends on water amount. No dependence on particle size as long as particles << λ, the GPS wavelength

Conversion to atmospheric variables (r , P, T, q)

- Can write in *N* in another form: use ideal gas law to convert from number density to pressure and temperature: $n = P/RT$ where P is the partial pressure of a particular constituent

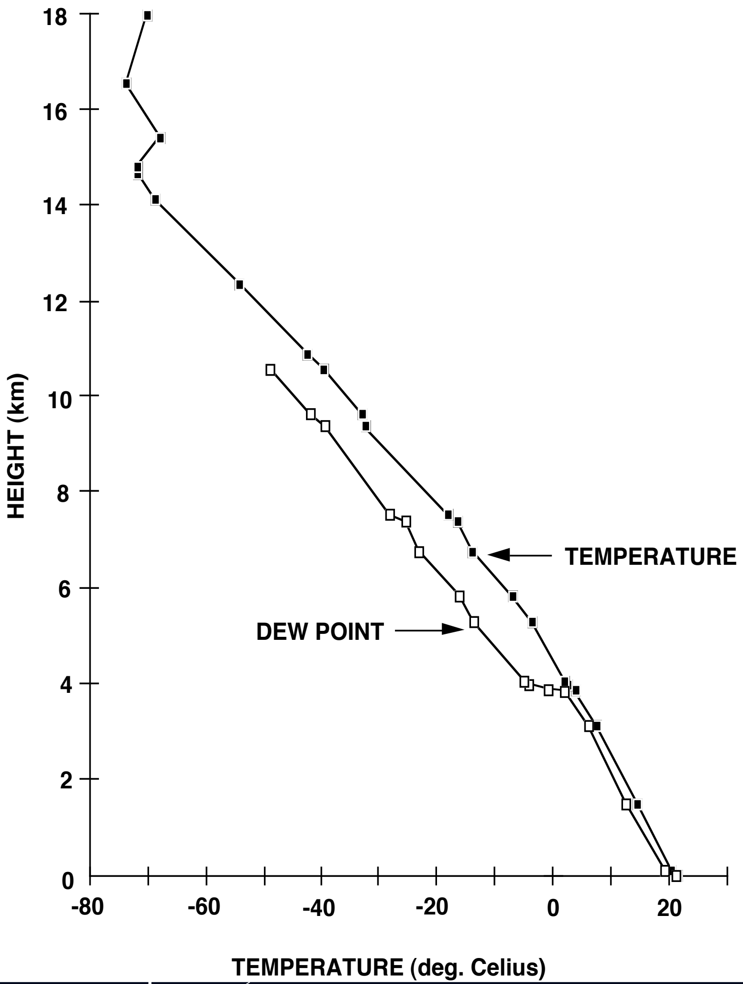
$$N = a_1 \frac{P}{T} + a_2 \frac{P_w}{T^2} - 40.3\times 10^6 \frac{n_e}{f^2} + O\left(\frac{1}{f^3}\right) + a_w W_W + a_i W_i \qquad (13)$$

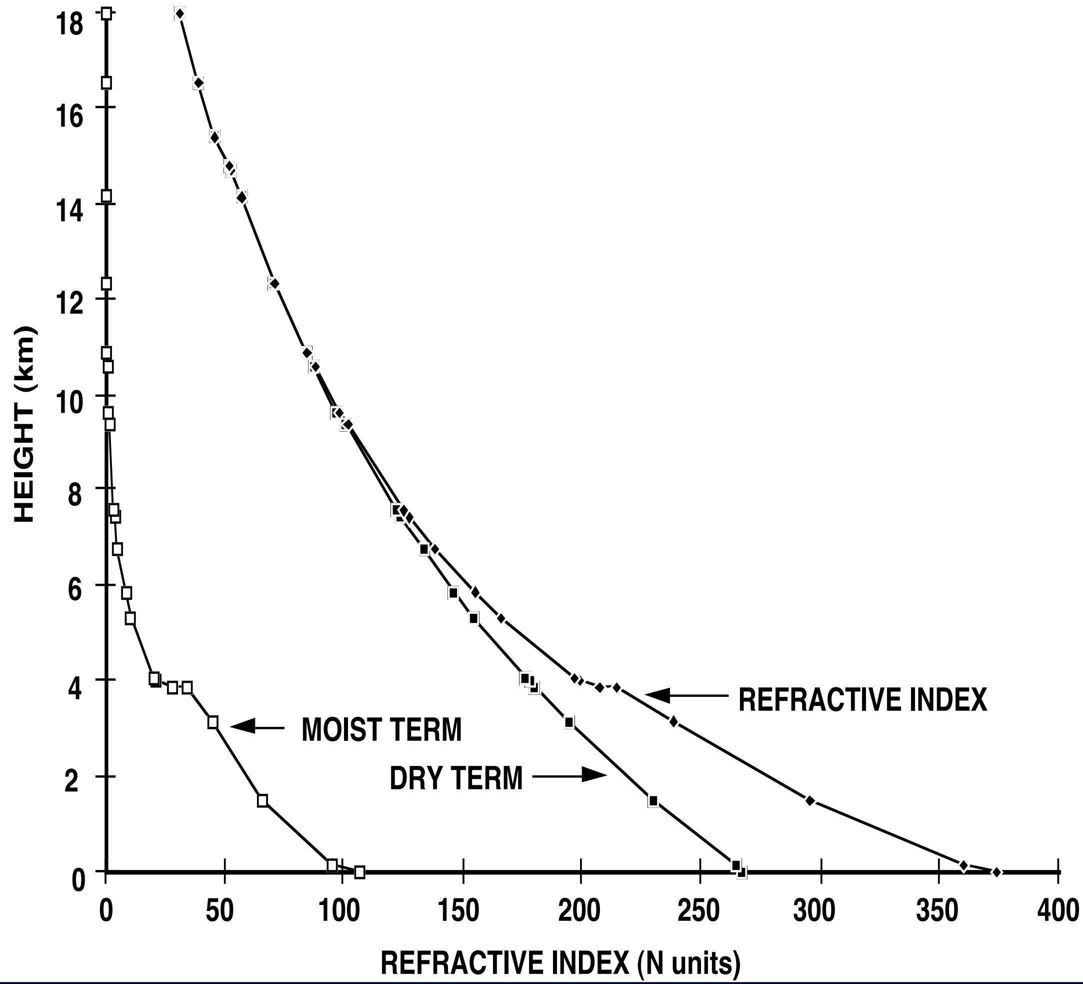
- The one dry and two wet terms have been combined into two terms with a_1 = 77.6 N-units K/mb and a_2 = 3.73e5 N-units K^2/mb
- Relative magnitude of terms
  - Ionospheric refractivity term dominates above 70–80 km altitude and ionospheric bending term dominates above 30-40 km. Can be calibrated using 2 GPS frequencies
  - Dry or hydrostatic term dominates at lower altitudes
  - Wet term becomes important in the troposphere for temperatures > 240K and contributes up to 30% of the total *N* in the tropical boundary layer. It often dominates bending in the lower troposphere
  - Condensed water terms are generally much smaller than water vapor term

Dry and Wet Contributions to Refractivity

Example of refractivity from Hilo radiosonde

- Water contributes up to one third of the total refractivity





Deriving Temperature & Pressure

- After converting f_d => $\alpha(a)$ => $n(r)$ and removing effects of ionosphere, from (12) we have a profile of dry molecule number density for altitudes between 50-60 km down to the 240K level in the troposphere:

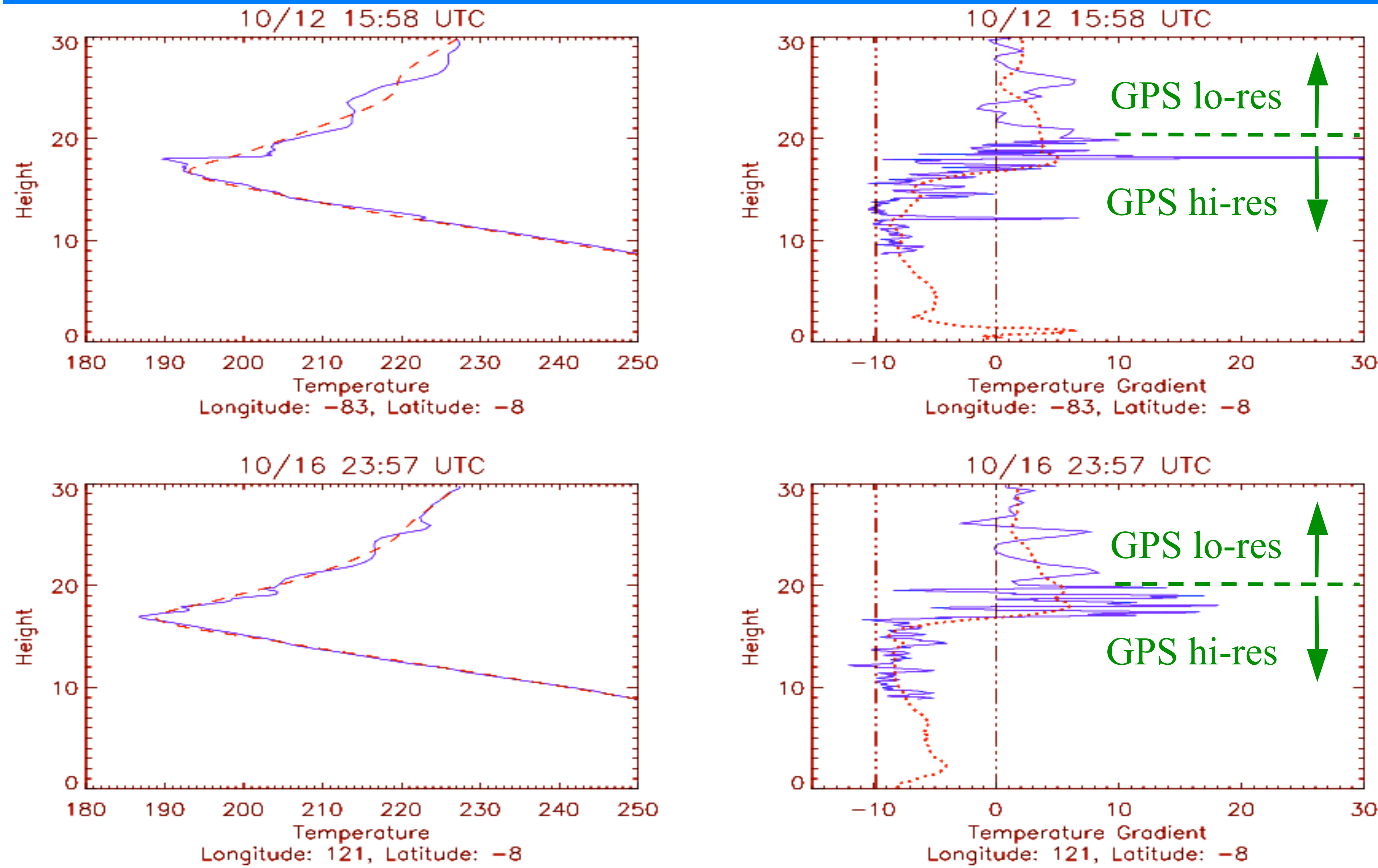
$$n_d(z) = N(z)/c_1 = [n(z)-1]*10^6/c_1$$

