Module 7 - Lecture 20

We will spend this lecture working out some humidity problems. This way you will learn more about the four humidity variables: mixing ratio (r), saturation mixing ratio (r_S), relative humidity (RH), and dew point temperature (T_D).

Example 1

We are given an air temperature of 90^0F and a mixing ratio (r) of 6 g/kg. We need to find the relative humidity and the dew point temperature. We will start by constructing a 4-column chart and fill in the information we are given.

We start by entering the data we were given in the table. Once you know the air temperature, you can look up the saturation mixing ratio value; it is 30 g/kg for 90^0F air. Air at this temperature can potentially hold 30 grams of water vapor per kilogram of dry air. The actual value in this example is 6 grams per kilogram. Once you know the mixing ratio and saturation mixing ratio, you can calculate the relative humidity as shown below.

Given mixing ratio, r = 6 g/kg
Air Temp = 90 °F

T_{air}	r	r_s	RH
90°F	6 g/kg	30 g/kg	20%

from chart

$$RH = 100\% \times \frac{r}{r_s} = 100\% \times \frac{6}{30} = 20\%$$

(A) We imagine cooling the air from 90^0F to 70^0F, then to 55^0F, and finally to 45^0F.

(B) At each step we look up the saturation mixing ratio and enter it on the chart. Note that the saturation mixing ratio values decrease as the air is cooling.

(C) The mixing ratio does not change as we cool the air because we are not adding or removing water vapor.

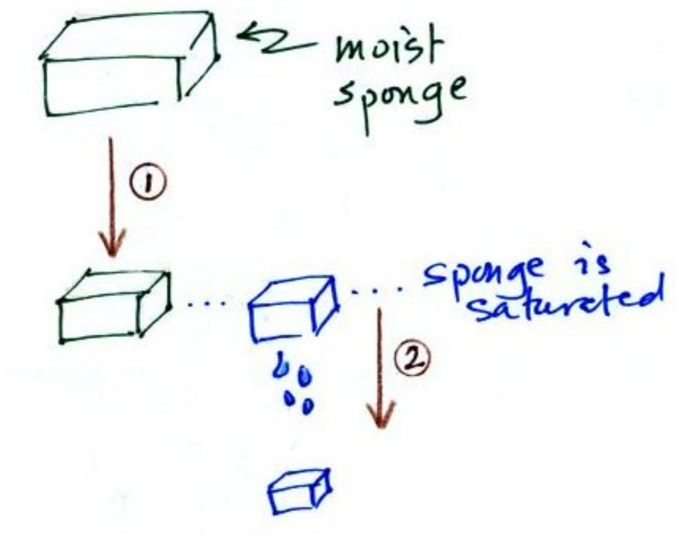
(D) Note how the relative humidity increases as we cool the air. The air still contains the same amount of water vapor but the air's capacity for water vapor is decreasing. Finally at 45^0F the relative humidity (RH) becomes 100%. We have cooled the air until it has become saturated. The dew point temperature, the temperature at which the air becomes saturated, is 45^0F.

A	T	C r	B r_s	RH	D
	90°F	6 g/kg	30 g/kg	20%	
	70	6	15	40%	
	55	6	9	67%	
	45	6	6	100%	

What would happen if we cooled the air further still, below the dew point temperature? 35^0F air cannot hold the 6 grams of water vapor that 45^0F air can. You can only "fit" 4 grams of water vapor into the 35^0F air. The remaining 2 grams would condense. If this happened at ground level, the ground would become wet with dew. If it happens above the ground, the water vapor condenses onto small particles in the air and forms fog or a cloud. The mixing ratio will decrease from 6 to 4 because water vapor is removed from the air through condensation into water.

T	r	r_s	RH
45°F	6 g/kg	6 g/kg	100%
35	4	4	100%

In many ways, cooling moist air is similar to squeezing a moist sponge. At first when you squeeze the sponge, no water drips out (1 below). Eventually you get to a point where the sponge is saturated (2 below). This is like reaching the dew point. If you squeeze the sponge any further (or cool air below the dew point) water will begin to drip out of the sponge (water vapor will condense from the air).



Example 2

In this problem, we will start with an air temperature of 90^0F and a relative humidity of 50%. We need to figure out the mixing ratio and the dew point temperature. The problem is worked out in detail below.

(A First you fill in the air temperature and the relative humidity data that you are given. We can look up the saturation mixing ratio because we know the air temperature. The value is 30 g/kg.

