Synoptic = Coincident in time

Continuum of scales	Examples (in context of course)
Microscale <1km, <1h	Atmospheric turbulence, PBL
Mesoscale 1-1000 km	Thunderstorms, sea breeze
Synoptic 1000-6000 km	Mid-latitude baroclinic waves
Global >6000 km	global circulation structure, Hadley Cell, polar jet.

Another way to think about demarcation of mesoscale vs. synoptic scale is via Rossby number (R_0), conceptually!

$$R_0 = \frac{\text{Advective terms}}{\text{Rotational (Coriolis) terms}}$$

Synoptic ~ 0.1 or below

Mesoscale ~ 1

Baroclinic vs. Barotropic

Barotropic: State of the atmosphere where surface p and ρ are roughly parallel, or isotherm's and isobars parellel. Little or no temperature advection

Implications for:
- Vertical motion: necessarily depends on mostly latent heat release from condensation (e.g. hurricane)
- Stability: Depends on pre-existent horizontal wind shear

Baroclinic: Constant p surfaces intersect constant ρ surfaces. Isobars and isotherms cross, and temperature advection occurs.

Implications for:
- Vertical motion: Depends on synoptic-scale temperature and differential vorticity advection, as well as latent heat release. These cause winds to be ageostrophic, which induces vertical motion
- Stability: Depends on vertical wind shear, since that implies a horizontal temperature gradient (baroclincity). A front is an area of intense baroclincity.

Brief review of basic dynamical concepts

Start with an equation of motion that is appropriate for synoptic scale motion:

$$\frac{\partial \vec{V}}{\partial t} = -\vec{V}\cdot\nabla\vec{V} - \frac{1}{\rho}\nabla_h p - f\hat{k}\times\vec{V} + \vec{F}_r$$

local time rate of change of wind

Advection of wind

Horizontal PGF

Coriolis force* ($f = 2\Omega\sin\phi$)

Friction

* Note: Coriolis force term here is simplification from $-2\vec{\Omega}\times\vec{V}$, where $\vec{\Omega}$ is Earth's rotation vector. Therefore the contributions of terms with $\hat{f}$ ($2\Omega\cos\phi$) and w are neglected.

What types of force balances can arise if wind speed is constant?

Balance	Application
1) Geostrophy Horiz. PGF = Coriolis force	Upper air flow where straight
2) Gradient Centripetal accel = Horizontal PGF + Coriolis	Flow around ridges and troughs
3) Cyclostrophic Centripetal = Horizontal PGF	Tight, small scale flows on mesoscale

In this class, we can consider

$$R_0 \sim 0.1 \quad \text{where} \quad R_0 = \frac{\text{Advective terms}}{\text{Rotational terms}} = \frac{U}{fL}$$

U = scale of wind $\quad$ L = length scale

f = Coriolis parameter

<u>CAVEAT</u>: This <u>does</u> <u>not</u> mean the advective terms are negligible! They are essential for generating the ageostrophic circulations necessary for vertical motion, as we will see later...

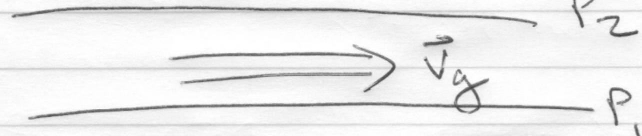
For the synoptic scale, in upper-atmosphere either geostrophic or gradient balance applies, and the latter necessarily implies vertical motion.