- We know the dry constituents are well mixed below ~100 km altitude so c_1 and the mean molecular mass, μ_d, are well known across this interval.
- We apply hydrostatic equation, $dP = -g\ r\ dz = -g\ n_d\ \mu_d\ dz$ to derive a vertical profile of pressure versus altitude over this altitude interval.

$$P(z) = \int_z^{z_{top}} g\rho dz + P(z_{top}) = \int_z^{z_{top}} g n_d \mu_d dz + P(z_{top}) \qquad (14)$$

- We need an upper boundary condition, $P(z_{top})$ which must be estimated from climatology, weather analyses or another source
- Given $P(z)$ and $n_d(z)$, we can solve for $T(z)$ over this altitude interval using the equation of state (ideal gas law): $T(z) = P(z) / (n_d(z)\ R)$ (15)

GPS Temperature Retrieval Examples



Refractivity Error

Fractional error in refractivity derived by GPS RO as estimated in Kursinski et al. 1997

Considers several error sources

Solar max, low SNR

Solar min, high SNR

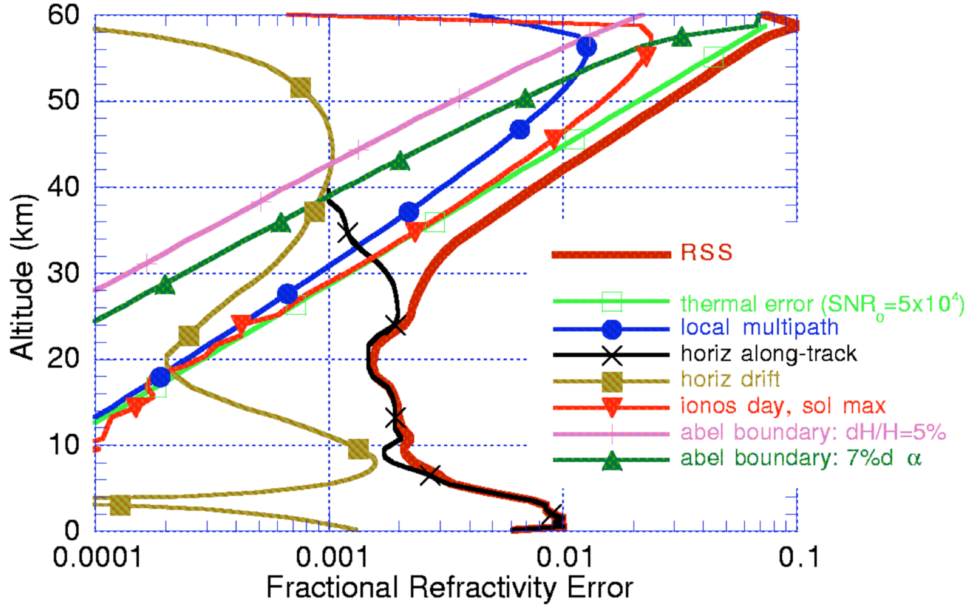


Figure 10

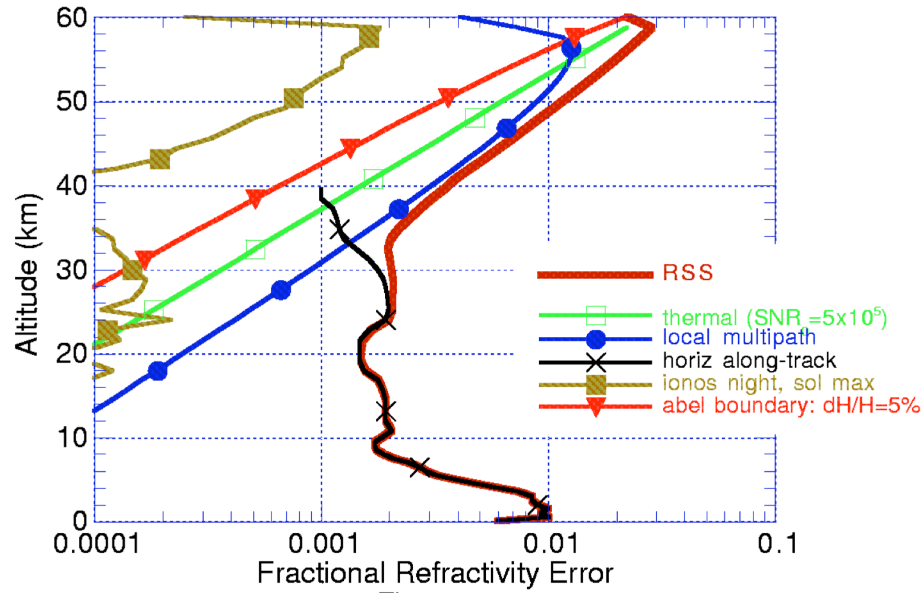


Figure 13

Error in the height of Pressure surfaces

- GPS RO yields pressure versus height
- Dynamics equations are written more compactly with pressure as the vertical coordinate where height of a pressure surface becomes a dependent variable

Solar max, low SNR

Solar min, high SNR

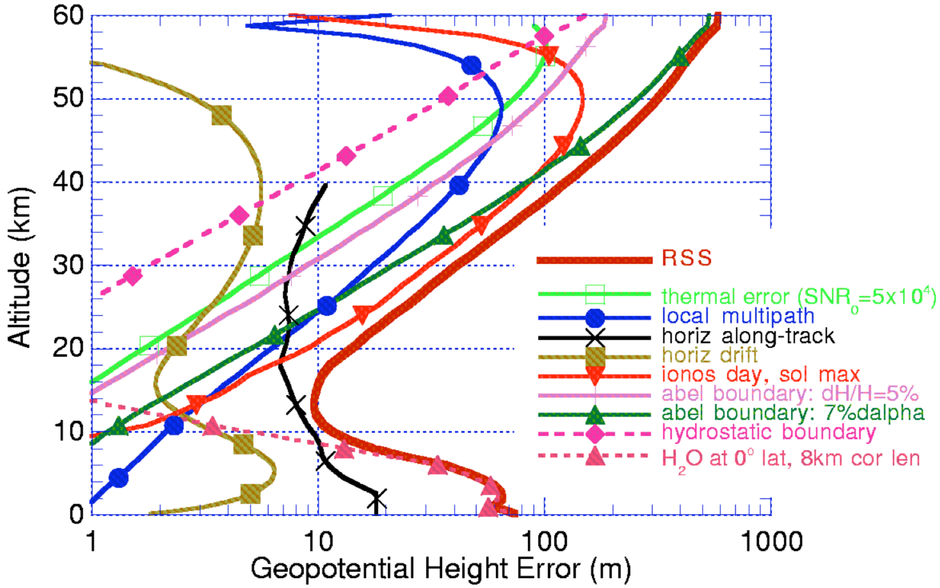


Figure 11

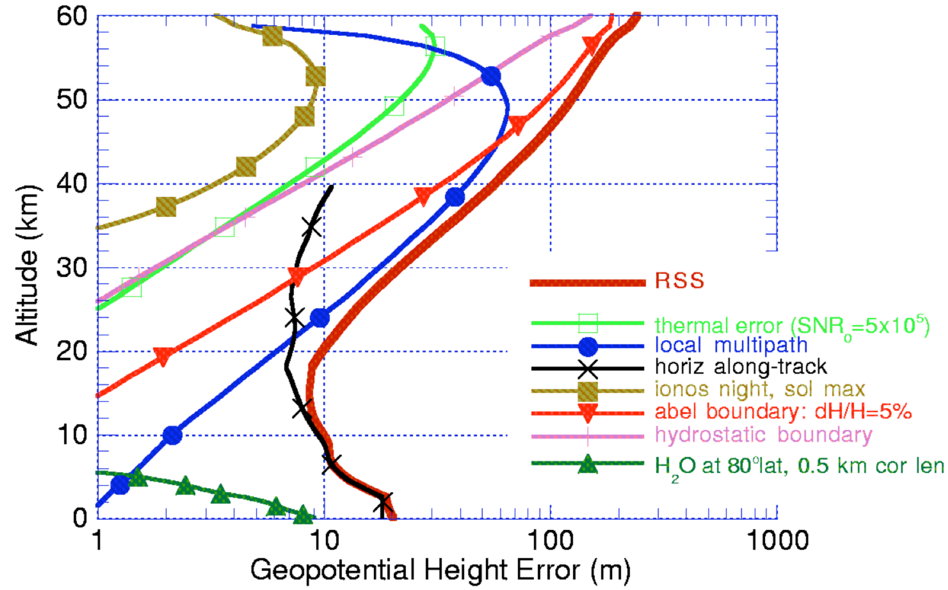


Figure 14

GPSRO Temperature Accuracy

- Temperature is proportional to Pressure/Density
- So $\varepsilon_T/T = \varepsilon_P/P - \varepsilon_\rho/\rho$
- Very accurate in upper troposphere/lower troposphere (UTLS)

Solar max, low SNR

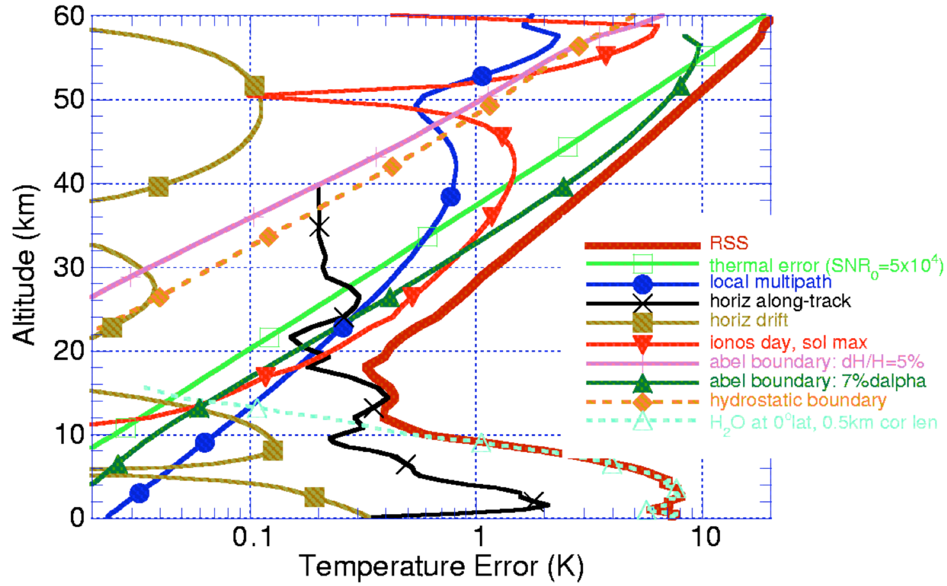


Figure 12

Solar min, high SNR

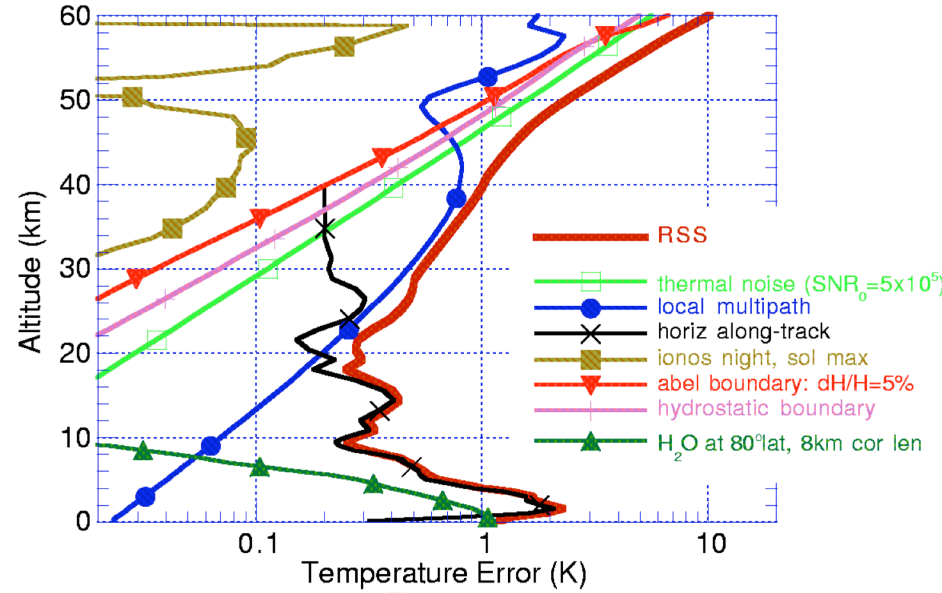


Figure 15

Deriving Water Vapor from GPS Occultations

• In the middle and lower troposphere, $n(z)$ contains dry and moist contributions
=>Need additional information