(B) Air that is filled to 50% of its capacity can hold up to 30 g/kg. 50% or half of 30 is 15, which is the mixing ratio.

Given $T_{air} = 90°F$

$RH = 50\%$

What is r?

What is Td?

Tair	r	rs	RH
90°F	15 g/kg	30 g/kg	50%

A from chart

B

$$RH = 100\% \times \frac{r}{r_s}$$

$$50\% = 100\% \times \frac{r}{30\ g/kg}$$

$$30\ g/kg \times \frac{50\%}{100\%} = 15\ g/kg$$

Finally, to determine the dew point, we imagine cooling the air. The saturation mixing ratio decreases, the mixing ratio stays constant, and the relative humidity increases. In this example the relative humidity reaches 100% when the air has cooled to 70^0F. This is the dew point temperature.

T	r	r_s	RH
90°F	15 g/kg	30 g/kg	50%
85	15	25	60%
80	15	21	71%
75	15	18	83%
70	15	15	100%

cool the air — constant — decr. — incr.

Td

We can use results from Example 1 and Example 2 to learn a useful rule. In the first example, the difference between the air and dew point temperatures was large (45^0F) and the relative humidity was low (20%). In the second example, the difference between the air and dew point temperatures was smaller (20^0F) and the relative humidity was higher (50%). The easiest way to remember this rule is to remember the case where there is no difference between the air and dew point temperatures. The relative humidity is then 100%.

Example #1

Tair = 90°F

Td = 45°F

Big difference

RH = 20%

Example #2

Tair = 90°F

Td = 70°F

small difference

RH = 50%

When there is a big difference between Tair & Td the RH is Low

small difference the RH is High

no difference the RH is 100%

Example 3

We are given a mixing ratio of 10.5 g/kg and a relative humidity of 50%. What are the air temperature and the dew point temperature?

(1) The air contains 10.5 g/kg of water vapor which is 50% or half of what the air could potentially hold. So the air capacity or saturation mixing ratio must be 21 g/kg.

(2) Once you know the saturation mixing ratio, you can look up the air temperature in a table. 80^0F air has a saturation mixing ratio of 21 g/kg.

(3) Then you imagine cooling the air until the relative humidity reaches 100%. This occurs at 60^0F. The dew point temperature is 60^0F.

Problem #3 $r = 10.5$ g/kg What is Tair?
$RH = 50\%$ What is Td?

T	r	r_s	RH
② 80°F	10.5	21 g/kg	50%
75			
70 ③			
65			
60	10.5	10.5	100%

Td

① $RH = 100\% \times \frac{r}{r_s}$

$50\% = 100\% \times \frac{10.5}{r_s}$

$r_s = \frac{100\%}{50\%} \times 10.5$

$\boxed{r_s = 21 \text{ g/kg}}$

Example 4

This is probably the most difficult problem. We are given the air temperature (90^0F) and the dew point temperature (50^0F) and asked to figure out the mixing ratio and the relative humidity. We enter the two temperatures onto a chart and look up the saturation mixing ratio for each.

Problem #4 T_{air} = 90°F what is r?

T_d = 50°F what is RH?

T	r	r_s	RH
90°F		30 g/kg	
50°F		7.5g/kg	

Recall the three earlier examples. When we cooled air to the dew point temperature, the mixing ratio did not change. So the mixing ratio must have been 7.5 g/kg all along. We can set the mixing ratio equal to the value of the saturation mixing ratio at 50 F, 7.5 g/kg.

T	r	r_s	RH
90	?	30 g/Kg	?
80			
70			
60			
50	7.5g/kg	7.5 g/kg	100%

Once we know the mixing ratio at 90^0F, it is a simple matter to calculate the relative humidity, which is 25%.

T	r	r_s	RH
90 °F	7.5 g/kg	30 g/kg	25%
50 °F	7.5	7.5 g/kg	100%

$$RH = 100\% \times \frac{7.5}{30} = 25\%$$

Here is a summary of the important facts and properties of the four humidity variables that we have been discussing.

1. the mixing ratio tells you the actual amount of WV in the air.

2. r only changes if you add or remove water vapor. / ordinarily cooling the air doesn't change the mixing ratio.

