Baroclinic Conversion

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ATMO558
7 May 2008

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Outline

1. Introduce Some Stuff
2. Talk About Some Other Stuff
3. Confuse Everyone With Some More Stuff
4. Show Some Pretty Pictures
5. Go To The Bar

(If it's not that evident, I don't like outline slides.)

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Warning & Apology

Eric Eady

Eady, E.T., 1949: Long waves and cyclone waves. *Tellus*, **1**, 33–52.

Eady Model

- We need a theory to account for origin of disturbances (both upper & lower) and describe the essence of their interaction
- We need to understand the conditions/mechanisms leading to the growth and the structure of observed systems
- We need to identify the structures that optimally extract energy from the basic state (i.e., growing unstable modes)
- We divide the atmosphere into a steady basic state and a possible growing perturbation/disturbance
- Eady Model
    i. Boussinesq approximation (i.e., basic state density is constant)
    ii. f-plane ($\beta = 0$)
    iii. Constant vertical shear
    iv. Rigid lids at top and bottom
    v. Adiabatic, Frictionless, Quasi-Geostrophic, Hydrostatic
- Zero mean PV in domain. So where do we get baroclinic conversion???

Eady Model

$$\frac{D\vec{V}}{Dt} = -f\hat{k} \times \vec{V} - \alpha \nabla p - g\hat{k} + \vec{F}$$

$$\frac{D\rho}{Dt} = -\rho(\nabla \cdot \vec{V})$$

$$\frac{D\theta}{Dt} = \frac{\theta}{T}\frac{Q}{c_p}$$

$$p\alpha = RT$$

$$\theta = T\left(\frac{p_s}{p}\right)^{\frac{R}{c_p}}$$

We want to "Eady-ize" the system...

- Linearize
- Hydrostatic-ize
- Anelastic-ize
- Adiabatic-ize
- Frictionless-ize
- Boussinesq-ize
- f-plane-ize
- QG-ize
- Perturb-ize
- Rigid lid-ize
- Periodic BC-ize
- Unbounded in y-ize
- Jazzercise?

Eady Model

$$\frac{\partial u_g}{\partial t} + u_g \frac{\partial u_g}{\partial x} + v_g \frac{\partial u_g}{\partial y} = f_0 v_a$$

$$\frac{\partial v_g}{\partial t} + u_g \frac{\partial v_g}{\partial x} + v_g \frac{\partial v_g}{\partial y} = -f_0 u_a$$

$$\frac{\partial b}{\partial t} + u_g \frac{\partial b}{\partial x} + v_g \frac{\partial b}{\partial y} = -N_0^2 w$$

$$\frac{\partial u_a}{\partial x} + \frac{\partial v_a}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\frac{\partial \phi}{\partial z} = b$$

$$u_g = U_g(z) + u_g' + u_a' = U\frac{z}{H} + u_a'$$

$$v_g = v_g' + v_a' = v_g'$$

$$w = w' = w'(x, z, t)$$

$$b = B(y) + b' = -\frac{f_0 U}{H} y + b'(x, z, t)$$

$$\phi = \Phi(y) + \phi' = -\frac{f_0 U z}{H} y + \phi'(x, z, t)$$

Eady Governing Equations

$$\frac{\partial v_g'}{\partial t} + U_g \frac{\partial v_g'}{\partial x} = -f_0 u_a'$$

$$\frac{\partial b'}{\partial t} + U_g \frac{\partial b'}{\partial x} = -v_g' \frac{\partial B}{\partial y} - w' N_0^2$$

$$\frac{\partial u_a'}{\partial x} + \frac{\partial w'}{\partial z} = 0$$

$$\frac{\partial \phi'}{\partial z} = b'$$

$$v_g' = \frac{1}{f_0} \frac{\partial \phi'}{\partial x}$$

Eady Model Energetics

KE

$$\left(\frac{\partial}{\partial t} + U_g \frac{\partial}{\partial x}\right) \frac{v_g'^2}{2} = \underbrace{-\frac{\partial}{\partial x}(u_a'\phi')}_{A} \underbrace{-\frac{\partial}{\partial z}(w'\phi')}_{B} + \underbrace{w'b'}_{C}$$