Either

• Add temperature from an analysis to derive water vapor profiles in lower and middle troposphere

or

• Perform variational assimilation combining $n(z)$ with independent estimates of T, q and $P_{surface}$ and covariances of each

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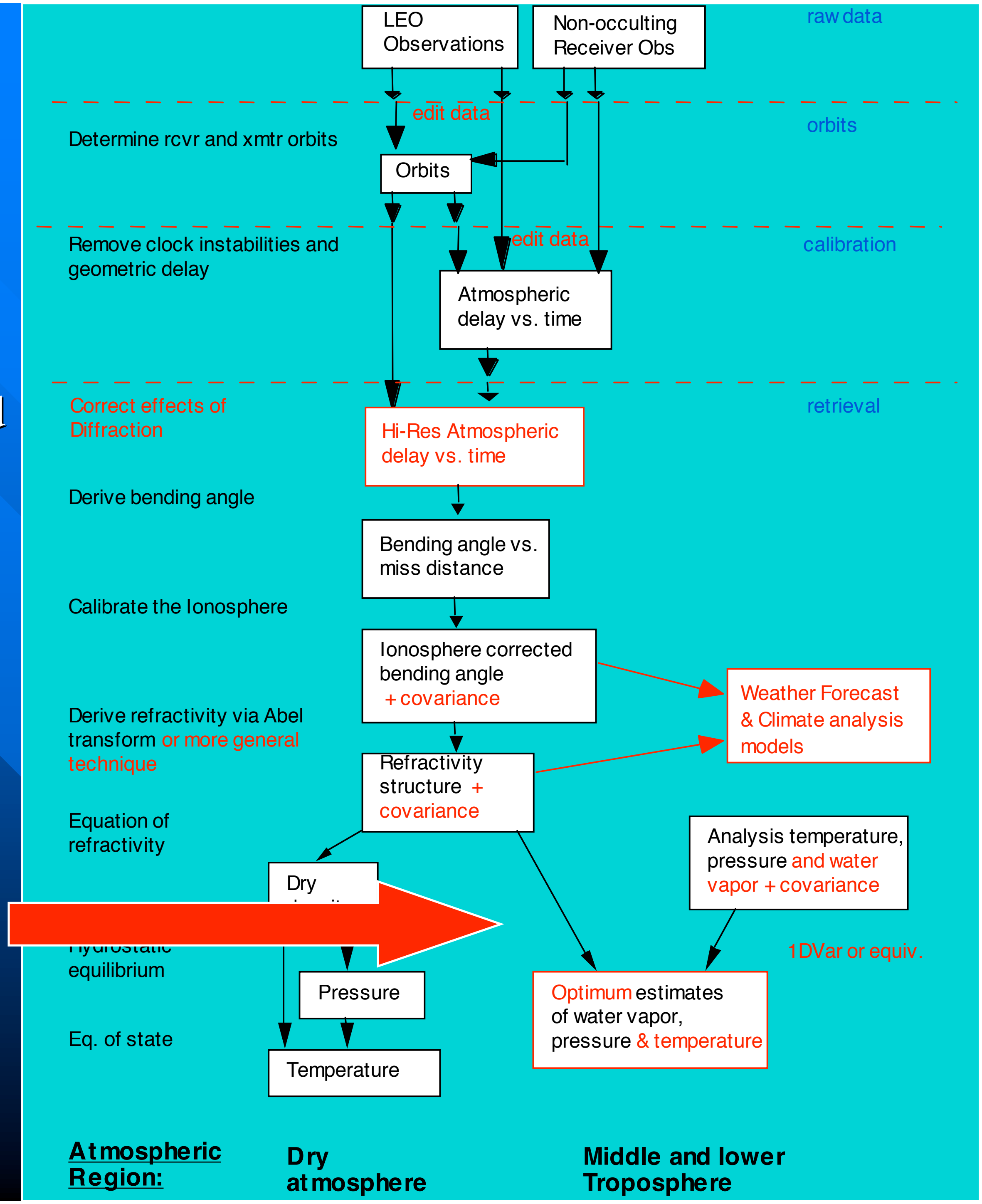


Refractivity of Condensed Water Particles

High dielectric constant (~80) of condensed liquid water particles suspended in the atmosphere slows light propagation via scattering
=> Treat particles in air as a dielectric slab

Particles are much smaller than GPS wavelengths

=> Rayleigh scattering regime

=> Refractivity of particles, N_p proportional to density of condensed water in atmosphere, W, and *independent* of particle size distribution.

- First order discussion given by *Kursinski* [1997].
- More detailed form of refractivity expression for liquid water given by *Liebe* [1989].

Liquid water drops:	$N_p \sim 1.4\, W$	[where W is in g/m^3]
Ice crystals:	$N_p \sim 0.6\, W$	[*Kursinski*, 1997].
Water vapor:	$N_w \sim 6\, \rho_v$	[where ρ_v is in g/m^3]

Refractivity of Condensed Water Particles

=> Same amount of water in vapor phase creates

~ 4.4 * refractivity of same amount of liquid water

~ 10 * refractivity of same amount of water ice

- Liquid water content of clouds is generally less than 10% of water vapor content (particularly for horizontally extended clouds)

=> Liquid water refractivity generally less than 2.2% of water vapor refractivity

=> Ice clouds generally contribute small fraction of water vapor refractivity at altitudes where water vapor contribution is already small

Clouds will *very* slightly increase apparent water vapor content

Deriving Humidity from GPS RO

- Two basic approaches
  - Direct method: use *N* & *T* profiles and hydrostatic B.C.
  - Variational method: use *N*, *T* & *q* profiles and hydrostatic B.C. with error covariances to update estimates of *T*, *q* and *P*.
- Direct Method
  - Theoretically less accurate than variational approach
  - Simple error model
  - (largely) insensitive to NWP model humidity errors
- Variational Method
  - Theoretically more accurate than simple method because of inclusion of apriori moisture information
  - Sensitive to unknown model humidity errors and biases
- Since we are evaluating a model we want water vapor estimates as independent as possible from models

=> We use the Direct Method

Direct Method: Solving for water vapor given N & T

$$N \equiv \{n-1\}\times 10^6 = a_1 \frac{P}{T} + a_2 \frac{P_w}{T^2} \qquad (1)$$

- Use temperature from a global analysis interpolated to the occultation location
- To solve for P and P_w given N and T, use constraints of hydrostatic equilibrium and ideal gas laws and one boundary condition

Solve for P by combining the hydrostatic and ideal gas laws and assuming temperature varies linearly across each height interval, i

$$P(z_{i+1}) = P(z_i)\left(\frac{T_i}{T_{i+1}}\right)^{\frac{\overline{m}_i \overline{g}_i}{R\dot{T}_i}} \qquad (2)$$

where:

z	height,
g	gravitation acceleration,
m	mean molecular mass of moist air
T	temperature
R	universal gas constant

Solving for water vapor given *N* & *T*

Given knowledge of $T(h)$ and pressure at some height for a boundary condition, then (1) and (2) are solved iteratively as follows:

1) Assume $P_w(h) = 0$ or 50% RH for a first guess

2) Estimate $P(h)$ via (2)

3) Use $P(h)$ and $T(h)$ in (1) to update $P_w(h)$

4) Repeat steps 2 and 3 until convergence.

Standard deviation of fractional P_w error (Kursinski et al., 1995):

$$\frac{\sigma_{P_w}}{P_w} = \left[(B+1)^2 \frac{\sigma_N^2}{N^2} + (B_s+2)^2 \frac{\sigma_T^2}{T^2} + B_s^2 \frac{\sigma_{P_s}^2}{P_s^2} \right]^{1/2}$$

where $B = a_1 TP \;/\; a_2 P_w$ and P_s is the surface pressure

Solving for Water Vapor given *N* & *T*

A moisture variable closely related to the GPS observations is specific humidity, q, the mass mixing ratio of water vapor in air.

Given P and P_w, q, is given by

$$q = \left[\frac{m_d}{m_w} \left\{ \frac{P}{P_w} - 1 \right\} + 1 \right]^{-1}$$

Note that GPS-derived refractivity is essentially a molecule counter for "dry" and water vapor molecules.

It is not a direct relative humidity sensor.

Estimating the Accuracy of GPS-derived Water Vapor

- Kursinski and Hajj, (2001) showed the standard deviation of the error in specific humidity, q, due to changes in refractivity (N), temperature (T) and pressure (P) from GPS is

$$\sigma_q = \left[\left(C+q\right)^2 \left(\frac{\sigma_N}{N}\right)^2 + \left(C+2q\right)^2 \left(\frac{\sigma_T}{T}\right)^2 + \left(C+q\right)^2 \left(\frac{\sigma_{P_s}}{P_s}\right)^2 \right]^{1/2}$$