3. The saturation mixing ratio tells you how much wv air can potentially hold. The air's capacity for w.v.

4. If you know T_{air} you can look up the saturation mixing ratio, r_s, in a table.

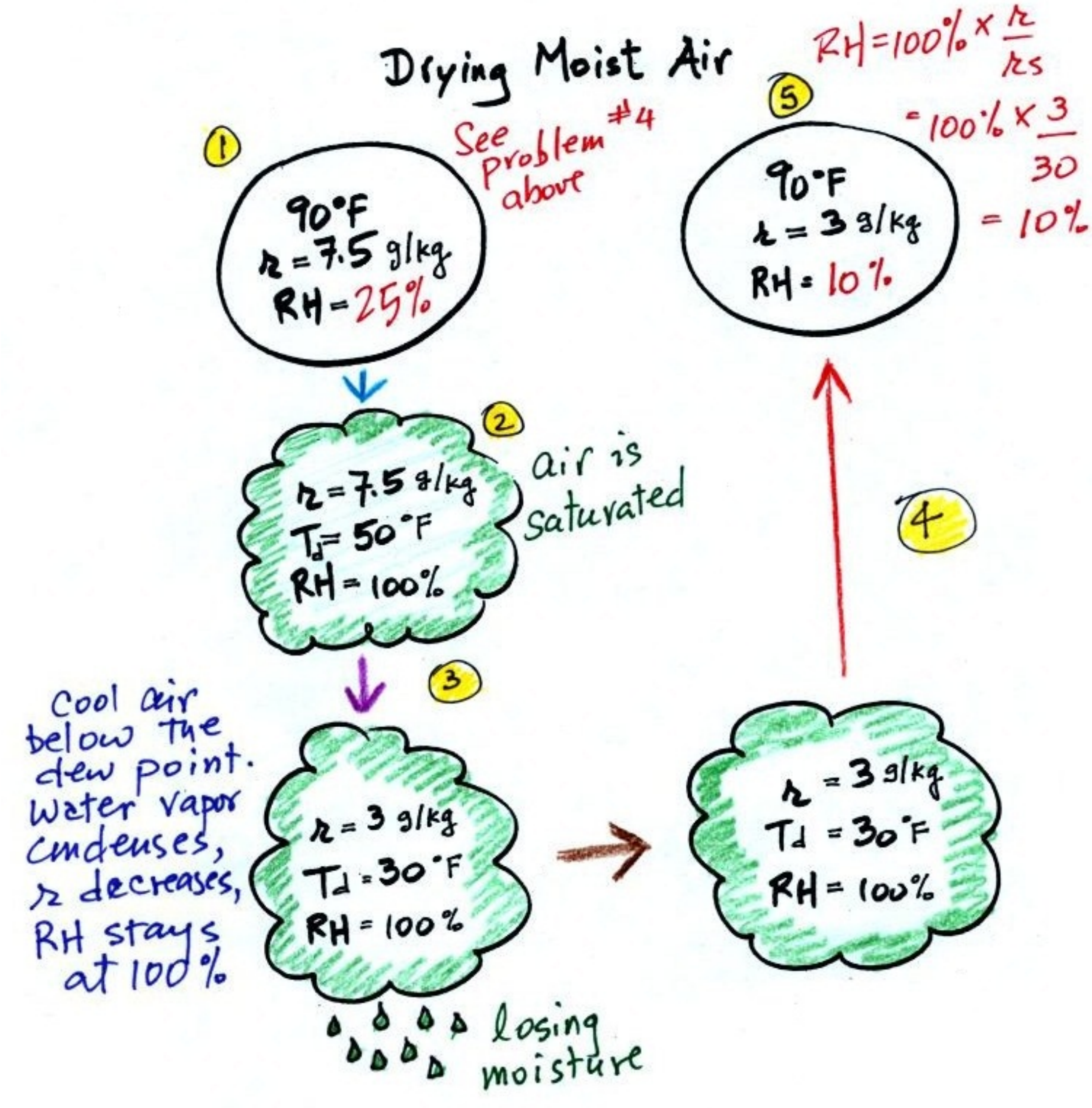
5. The RH tells you how close the air is to being filled with water vapor $RH = 100\% \times \frac{r}{r_s}$

6. cooling moist air causes the RH to increase.