<u>Geostrophy</u>

$$\vec{V}_g = \hat{k} \times \frac{1}{\rho f} \nabla p$$

$$u_g = -\frac{1}{\rho f}\frac{\partial p}{\partial y}$$

$$v_g = \frac{1}{\rho f}\frac{\partial p}{\partial x}$$



$P_1 > P_2$

- Parallel to isobars
- Non divergent $\rightarrow \nabla \cdot \vec{V}_g = 0$

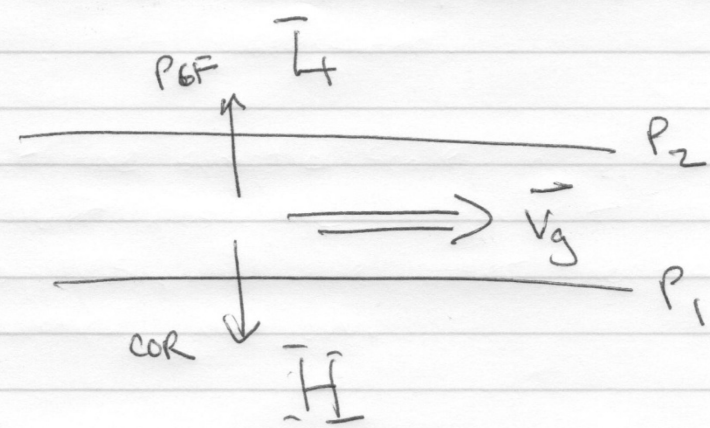
If looking on upper air map, it is more convenient to consider geostrophic wind in terms of geopotential (Φ)

$$\Phi = gz \qquad g = \text{gravitational accel. } 9.8\,m/s^2$$

Note! If assume g is constant, geopotential height is just z. Okay for troposphere.

Properties of geostrophic wind

1)



Parallel to isobars or lines of constant geopotential height.

Okay for mid-latitudes but not tropics. Coriolis force is so small there that geostrophy doesn't apply as strongly, so atmospheric circulations tend to be more on mesoscale there.

2) Geostrophic wind is non-divergent

$$\nabla \cdot \vec{V}_g = 0$$

But, if no divergence/convergence, no vertical motion!!
Therefore, other terms in the equation of motion, though small are necessary to explain vertical motion and weather!

3) Synoptic scale applies

$$R_o << 1 \qquad R_o = \frac{u}{fL}$$

Gradient balance:

Now include rotation to geostrophy

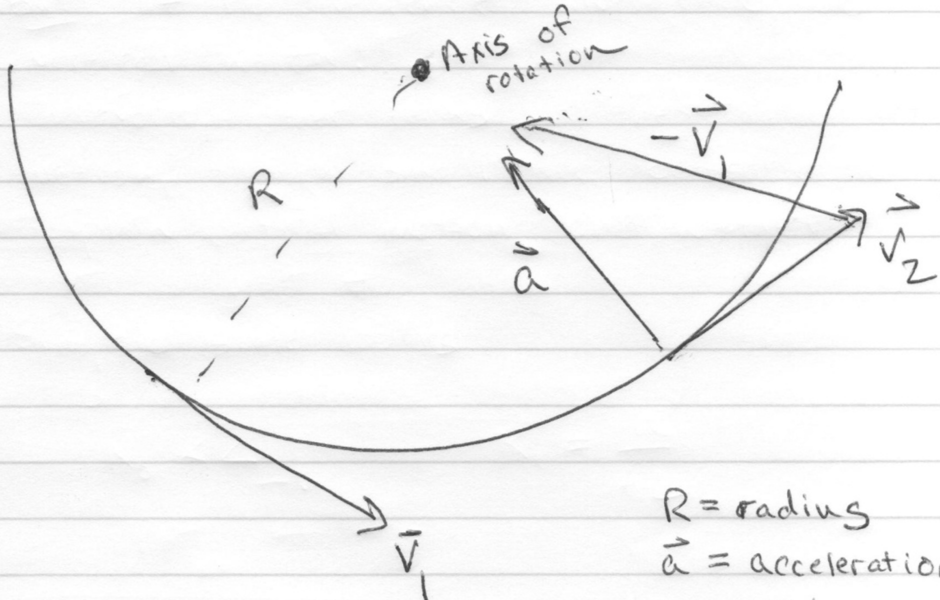
$$0 = \frac{\partial \vec{V}}{\partial t} - f\hat{k} \times \vec{V} - \frac{1}{\rho} \nabla p$$

Acceleration term due to rotational effects

Coriolis

PGF

Accounting for rotational acceleration with constant speed around an axis of rotation (e.g. trough, ridge)



R = radius
$\vec{a}$ = acceleration vector (towards axis of rotation)

$$\vec{a} = \frac{\vec{V}_2 - \vec{V}_1}{\Delta t} = \text{Centripetal acceleration} = \vec{a}_{cent}$$

Mathemathically

$$\vec{a}_{cent} = \frac{-|\vec{V}|^2}{R} \quad \text{Centripetal acceleration}$$

Centrifugal acceleration → fictitious acceleration acoounted for by inertia going around a curve

$$\vec{a}_{Centrifugal} = \frac{|\vec{V}|^2}{R}$$ directed <u>away</u> from axis of rotation.