where $C = a_1 T m_w / a_2 m_d$,

Similarly, the error in relative humidity, U $(=e/e_s)$, is

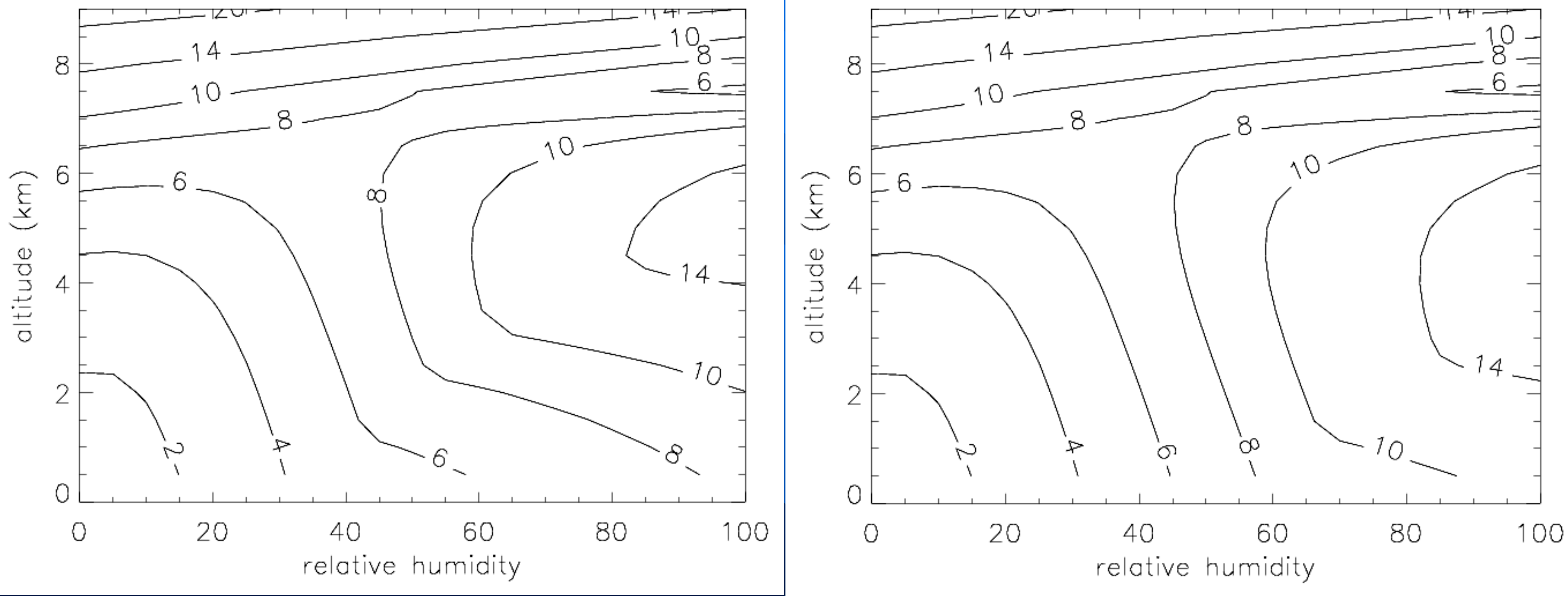
$$\sigma_U = \left[\left(B_s + U\right)^2 \frac{\sigma_N^2}{N^2} + \left(B_s + U\left(2 - \frac{L}{R_v T}\right)\right)^2 \frac{\sigma_T^2}{T^2} + B_s^2 \frac{\sigma_P^2}{P^2} \right]^{1/2}$$

where L is the latent heat and $B_s = a_1 TP / a_2 e_s$.

- The temperature error is particularly small in the tropics (1 –1.25 K)

Relative Humdity Error

- *Humidity errors in % for tropical conditions using Kuo et al. (2004) errors with a maximum refractivity error of 2 and 3% respectively in panels a and b*



Variational Estimation of Water

- Given the GPS observations alone, we have an *underdetermined* problem in solving for P_w
- Previously, we assumed knowledge of temperature to provide the missing information & solve this problem.
- However, temperature estimates have errors that we should incorporate into our estimate
- If we combine apriori estimates of temperature *and* water from a forecast or analysis with the GPS refractivity estimates we create an *overdetermined* problem

=> We can use a least squares approach to find the optimal solutions for T, P_w and P.

Variational Estimation of Water

In a variational retrieval, the most probable atmospheric state, *x*, is calculated by combining *a priori* (or background) atmospheric information, x^b, with observations, y^o, in a statistically optimal way.

The solution, *x*, gives the best fit - in a least squared sense - to both the observations and *a priori* information.

For Gaussian error distributions, obtaining the most probable state is equivalent to finding the *x* that minimizes a cost function, *J(x),* given by

$$J(x) = \frac{1}{2}\left\{x - x^b\right\}^T B^{-1}\left\{x - x^b\right\} + \frac{1}{2}\left\{y^o - H(x)\right\}^T \left\{E + F\right\}^{-1}\left\{y^o - H(x)\right\}$$

where:

- *B* is the background error covariance matrix.
- *H(x)* is the forward model, mapping the atmospheric information *x* into measurement space.
- *E* and *F* are the error covariances of measurements and forward model respectively.
- Superscripts *T* and –1 denote matrix transpose and inverse.

Variational Estimation of Water

- The model consists of T, q and $P_{surface}$ all of which are improved when GPS refractivity information is added
- The normalized form has allowed us to combine "apples" (an atmospheric model state vector) and "oranges" (GPS observations of bending angles or refractivity).
- The variational approach makes optimal use of the GPS information relative to the background information so it uses the GPS to solve for water vapor when appropriate and dry density when when appropriate in colder, drier conditions
- The error covariance of the solution, x, is

$$B' = \left[B^{-1} + K^T \{E + F\}^{-1} K \right]^{-1}$$

where K is the gradient of y^o with respect to x.

NOTE: As will be discussed in following lectures, the distinction between E and F is important

• F is important if the forward model is not as good as the observations so that $F \geq E$

Variational Estimation of Water: Advantages & Disadvantages

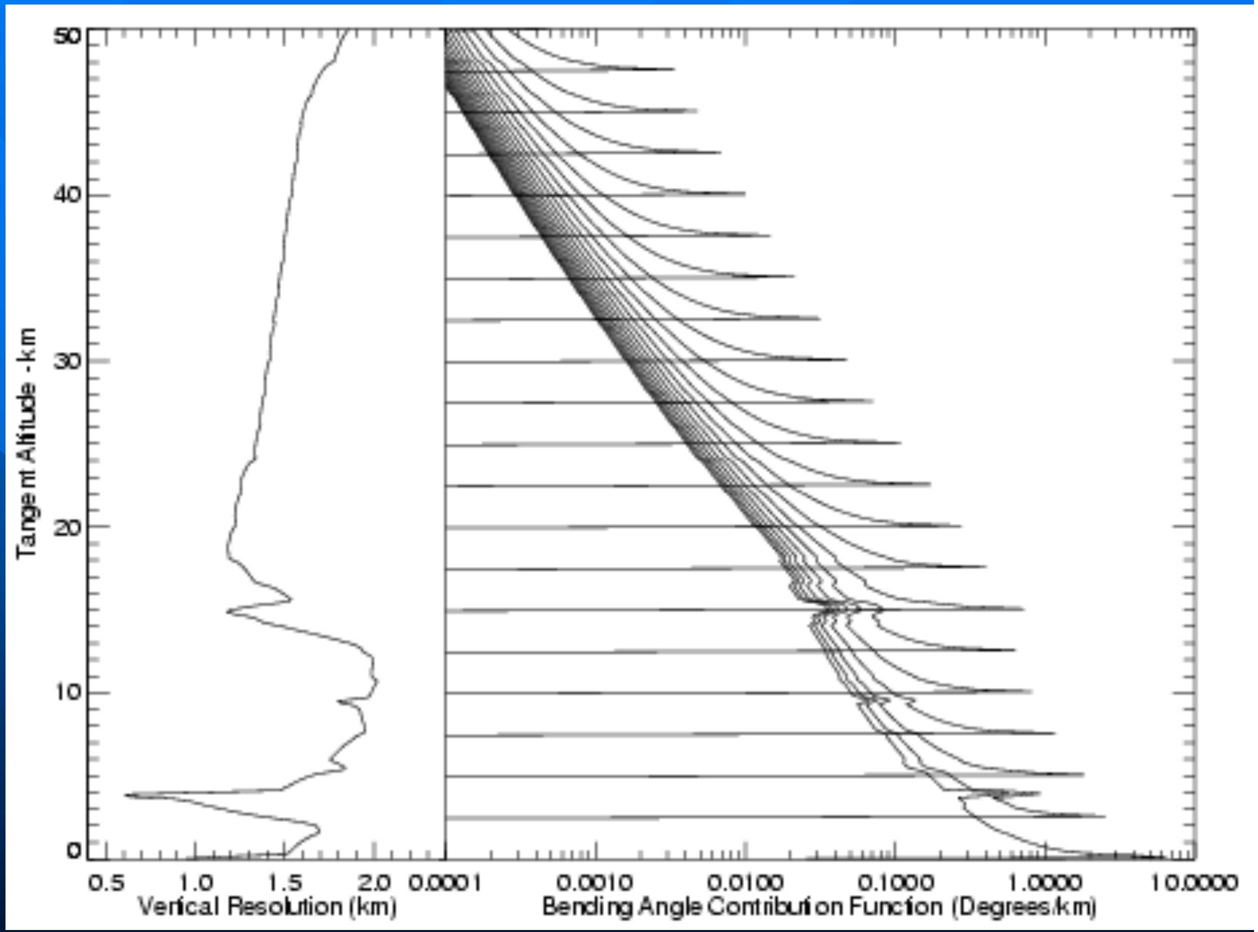
- The solution is theoretically better than the solution assuming only temperature
- The solution is limited to the model levels and GPS generally has higher vertical resolution than models
- The solution is as good as its assumptions
  - Unbiased apriori
  - Correct error covariances
- Model constraints are significant in defining the apriori water estimates and may therefore yield unwanted model biases in the results
- Temperature approach provides more independent estimate of water vapor

Vertical and Horizontal Resolution of RO

- Resolution associated with distributed bending along the raypath
- Diffraction limited vertical resolution
- Horizontal resolution

Resolution: Bending Contribution along the Raypath

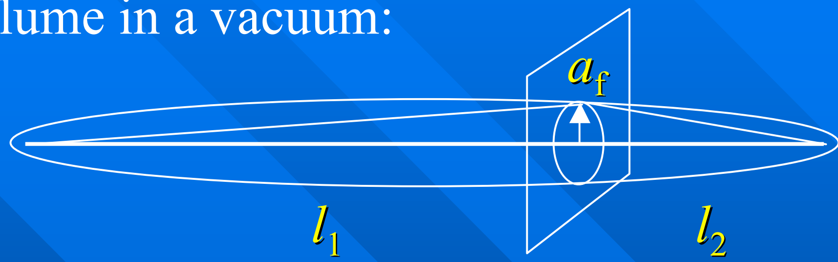
- Contribution to bending estimated from each 250 m vertical interval



- Results based on a radiosonde profile from Hilo, Hawaii
- Left panel shows the vertical interval over which half the bending occurs
- Demonstrates how focused the contribution is to the tangent region

Fresnel's Volume: Applicability of Geometric Optics.