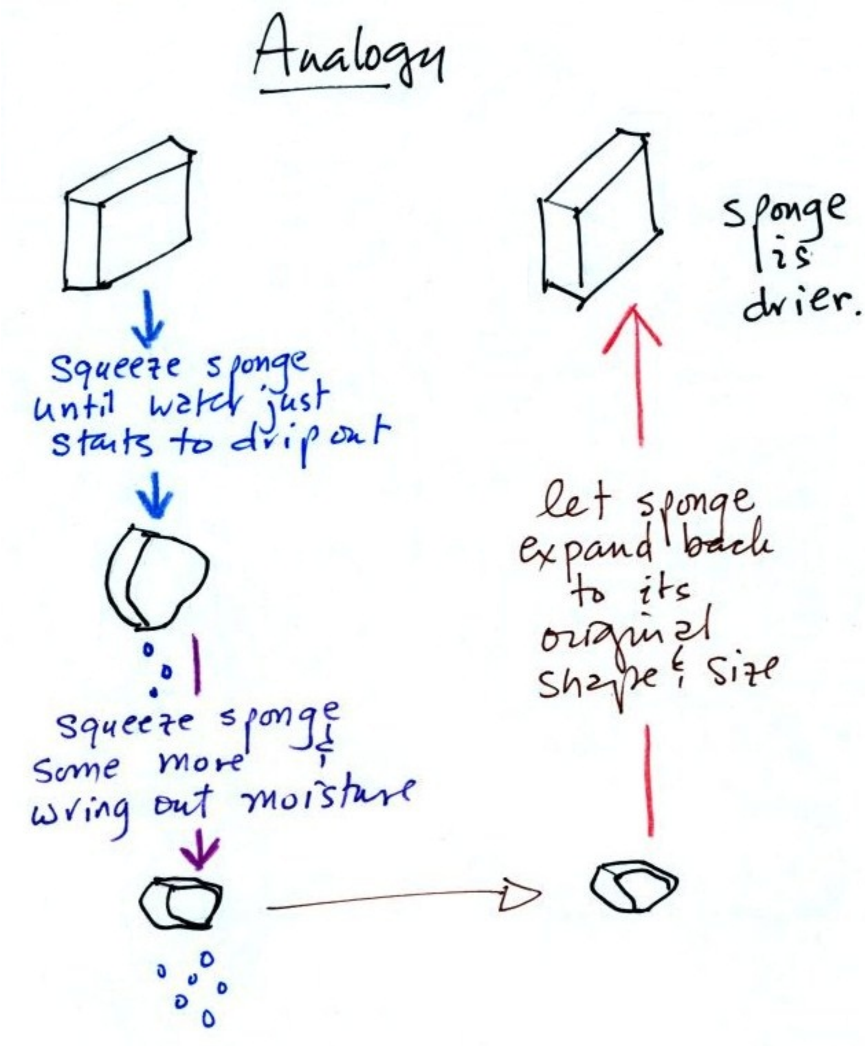
7. If you cool moist air to the dew point the RH will become 100%.

8. Dew point tells you how much water vapor is in the air | if you know T_d you can figure out r.

Here is a picture illustrating how the relative humidity of an air parcel (a balloon of air) changes within the atmosphere. At Point 1, the air parcel has a temperature of 90^0F, a mixing ratio of 7.5 g/kg and a somewhat low relative humidity of 25%. These are the same data used for Problem 4 above. At Point 2, the air parcel has been cooled to its dew point temperature. The relative humidity is now 100% and a cloud forms. When the air parcel is cooled below the dew point temperature to 30^0F, it cannot hold the 7.5 g/kg of water vapor that it started with (Point 3). The excess moisture condenses into water droplets and falls out of the air parcel as rain or snow until the air parcel contains only 3 g/kg of water vapor. Finally the air parcel is warmed back to its starting temperature of 90^0F (Point 4). Now the air parcel has a relative humidity of only 10% because it contains only half of the water vapor it began with (Point 5).