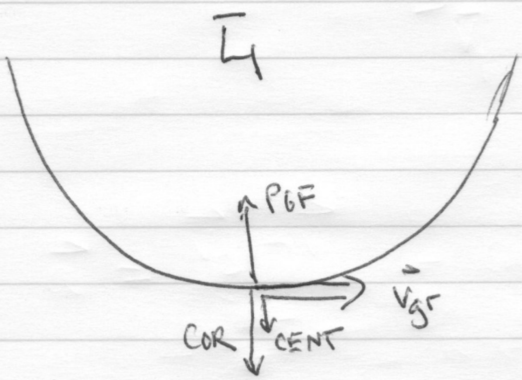
We'll consider this in the force balance diagrams that follow.

$$0 = CENT + COR + PGF$$ Gradient balance

$\vec{v}_{gr}$ = gradient wind

Cyclonic rotation

<u>Trough (R positive)</u>



$$COR + CENT = PGF$$

or

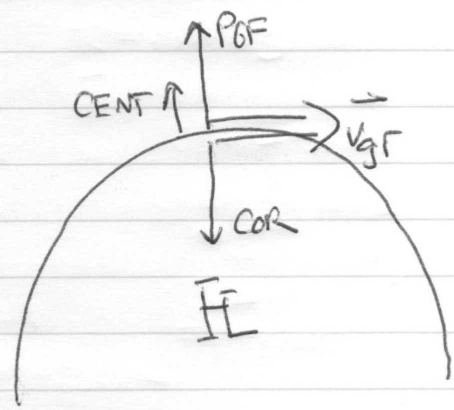
$$COR = PGF - CENT$$

Effectively reduces PGF, slowing wind down

<u>Subgeostrophic</u>

Anticyclonic rotation

<u>Ridge (R negative)</u>



$$PGF + CENT = COR$$

Effectively increase PGF, speeding wind up

<u>Supergeostrophic</u>

Solution to geostrophic wind:

$$V_g = \frac{1}{f} \frac{\partial \Phi}{\partial n}$$

n = distance in direction normal to isobars

Φ = geopotential (can also be written in terms of pressure)

Solution to gradient wind:

Inclusion of centrifugal acceleration terms makes solution quadratic.

$$V_{gr} = \frac{1}{f} \left(\frac{\partial \Phi}{\partial n} + \frac{V_{gr}^2}{R_T} \right)$$

$R_T > 0$ Cyclonic
$R_T < 0$ Anticyclonic

Solution:

$$V_{gr} = \frac{-fR}{2} \pm \left(\frac{f^2 R^2}{4} - R_T \frac{\partial \Phi}{\partial n} \right)^{1/2}$$

Radical second term.

Net positive for $R_T > 0$
Net negative for $R_T < 0$

For physical solution:

1) V_{gr} must be real, non-negative

2) $\partial \Phi / \partial n < 0$ for normal highs and lows in atmosphere

See Holton, Ch. 3

3) Consider positive root for low pressure (cyclonic)

4) Consider negative root for high pressure (anticyclonic)

Important Physical Implications to Gradient Wind Solution

1) Solutions to gradient wind equation are unlimited for low pressure, therefore radical will always have a real solution ($\partial \Phi / \partial n < 0$, $R_T > 0$)

But, for high pressure ($R_T < 0$), limit on strength of PGF

$$\left| \frac{\partial \Phi}{\partial n} \right| < \frac{|R_T| f^2}{4}$$

Explains the assymetry between lows and highs on a weather map.