Generally, EM field at receiver depends on the refractivity in all space.
In practice, it depends on the refractivity in the finite volume around the geometric-optical ray (Fresnel's volume).
The Fresnel's volume characterizes the physical "thickness" of GO ray.
The Fresnel's volume in a vacuum:



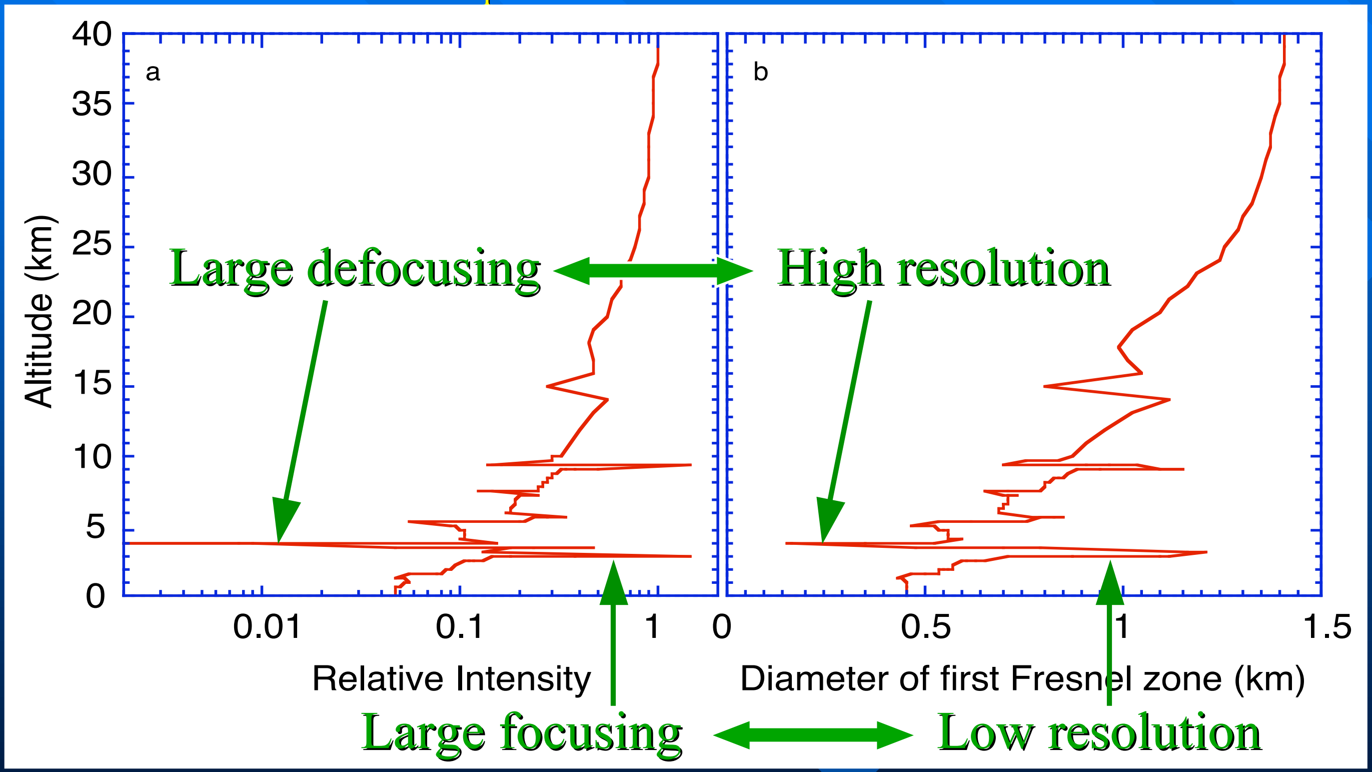
$$\sqrt{l_1^2 + a_f^2} + \sqrt{l_2^2 + a_f^2} - l_1 - l_2 = \lambda / 2$$

The Fresnel's zone (cross-section of the Fresnel's volume): $a_f = \sqrt{\dfrac{\lambda l_1 l_2}{l_1 + l_2}}$

Two rays may be considered independent when their Fresnel's volumes do not overlap.
Geometric optics is applicable when transverse scales of N-irregularities are larger than the diameter of the first Fresnel zone.

Atmospheric Effects on the First Fresnel Zone Diameter

- Without bending, 2 a_f ~ 1.4 km for a LEO-GPS occultation
- The atmosphere affects the size of the first Fresnel zone, generally making it smaller
- The bending gradient, $d\alpha/da$, causes defocusing which also causes a more rapid increase in length vertically away from the tangent point such that the $\lambda/2$ criterion is met at a smaller distance than a_f.



=> The Fresnel zone diameter and resolution is estimated from the amplitude data

GPS Vertical Resolution

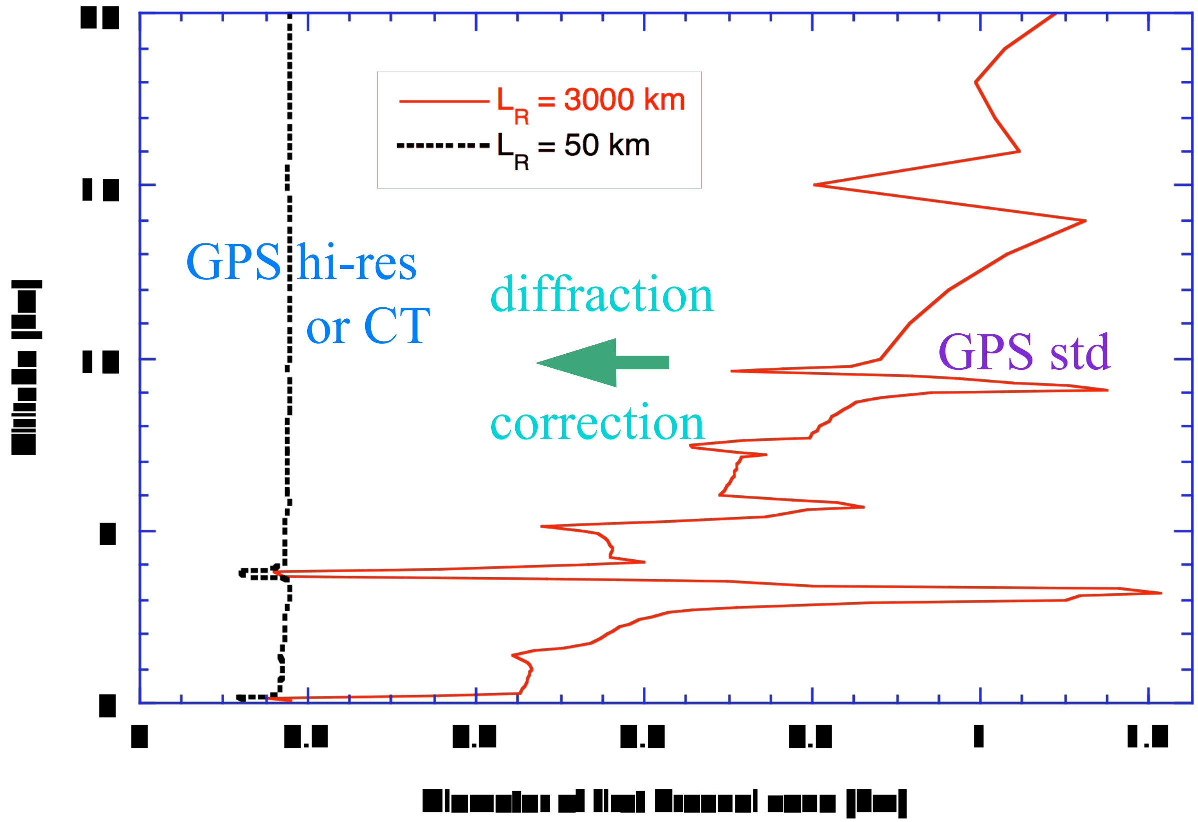


Figure estimated from Hilo radiosonde profile shown previously

- Natural vertical resolution of GPS: typically ~0.5 to 1.4 km
- Diffraction correction can improve this to ~200 m (or better)
  - þ"GPS hi-res" or CT data

Horizontal Resolution

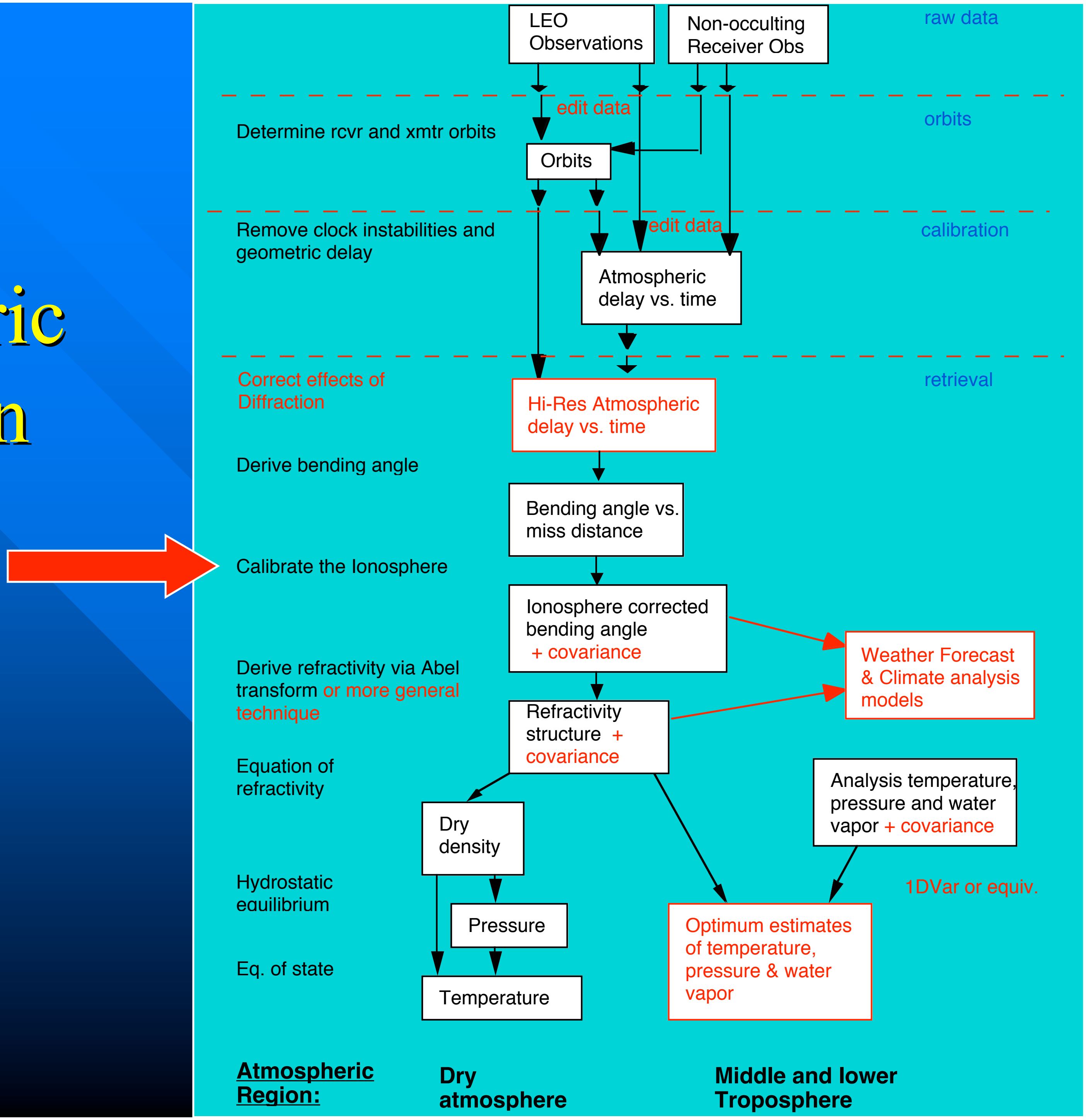
- Different approaches to estimating it
  - Gaussian horizontal bending contribution: $\pm$300 km
  - Horizontal interval of half the bending occurs: ~300 km
  - Horizontal interval of natural Fresnel zone: ~250 km
  - Horizontal interval of diffraction corrected, 200 m Fresnel zone: (Probably not realistic) ~100 km

- *Overall approximate estimate is ~ 300 km*

Intro to Difficulties of RO soundings

- *Residual ionospheric noise*
- *Multipath,*
- *Superrefraction*
- *Upper boundary conditions*