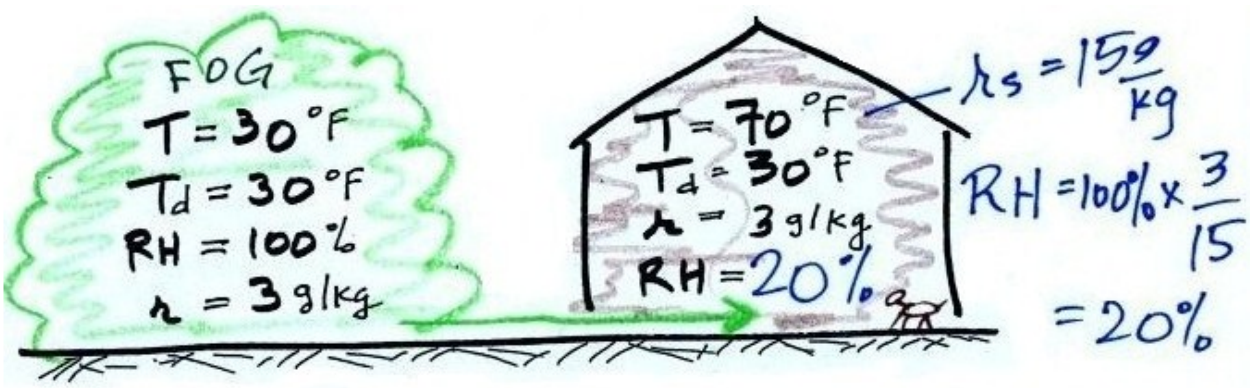


Drying moist air is like wringing moisture from a wet sponge. When you start to squeeze the sponge, no water comes out at first. After sufficient squeezing, water is released from the sponge. This is similar to cooling an air parcel because the mixing ratio remains constant until the dew point temperature is reached. The sponge resumes its original shape and size when you release the sponge and it is contains less water than before. In a similar fashion, an air parcel is drier when it is cooled to a temperature below the dew point and then warmed back to its original temperature.



This kind of process ("squeezing" water vapor out of moist air by cooling the air below its dew point temperature) happens frequently. Here are some examples.

In the winter, cold air is brought inside your house or apartment and warmed to a comfortable temperature. Imagine 30^0F air with a relative humidity of 100%. This is a best case scenario because cold winter air usually has a lower dew point and is drier. Bringing the air inside and warming it will cause the relative humidity to drop from 100% to 20%. During the winter, indoor air often has a low relative humidity which is especially evident when the outdoor temperature is below zero degrees Fahrenheit.



The air outside air an airplane is very cold (-60^0F perhaps) and dry (even if the -60^0F air is saturated it would contain essentially no water vapor). When outside air is brought inside the plane and warmed to a comfortable temperature, the relative humidity is very close to 0%. This is why people can become dehydrated on long airplane flights.

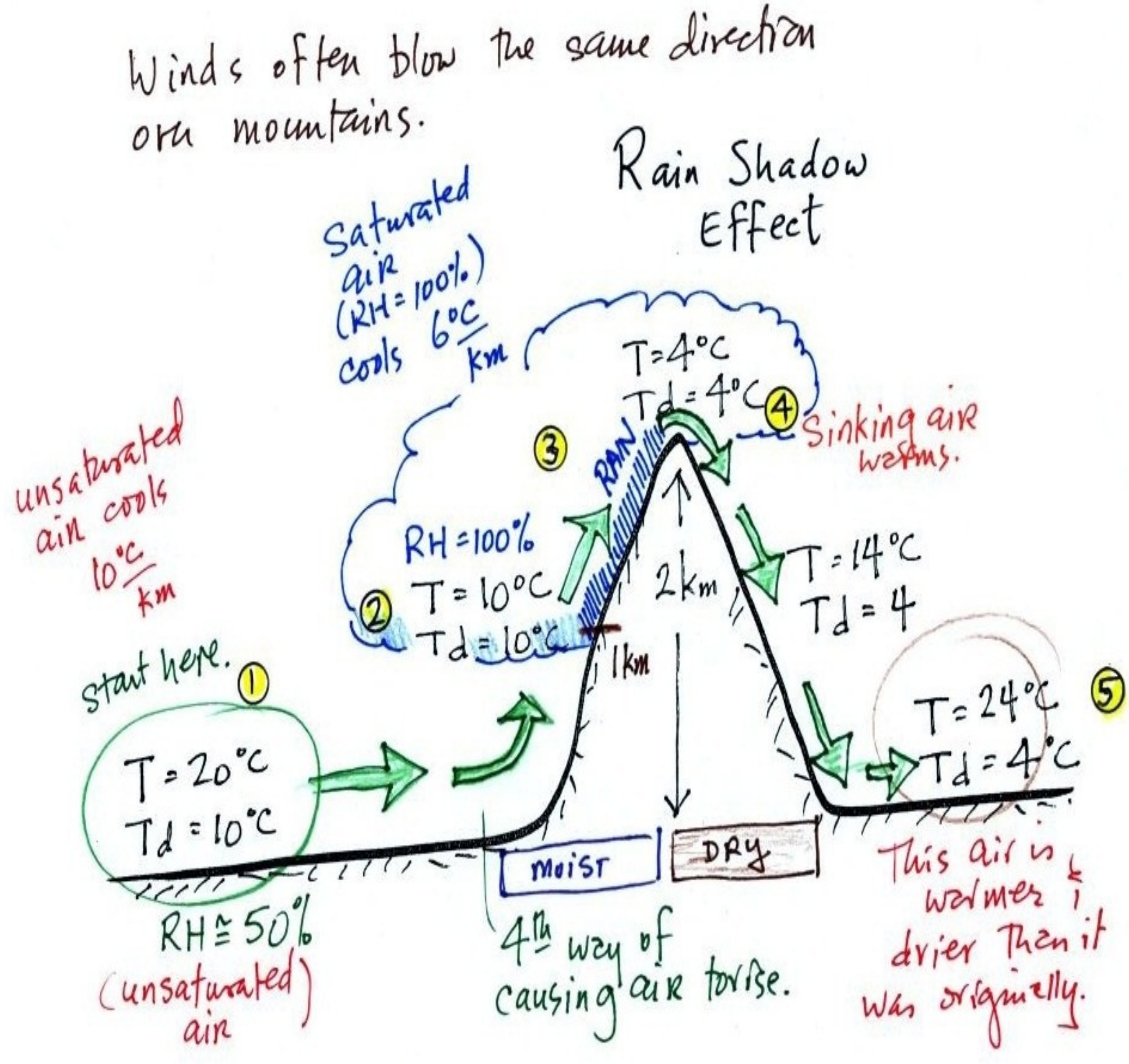
Orographic lifting, the lifting of air parcels by a mountain, is a very important example of how cooling air removes moisture. Orographic lifting is the fourth mechanism which causes air to rise. The other three mechanisms are convergence into surface low pressure centers, the movement of warm and cold fronts and convection. When air is forced to rise by one of these processes, it cools and expands. If the dew point temperature is reached, clouds will form.