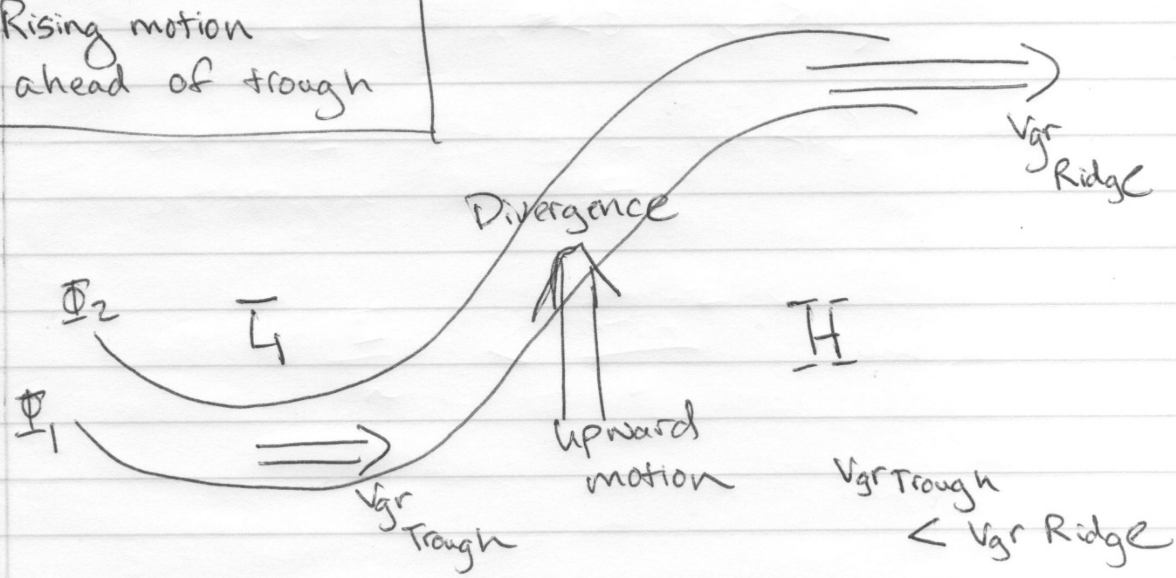
Lows → Have <u>no limit</u> to how deep they can get, because always a gradient wind solution

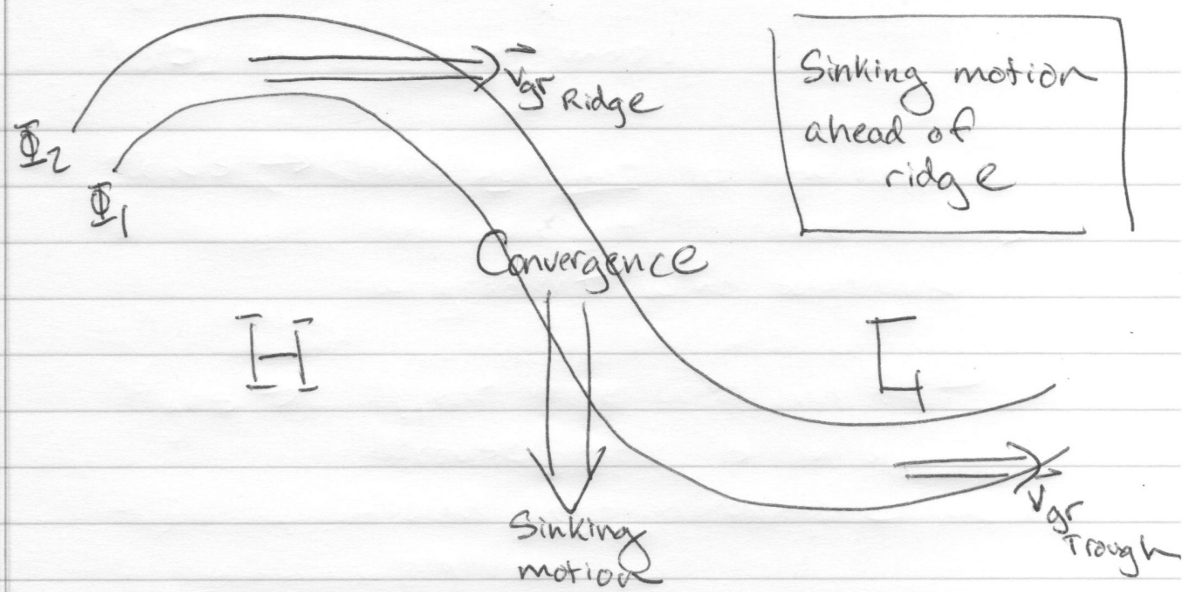
Highs → Appear more broad and flat because there is an upper bound on supergeostrophic wind solution.

2) Accounting for rotation will necessarily create divergence and convergence in the atmosphere because $|\vec{V}_{gr}|$ is changing.