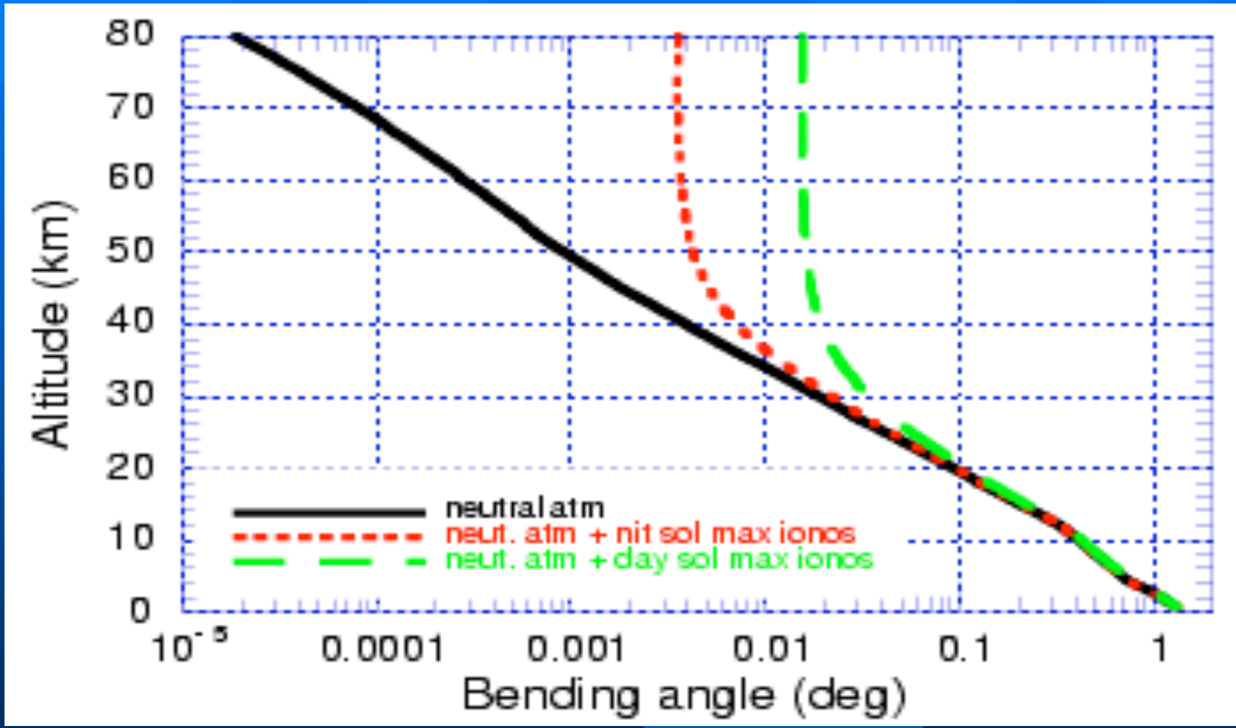
Ionospheric Correction



April 16, 2009

Ionosphere Correction

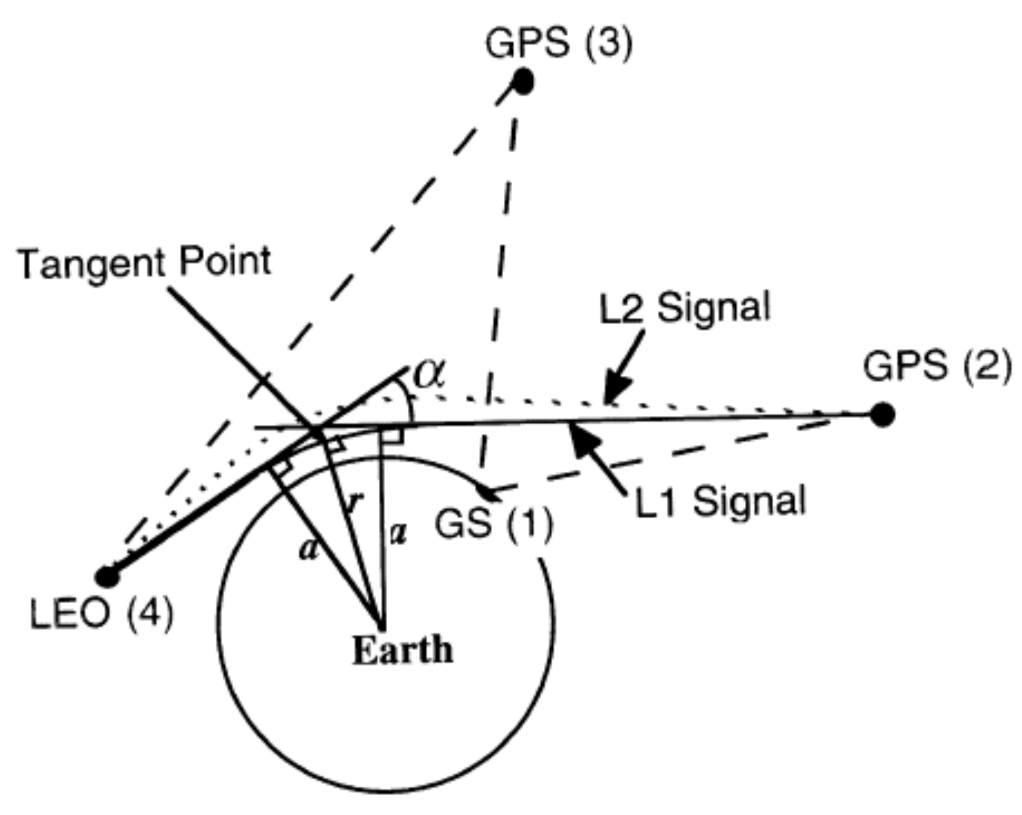
- The goal is to isolate the neutral atmospheric bending angle profile to as high an altitude as possible
- Problem is the bending of the portion of the path within the ionosphere dominates the neutral atmospheric bending for raypath tangent heights above 30 to 40 km depending on conditions (daytime/nightime, solar cycle)



- Need a method to remove unwanted ionospheric effects from the bending angle profile
- This will be discussed briefly here and in more detail in S. Syndergaard's talk

Ionosphere Correction

- Ionospheric refractivity scales to first order as $1/f^2$ where f is the signal frequency
- With two frequencies one can estimate & remove 1st order ionosphere effect as long as paths for L1 and L2 are coincident
- However, occultation signal paths at the two GPS frequencies, L1 and L2, differ and do not sample the same regions of the atmosphere and ionosphere

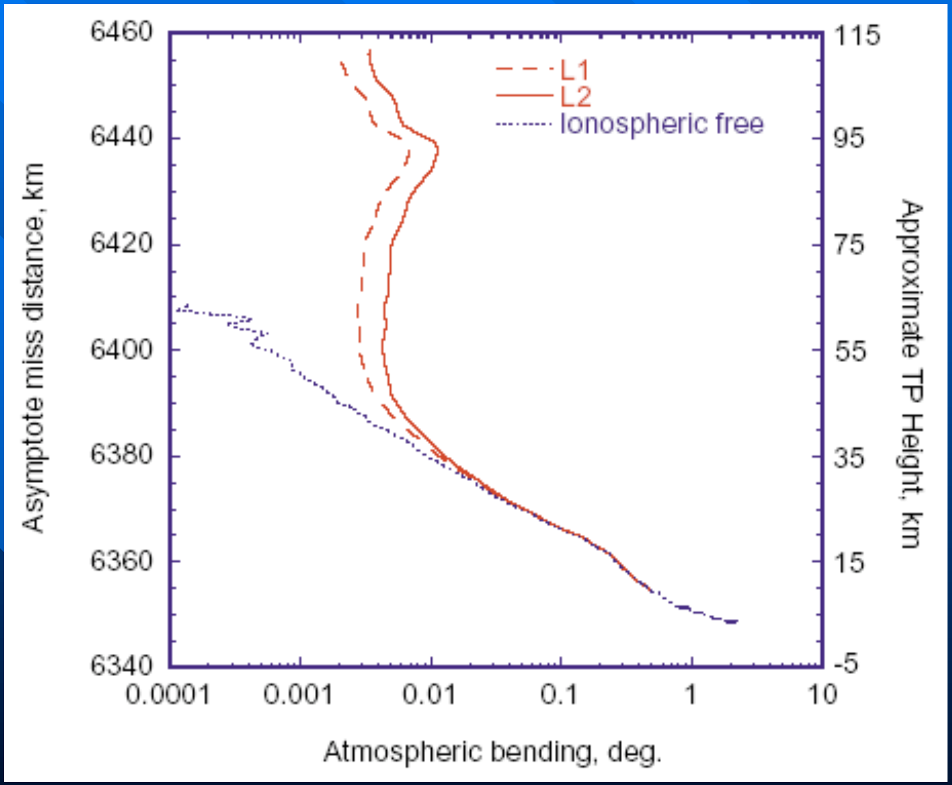


- So $a_{L1}(t) >< a_{L2}(t)$

Ionosphere Correction

- Vorob'ev & Krasil'nikova (1994) developed a relatively simple solution
- The trick to first order is to interpolate the $\alpha(a_{L2})$ such that the asymptotic miss distances of the L2 observations match those of the L1 observations ($a_{L2} = a_{L1}$) before applying the ionospheric correction
- This causes the L1 and L2 signal path tangent regions to be coincident when the correction is applied

Example of ionospheric correction



Refractivity Error

Fractional error in refractivity derived by GPS RO as estimated in Kursinski et al. 1997