A. Ageostrophic Geopotential Flux
B. Vertical Geopotential Flux
C. Baroclinic Conversion

- No basic state conversion term

PE

$$\left(\frac{\partial}{\partial t} + U_g \frac{\partial}{\partial x}\right) \left(\frac{b'^2}{2N_0^2}\right) = -\underbrace{w'b'}_{D} \underbrace{-\frac{v_g'b'}{N_0^2}\frac{\partial B}{\partial y}}_{E}$$

D. Vertical Heat Flux
E. "Creates" PE with northward heat flux

- Converts basic state energy

Eady Model Energetics

Total $$\left(\frac{\partial}{\partial t}+U_g\frac{\partial}{\partial x}\right)\left(\frac{v_g'^2}{2}+\frac{b'^2}{2N_0^2}\right)=-\frac{\partial}{\partial x}(u_a'\phi')-\frac{\partial}{\partial z}(w'\phi')-\frac{v_g'b'}{N_0^2}\frac{\partial B}{\partial y}$$

- Only term that can take energy from basic state
- When we integrate over the entire Eady domain, it is evident that the meridional heat flux is where all the energy comes from

$$\int_0^L \frac{\partial}{\partial x}U_g(E_E)\,\mathrm{d}x=0$$ Because of cyclic BCs

$$\int_0^L \frac{\partial}{\partial x}(u_a'\phi')\,\mathrm{d}x=0$$ Because of cyclic BCs

$$\int_0^H \frac{\partial}{\partial z}(w'\phi')\,\mathrm{d}x=0$$ Because w' = 0 @ z = 0, L

$$\therefore\frac{\partial}{\partial t}\int_0^L\int_0^H\left(\frac{v_g'^2}{2}+\frac{b'^2}{2N_0^2}\right)\mathrm{d}z\mathrm{d}x=-\frac{1}{N_0^2}\frac{\partial B}{\partial y}\int_0^L\int_0^H v_g'b'\,\mathrm{d}z\mathrm{d}x$$

Eady Model Energetics

$$c(P_{BS},P_E)=-\int_0^L\int_0^H\frac{v_g'b'}{N_0^2}\frac{\partial B}{\partial y}\mathrm{d}z\mathrm{d}x$$

$$c(P_E,K_E)=\int_0^L\int_0^H w_g'b'\,\mathrm{d}z\mathrm{d}x$$

- Now we have a path that can convert basic state potential energy into kinetic energy
- Now we want to find out if the disturbances grow
- But first, another quick review of QG theory...

QG Theory

$$\underbrace{\left[\nabla^2 + \frac{\partial}{\partial p}\left(\frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\right)\right]\chi}_{A} = \underbrace{-f_0\vec{V}_g\cdot\nabla\left(\frac{1}{f_0}\nabla^2\Phi + f\right)}_{B}\underbrace{-\frac{\partial}{\partial p}\left[-\frac{f_0^2}{\sigma}\vec{V}_g\cdot\nabla\left(-\frac{\partial\Phi}{\partial p}\right)\right]}_{C}$$

Geopotential Tendency Equation

Term C $$-\left[\vec{V}_g\cdot\nabla\frac{\partial}{\partial p}\left(\frac{f_0^2}{\sigma}\frac{\partial\Phi}{\partial p}\right) + \frac{f_0^2}{\sigma}\frac{\partial\vec{V}_g}{\partial p}\cdot\nabla\left(\frac{\partial\Phi}{\partial p}\right)\right]$$

But... $$f_0\frac{\partial\vec{V}_g}{\partial p} = \hat{k}\times\nabla\left(\frac{\partial\Phi}{\partial p}\right)$$

Add in B... $$-f_0\vec{V}_g\cdot\nabla\left[f + \frac{1}{f_0}\nabla^2\Phi + \frac{\partial}{\partial p}\left(\frac{f_0}{\sigma}\frac{\partial\Phi}{\partial p}\right)\right]$$

$$\left(\frac{\partial}{\partial t} + \vec{V}_g\cdot\nabla\right)\left[f + \frac{1}{f_0}\nabla^2\Phi + \frac{\partial}{\partial p}\left(\frac{f_0}{\sigma}\frac{\partial\Phi}{\partial p}\right)\right] = \frac{D_g q}{Dt} = 0$$