Assume channel of constant $\partial\Phi/\partial n$

Rising motion ahead of trough

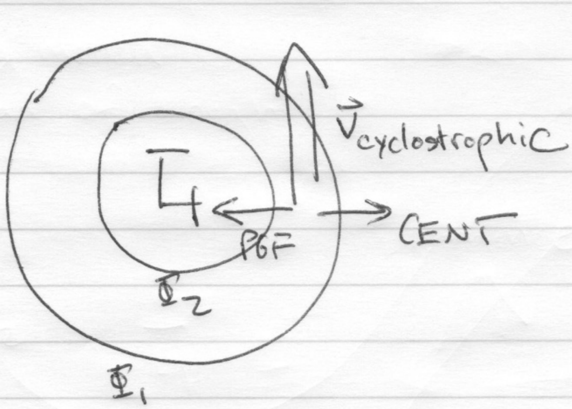




$\boxed{\text{Sinking motion ahead of ridge}}$

Cyclo<u>strophic</u> <u>flow</u>: Applies to very small rotations on mesoscale ($R_o \geq 1$)

$0 = PGF + CENT$



Rotational effects <u>not</u> important, so no hemispheric dependence

<u>Examples</u>: Hurricanes (at peak intensity), tornadoes, dust devils, flushing toilets, draining sinks

Gradient with Friction

$0 = PGF + COR + CENT + FRICT$

Form to express frictional drag

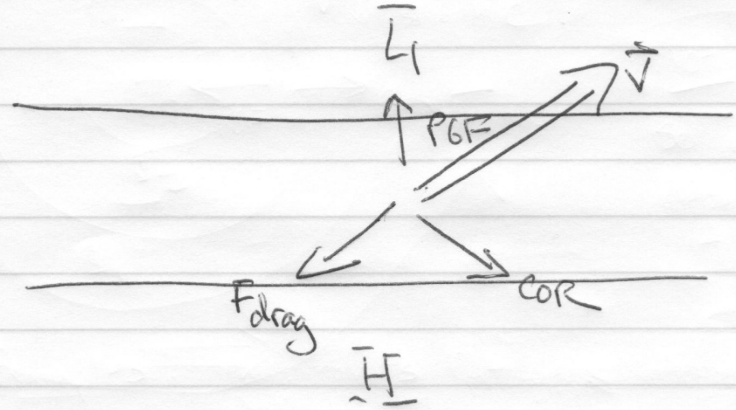
$$\tau_s = -\rho c_D |\vec{V}|^2$$

Shear stress

Drag coefficient

$$\vec{F}_{drag} = -\frac{1}{\rho} \partial \tau / \partial z$$

Friction is change in wind shear stress with height. Has an Eckman Spiral solution in vertical from top of PBL down to surface.



Causes wind to curve into low pressure
Source of convergence / divergence → vertical motion

3.2.4 Cyclostrophic Flow

If the horizontal scale of a disturbance is small enough, the Coriolis force may be neglected in (3.10) compared to the pressure gradient force and the centrifugal force. The force balance normal to the direction of flow is then

$$\frac{V^2}{R} = -\frac{\partial \Phi}{\partial n}$$

If this equation is solved for V, we obtain the speed of the *cyclostrophic wind*

$$V = \left(-R\frac{\partial \Phi}{\partial n}\right)^{1/2} \tag{3.14}$$

As indicated in Fig. 3.4, the cyclostrophic flow may be either cyclonic or anticyclonic. In both cases the pressure gradient force is directed toward the center of curvature and the centrifugal force away from the center of curvature.

The cyclostrophic balance approximation is valid provided that the ratio of the centrifugal force to the Coriolis force is large. This ratio V/fR is equivalent to the Rossby number discussed in Section 2.4.2. As an example of cyclostrophic scale motion we consider a typical tornado. Suppose that

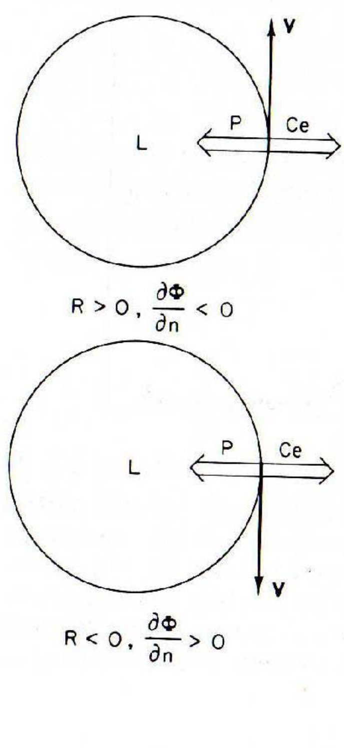


Fig. 3.4 Force balance in cyclostrophic flow; *P* designates the pressure gradient, *Ce* the centrifugal force.