Considers several error sources

Solar max, low SNR

Solar min, high SNR

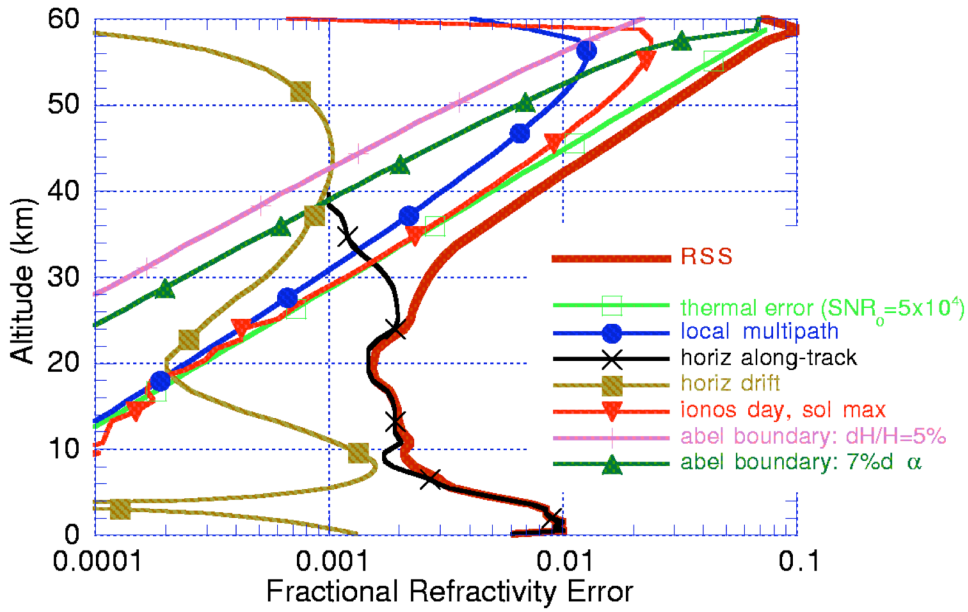


Figure 10

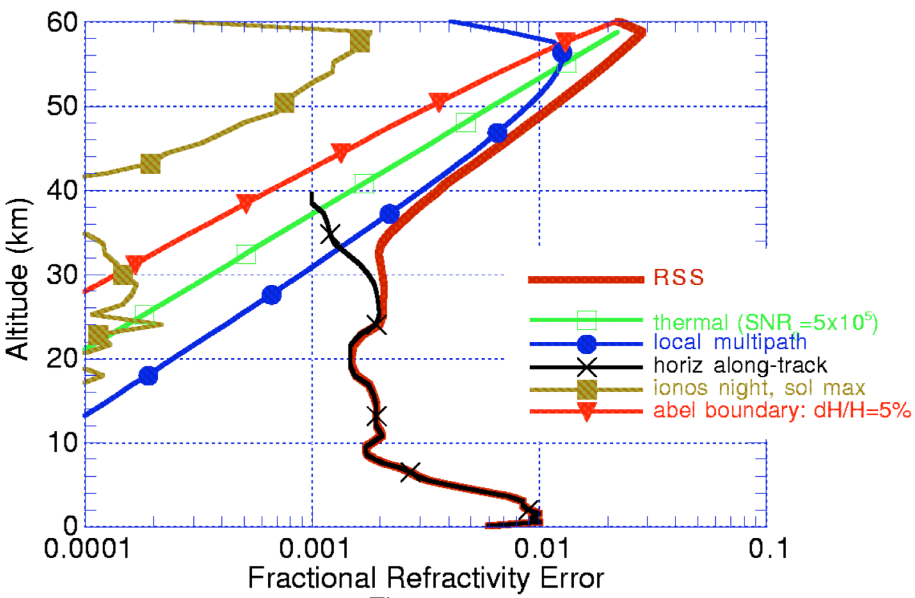


Figure 13

GPSRO Temperature Accuracy

- Temperature is proportional to Pressure/Density
- So $\varepsilon_T/T = \varepsilon_P/P - \varepsilon_\rho/\rho$
- Very accurate in upper troposphere/lower troposphere (UTLS)

Solar max, low SNR

Solar min, high SNR

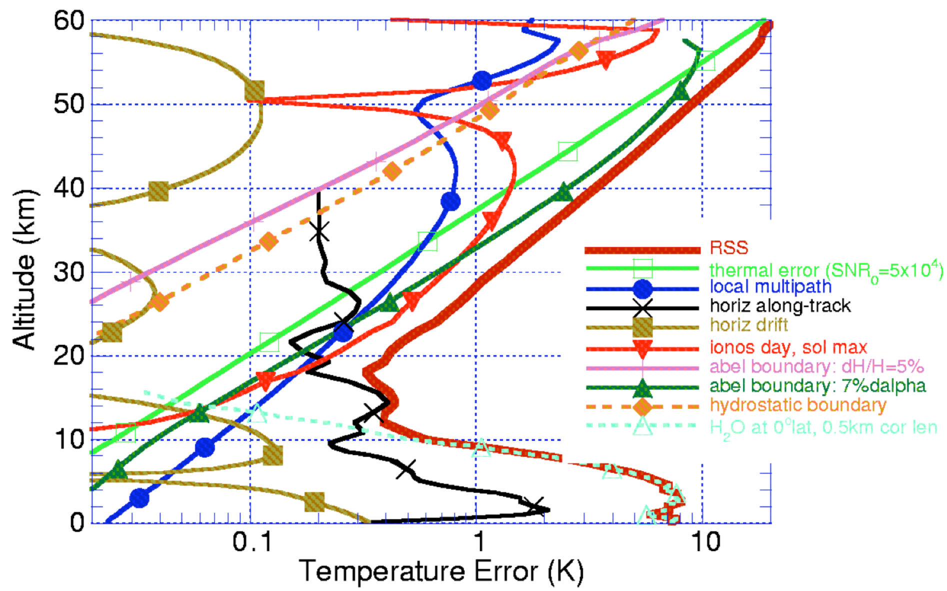


Figure 12

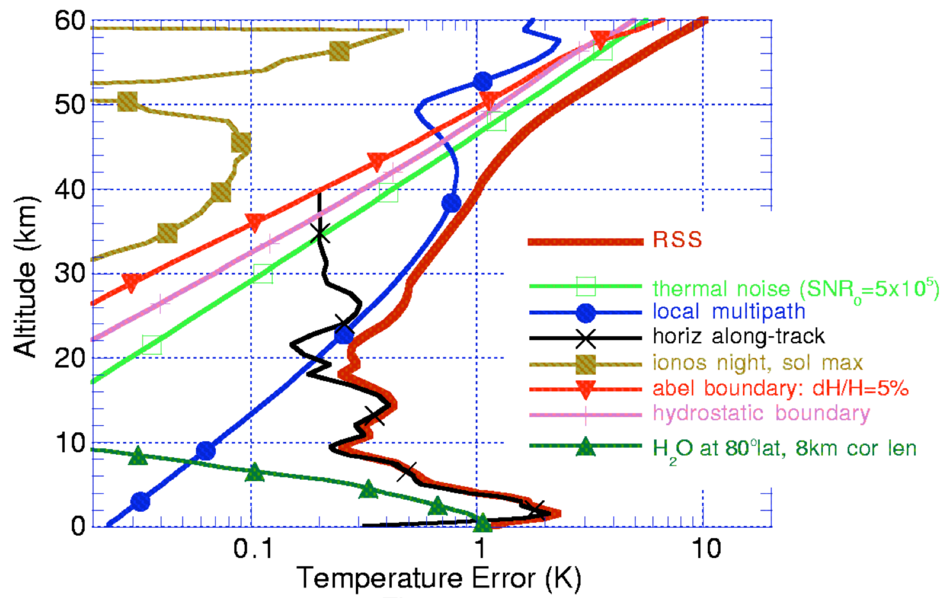


Figure 15

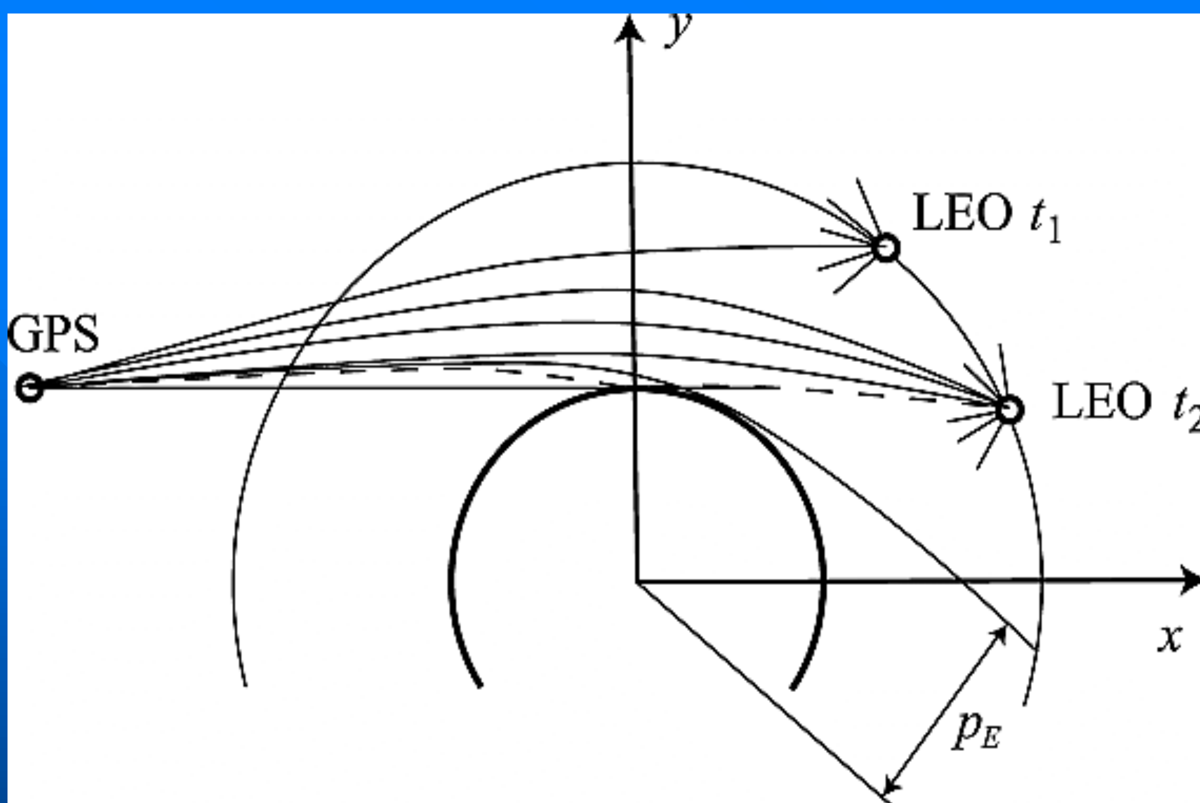
Reducing the ionospheric effect of the solar cycle

- With the current ionospheric calibration approach, a subtle systematic ionospheric residual effect is left in the bending angle profile
- This effect is large compared to predicted decadal climate signatures ~0.1K/decade
- The residual ionosphere effect is due to an overcorrection of the ionospheric effect
- This causes the ionospherically corrected bending angle to change sign and become slightly negative.
- This negative bending can be averaged and subtracted from the bending angle profile to largely remove the bias
- This idea needs further work but appears promising

Upper Boundary Conditions

- We have two upper boundary conditions to contend with: the Abel integral and the hydrostatic integral.
- For the Abel, we can either extrapolate the bending angle profile to higher altitudes or combine the data with climatological or weather analysis information
- Hydrostatic integral requires knowledge of pressure near the stratopause. Typical approach is to use an estimate of temperature combined with refractivity derived from GPS to determine pressure.
- Problem with using a climatology is it may introduce a bias
- Also a basic challenge is to determine, based on the data accuracy, at what altitude to start the abel and hydrostatic integrals