PV equation for QG system

Eady Model Growth

For our Eady Model... $$q = Q + q' = f_0 + \left[\frac{1}{f_0}\nabla^2\phi' + \frac{f_0}{N_0^2}\frac{\partial^2\phi'}{\partial z^2}\right]$$

$$\frac{1}{f_0}\frac{\partial^2\phi'}{\partial x^2} + \frac{f_0}{N_0^2}\frac{\partial^2\phi'}{\partial x^2} = 0$$

- q' = 0
- PV is conserved
- At *t* = 0, we have no disturbance

$$\phi' = \hat{\phi}(z)e^{ik(x-ct)}$$

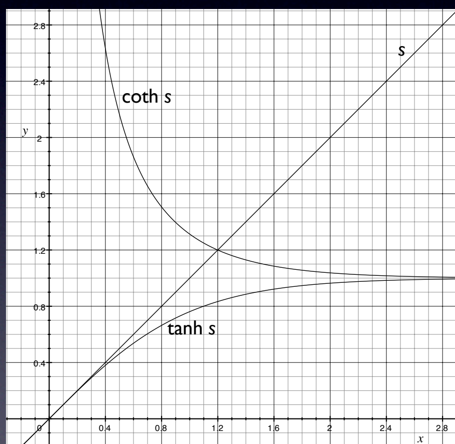
- Assume wavelike solution

LOTS OF REALLY UGLY MATH
I'M TALKIN BEAT-WITH-THE-UGLY-STICK BAD

$$c = \frac{U}{2} \pm \frac{U}{2s}\left[(s - \coth s)(s - \tanh s)\right]^{\frac{1}{2}}$$ where $$s = \frac{N_0 kH}{2f_0}$$

Eady Model Growth

$$c = \frac{U}{2} \pm \frac{U}{2s}\left[(s - \coth s)(s - \tanh s)\right]^{\frac{1}{2}}$$



- s - tanh s > 0 always
- s - coth s < 0 we get growth
- We want complex part to *c*
- There exists a "shortwave cutoff"
- If $\lambda < \lambda_c$, then we don't get growth

$$L_R = \frac{N_0 H}{f_0} \qquad \therefore\ s = \frac{L_R}{2} k$$

$$k c_i \propto \frac{U}{L_R}$$

If λ is too short, the Rossby depth is too shallow and the circulation won't reach far into the interior (i.e., no phase locking)

For rapid growth, we want...

- large shear (i.e., big U)
- small N_0 (i.e., weak stability)
- shallow troposphere (i.e., small H)
- strong rotation (i.e., small L_R)

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Eady Model

- In terms of energetics, we have thermally direct circulation (i.e., warm air rising, cold air sinking)
- In terms of energetics, we have meridional transport of heat (i.e., cold air to the north, warm air to the south)
- Westward phase tilt of disturbance against basic state shear provides mechanism for baroclinic conversion
- PV = 0 in the interior, so all PV is at the boundaries. Therefore, we expect decay as we move away from the surface/top
- We see this in the real atmosphere: all PV is in the tropopause, and if we invert the PV on boundary we get a distrubance in the interior (c.f., Bretherton 1966)
- Maximum amplitudes are near boundaries, much like tropopause extrusions
- So we have a realistic representation of a baroclinic disturbance from a very simple model
- Cyclogenesis results from exponential growth of the unstable normal mode