the tangential velocity is 30 m s^{-1} at a distance of 300 m from the center of the vortex. Assuming that $f = 10^{-4}\ s^{-1}$, we obtain a Rossby number of $Ro = V/|fR| \approx 10^3$, which implies that the Coriolis force can be neglected in computing the balance of forces for a tornado. However, the majority of tornadoes in the Northern Hemisphere are observed to rotate in a cyclonic (counterclockwise) sense. This is apparently because they are imbedded in environments that favor cyclonic rotation (see Section 9.6.1). Smaller-scale vortices, on the other hand, such as dust devils and water spouts, do not have a preferred direction of rotation. According to data collected by Sinclair (1965), they are observed to be anticyclonic as often as cyclonic.

3.2.5 The Gradient Wind Approximation

Horizontal frictionless flow that is parallel to the height contours so that the tangential acceleration vanishes ($DV/Dt = 0$) is called *gradient flow*. Gradient flow is a three-way balance between the Coriolis force, the centrifugal force, and the horizontal pressure gradient force. Like geostrophic flow, pure gradient flow can exist only under very special circumstances. It is always possible, however, to define a gradient wind, which at any point is just the wind component parallel to the height contours that satisfies (3.10). For this reason (3.10) is commonly referred to as the gradient wind equation. Because (3.10) takes account of the centrifugal force owing to the curvature of parcel trajectories, the gradient wind is often a better approximation to the actual wind than is the geostrophic wind.

The gradient wind speed is obtained by solving (3.10) for V to yield

$$V = -\frac{fR}{2} \pm \left(\frac{f^2R^2}{4} - R\frac{\partial\Phi}{\partial n}\right)^{1/2} \tag{3.15}$$

Not all the mathematically possible roots of (3.15) correspond to physically possible solutions since it is required that V be real and nonnegative. In Table 3.1 the various roots of (3.15) are classified according to the signs of R and $\partial\Phi/\partial n$ in order to isolate the physically meaningful solutions.

The force balances for the four permitted solutions are illustrated in Fig. 3.5. Equation (3.15) shows that in the cases of both the regular and anomalous highs the pressure gradient is limited by the requirement that the quantity under the radical be nonnegative; that is,

$$\left|\frac{\partial\Phi}{\partial n}\right| < \frac{|R|f^2}{4} \tag{3.16}$$

Table 3.1 *Classification of Roots of the Gradient Wind Equation in the Northern Hemisphere*[a]

Sign $\partial\Phi/\partial n$	$R>0$	$R<0$
Positive	Positive root: unphysical	Positive root: antibaric flow (anomalous low)
	Negative root: unphysical	Negative root: unphysical
Negative	Positive root: cyclonic flow (regular low)	Positive root: $(V>-fR/2)$: anticyclonic flow (anomalous high)
	Negative root: unphysical	Negative root: $(V<-fR/2)$: anticyclonic flow (regular high)

[a] The terms *positive root* and *negative root* in columns 2 and 3 refer to the sign taken in the final term in (3.15).

Thus, the pressure gradient in a high must approach zero as $|R|\rightarrow 0$. It is for this reason that the pressure field near the center of a high is always flat and the wind gentle compared to the region near the center of a low.

The absolute angular momentum about the axis of rotation for the circularly symmetric motions shown in Fig. 3.5 is given by $VR+fR^2/2$.