The figure below illustrates orographic lifting. At point 1, the air is moist but unsaturated with a relative humidity of about 50%. The air encounters a mountain and is forced to rise. It expands and cools at a rate of about 10^0C (50^0F) for every kilometer of altitude gain. This is known as the dry adiabatic lapse rate. Adiabatic means that the air does not absorb energy from its surroundings or release energy.

A cloud appears when the air reaches its dew point temperature and becomes saturated (Point 2). As the air continues to rise and is cooled below its dew point temperature, water vapor condenses into water droplets and fall to the ground as rain (Point 3). Rising saturated air cools at a slower rate than unsaturated air because it releases latent heat which partially offsets the cooling caused by expansion. Now the air cools at a rate of 6^0C (43^0F) for every kilometer.

At the top of the mountain (Point 4), the air now has a temperature of 4^0C. As the air begins to descend along the other side of the mountain, it is compressed because the atmospheric pressure is increasing and becomes warmer. Soon the relative humidity drops below 100% and the air warms at the rate of 10^0C per kilometer of altitude lost. When the air reaches the ground (Point 5), it is warmer (24^0C vs. 20^0C) and drier (Td = 4 C vs. Td = 10 C) than it was before it ascended the mountain.

The downwind side of the mountain is referred to as a "rain shadow" because rain is less likely to occur on than on the upwind side of the mountain. In Arizona for most of the year, the air that arrives in Arizona originates from the Pacific Ocean. By the time the air reaches Arizona, it has already travelled up and down over the Sierra Nevada mountains in California and the Sierra Madre mountains further south in Mexico. The air loses much of its moisture on the western slopes of those mountains. Another example is the eastern half of Oregon and Washington, which is drier than the western half because air travels from the Pacific up and over the Cascade mountains.


Winds often blow the same direction
over mountains.
Rain Shadow Effect
Saturated air (RH = 100%) cools 6°C/km
unsaturated air cools 10°C/km
start here.
1
T = 20°C
Td = 10°C
RH ≅ 50% (unsaturated) air
2
RH = 100%
T = 10°C
Td = 10°C
1km
3
RAIN
T = 4°C
Td = 4°C
4
Sinking air warms.
2km
T = 14°C
Td = 4
5
T = 24°C
Td = 4°C
MOIST
DRY
4th way of causing air to rise.
This air is warmer & drier than it was originally.