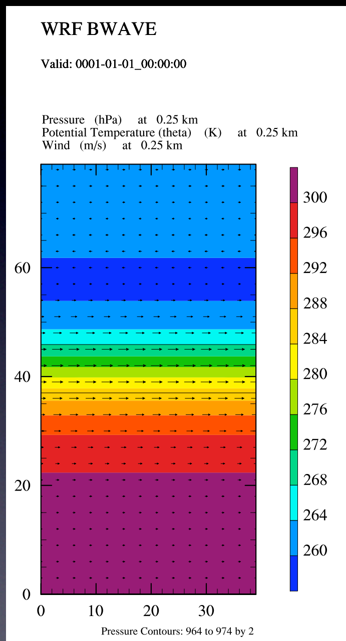
Eady Model

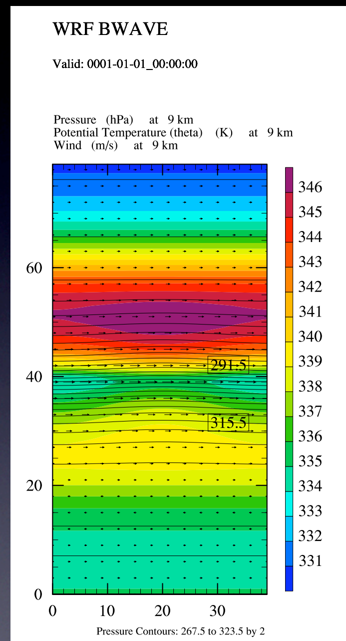
- Unrealistic Aspects of Eady Model
  - Growth rate too small
  - Growth results from infinitesimal perturbations, not precursors
  - No interior PV
  - Phase tilt constant in time
  - Linear shear everywhere
  - Fixed tropopause
  - Fixed, flat top and bottom boundaries
  - β effect ignored
  - No diabatic processes

WRF B-Wave

- Idealized baroclinic wave <u>built into WRF</u> (::cough:: black box ::cough::)
- Baroclinically unstable jet $u = u(y,z)$ on a *f*-plane
- Periodic BCs in *x*-direction (what goes out comes back in)
- Symmetric BCs in *y*-direction
- 4km Rayleigh Damping layer at top; not rigid lid but close enough
- Non-hydrostatic (though Hydrostatic wasn't much different)
- <u>No</u> physics
- $\Delta x = \Delta y =$ 100km; 41 x 81 grid points
- z_{BOT} = 0km; z_{TOP} = 16km; 65 vertical levels
- Δt = 600s; run for 7 days (~1hr on dual Athlon MP 2400+ 2.0GHz)

B-Wave Initialization





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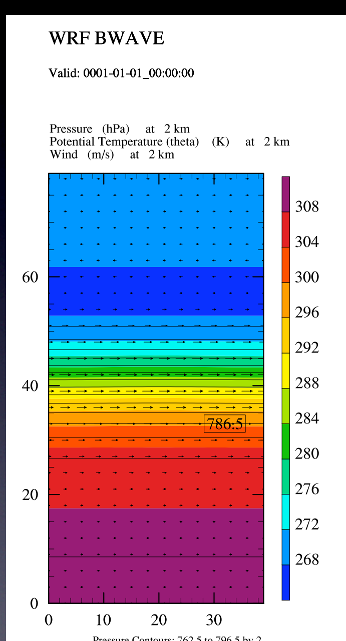
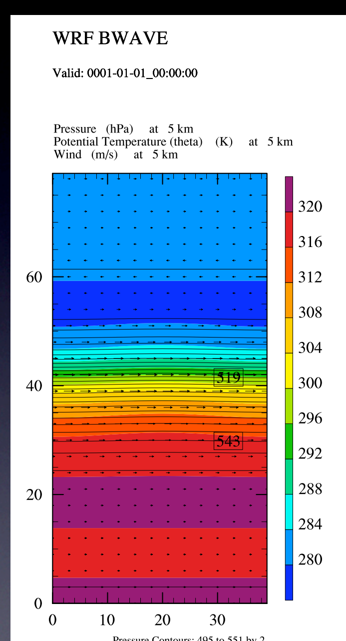
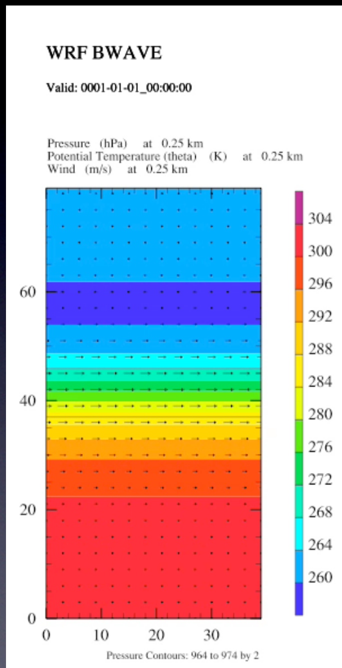
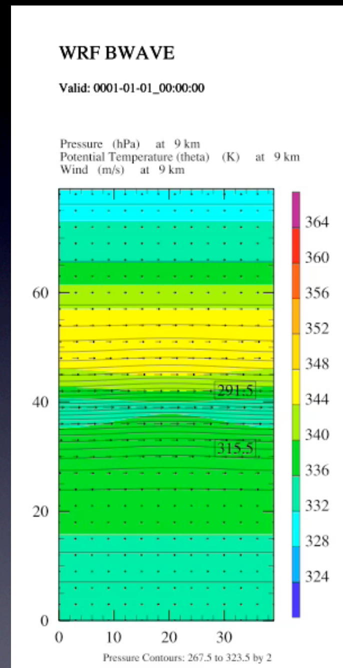