Atmospheric Multipath

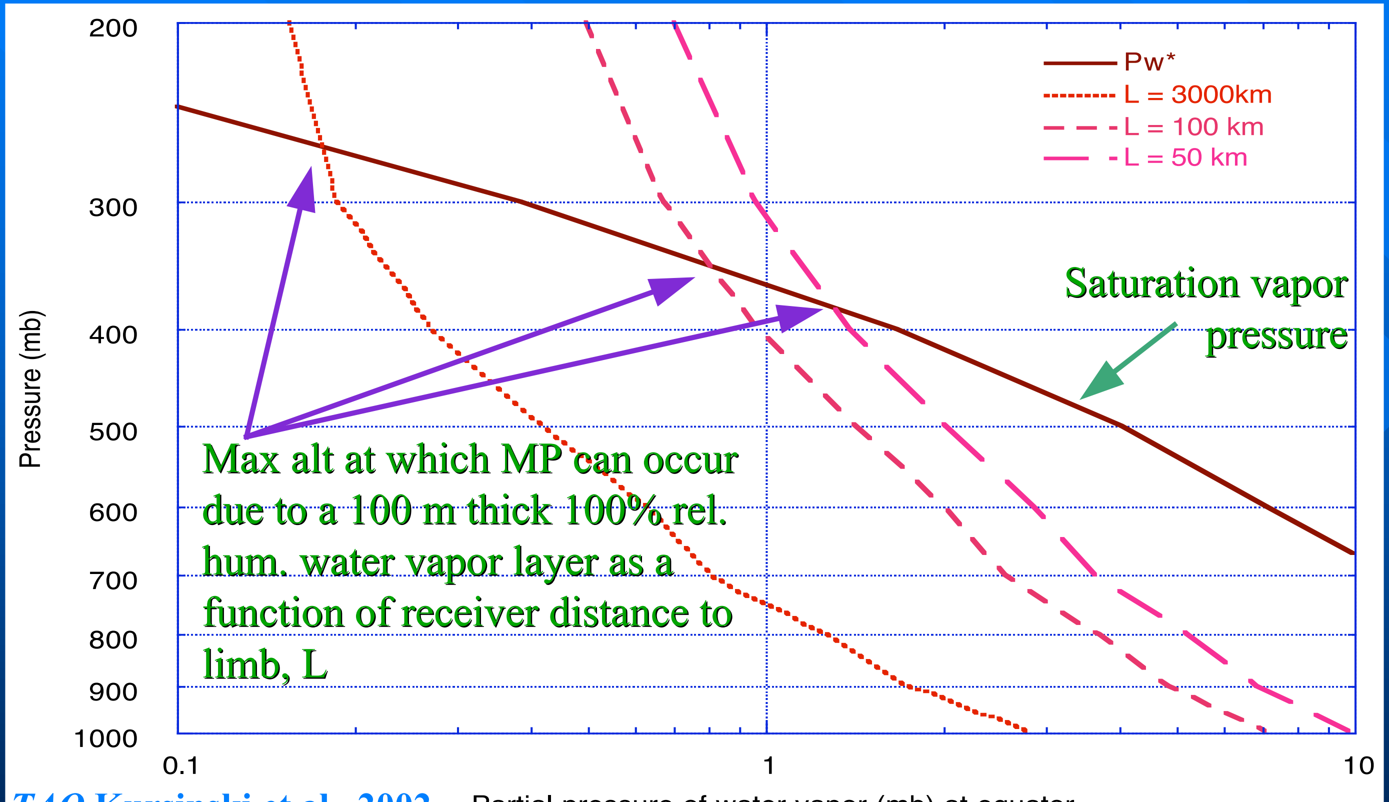


- Standard retrievals assume only a single ray path between GPS and LEO
- CT retrievals move the virtual receiver closer to the limb where the rays do not cross so each ray can be accounted for.
- Since Standard retrievals miss some of the paths, they will systematically *underestimate* refractivity in regions where multipath occurs.

Existence & Mitigation of Atmospheric Multipath

For atmospheric multipath to occur, there must be large vertical refractivity gradients that vary rapidly with height with substantial horizontal extent

One expects multipath in regions of high absolute humidity



TAO, Kursinski et al., 2002

Moving the receiver closer to the limb reduces the maximum altitude at which multipath can occur but it does not eliminate ray crossings

Super-refraction

Super-refraction: when the vertical refractivity gradient becomes so large that the radius of curvature of the ray is smaller than the radius of curvature of the atmosphere, causing the ray to curve down toward the surface.

No raypath connecting satellites can exist with a tangent height in this altitude interval.

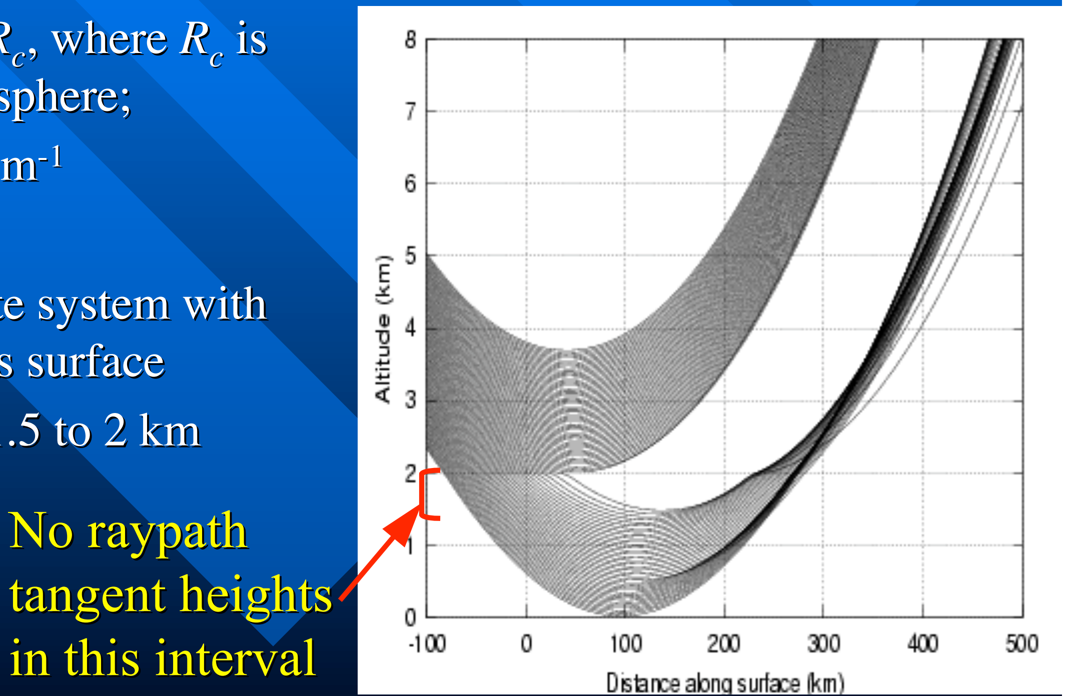
- A signal launched horizontally at this altitude will be trapped or ducted
- This presents a serious problem for our abel transform pair

Super-refractive conditions occur when the refractivity gradient $dN/dr < -10^6/R_c$, where R_c is the radius of curvature of the atmosphere;

- Critical $dN/dr \sim -0.16$ N-units m^{-1}

Figure shows raypaths in the coordinate system with horizontal defined to follow Earth's surface

Ducting layer in Figure extends from 1.5 to 2 km altitude



Super-refraction

The vertical atmospheric gradients required to satisfy this inequality can be found by differentiating the dry and moist refractivity terms of the *N* equation:

$$\frac{dN}{dr} = -\frac{b_1 P}{H_P T} - \left(\frac{b_1 P}{T^2} + \frac{2 b_2 P_W}{T^3}\right)\frac{dT}{dr} + \frac{b_2}{T^2}\frac{dP_W}{dr}$$

where H_P is the pressure scale height.

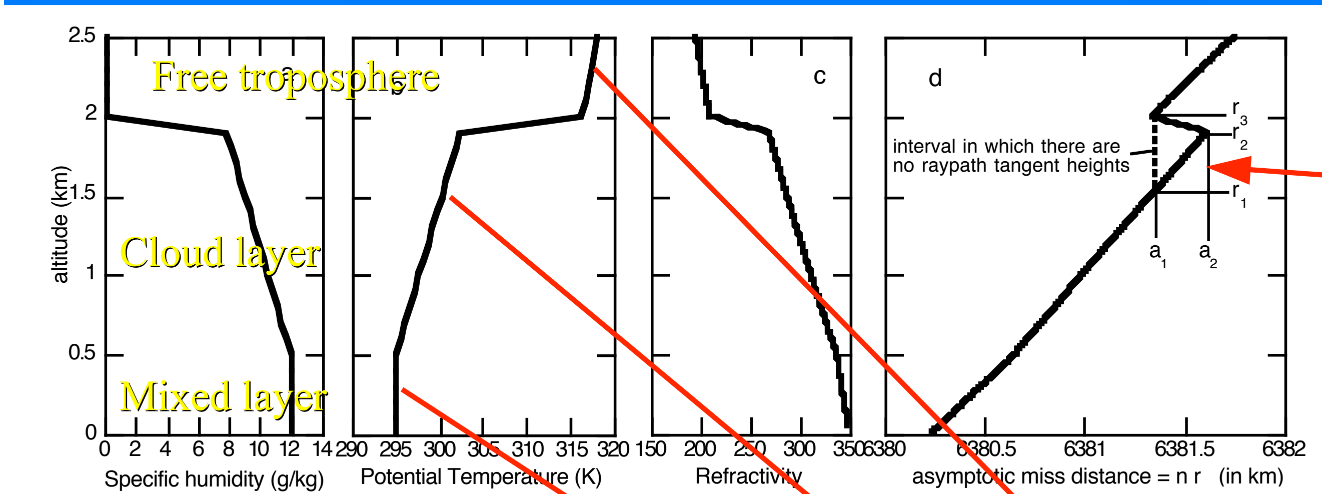
The three terms on the RHS represent the contributions of the vertical pressure, temperature, and water vapor mixing ratio gradients to *dN*/*dr*.

P gradients are too too small to produce critical *N* gradients.

Realistic *T* gradients are smaller than +140 K/km needed to produce critical *N* gradients

P_w gradients can exceed the critical -34 mbar/km gradient in the warm lowermost troposphere and therefore can produce super-refraction.

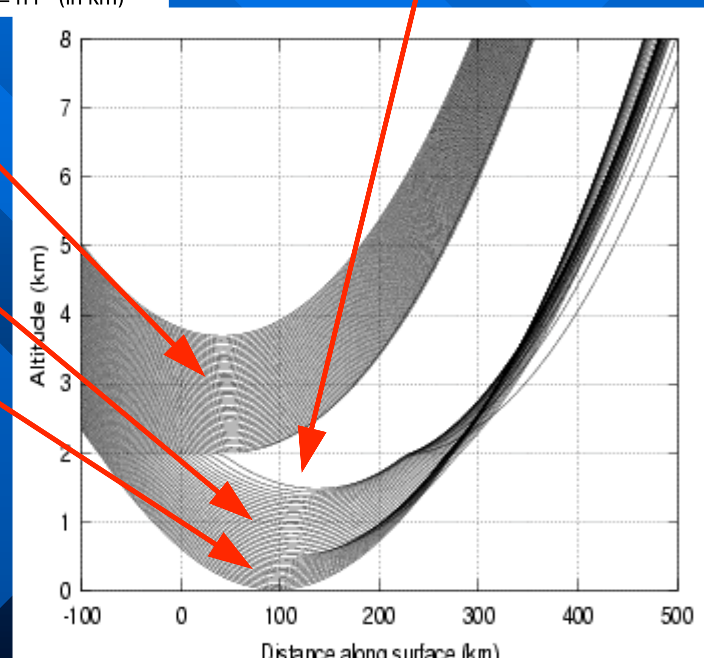
Super-refraction



$$\sqrt{n^2 r^2 - a^2}$$

becomes imaginary within the interval

PBL schematic



Feiqin Xie did his thesis here on developing a solution to the super-refraction problem

Occultation Features Summary

Occultation signal is a point source

- Þ Fresnel Diffraction limited vertical resolution
- Þ Very high vertical resolution

We control the signal strength and therefore have much more control over the SNR than passive systems

- Þ Very high precision at high vertical resolution

Self calibrating technique

- Þ Source frequency and amplitude are measured immediately before or after each occultation so there is no long term drift
- Þ Very high accuracy

Occultation Features Summary

Simple and direct retrieval concept

Þ Known point source rather than unknown distributed source that must be solved for

Þ Unique relation between variables of interest and observations (unlike passive observations)

Þ Retrievals are independent of models and initial guesses

Height is independent variable

Þ Recovers geopotential height of pressure surfaces remotely completely independent of radiosondes

Occultation Features Summary

Microwave system

Þ Can see into and below clouds, see cloud base and multiple cloud layers

Þ Retrievals only slightly degraded in cloudy conditions

Þ Allows all weather global coverage with high accuracy and vertical resolution

Complementary to Passive Sounders

Þ Limb sounding geometry and occultation properties complement passive sounders used operationally