Go B-Wave, Go!





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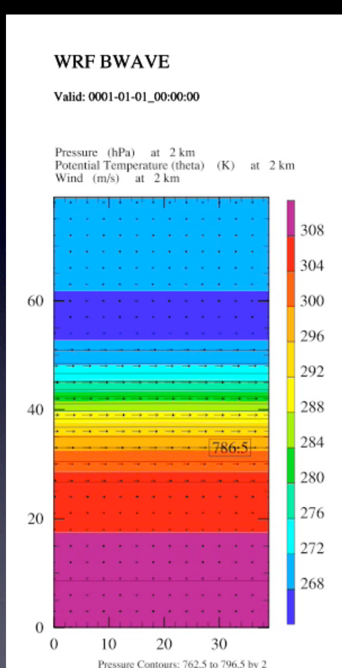

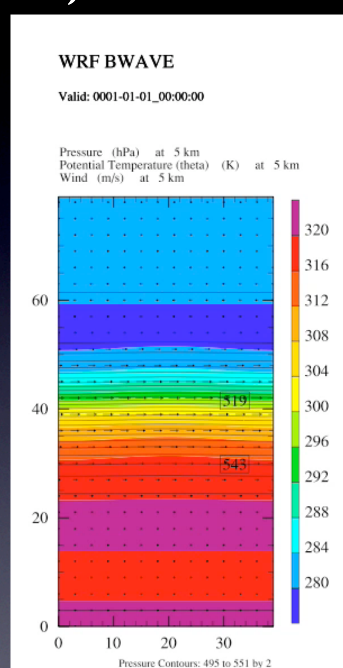

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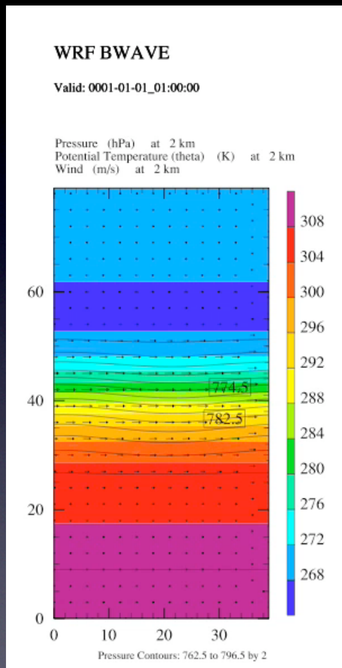

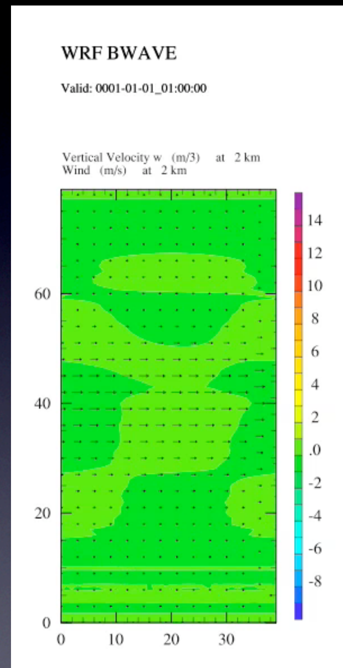

Questions?

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