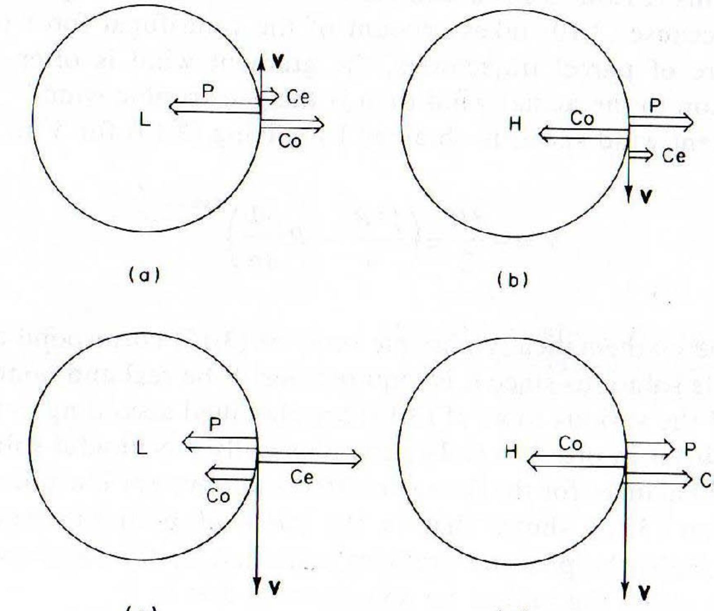


Fig. 3.5 Force balances in the Northern Hemisphere for the four types of gradient flow: (a) regular low; (b) regular high; (c) anomalous low; (d) anomalous high. *P* designates the pressure gradient, *Ce* the centrifugal force, and *Co* the Coriolis force.

From (3.15) it is readily verified that the regular gradient wind balances have positive absolute angular momentum in the Northern Hemisphere, while the anomalous cases have negative absolute angular momentum. Since the only source of negative absolute angular momentum is the Southern Hemisphere, the anomalous cases are unlikely to occur except perhaps close to the equator.

In all cases except the anomalous low (Fig. 3.5c) the horizontal components of the Coriolis and pressure gradient forces are oppositely directed. Such flow is called *baric*. The anomalous low is antibaric; the geostrophic wind V_g defined in (3.11) is negative for an anomalous low and clearly not a useful approximation to the actual speed.[4] Furthermore, as shown in Fig. 3.5, gradient flow is cyclonic only when the centrifugal force and the horizontal component of the Coriolis force have the same sense ($Rf > 0$); it is anticyclonic when these forces have the opposite sense ($Rf < 0$). Since the direction of anticyclonic and cyclonic flow is reversed in the Southern Hemisphere, the requirement that $Rf > 0$ for cyclonic flow holds regardless of the hemisphere considered.

The definition of the geostrophic wind (3.11) can be used to rewrite the force balance normal to the direction of flow (3.10) in the form

$$V^2/R + fV - fV_g = 0$$

Dividing through by fV shows that the ratio of the geostrophic wind to the gradient wind is

$$\frac{V_g}{V} = 1 + \frac{V}{fR} \tag{3.17}$$

For normal cyclonic flow ($fR > 0$) V_g is larger than V, while for anticyclonic flow ($fR < 0$) V_g is smaller than V. Therefore, the geostrophic wind is an overestimate of the balanced wind in a region of cyclonic curvature and an underestimate in a region of anticyclonic curvature. For midlatitude synoptic systems, the difference between the gradient and geostrophic wind speeds generally does not exceed 10–20%. (Note that the magnitude of $V/(fR)$ is just the Rossby number.) For tropical disturbances, the Rossby number is in the range of 1–10, and the gradient wind formula must be applied rather than the geostrophic wind. Equation (3.17) also shows that the antibaric anomalous low, which has $V_g < 0$, can exist only when $V/(fR) < -1$. Thus, antibaric flow is associated with small-scale intense vortices such as tornadoes.

[4] Remember that in the natural coordinate system the speed V is positive definite.

Brief review of where we're at in the evaluation of horizontal momentum equation:

$$\frac{\partial \vec{V}}{\partial t} = -\vec{V} \cdot \nabla \vec{V} \cdot \frac{1}{\rho} \nabla_h P - f\hat{k} \times \vec{V} + \vec{F_r}$$

Curvature term $|V^2|/R$ that leads to supergeostrophic, subgeostrophic wind

Advection × NOT accounted for yet!

PGF COR Geostrophic terms

Friction, leads to Eckman spiral in PBL

So, thus far we've established that both curvature and friction can lead to vertical motion.

What about advection??

→ We have not accounted for it yet, but the way we will do it is by accounting for advection by only the geostrophic wind.

That is the basis of quasi-geostrophic theory. Accounting for advection will lead to ageostrophic circulations and vertical motion ... much more later!!

Thickness and Hypsometric Equation

Start with hydrostatic equation

$$\frac{dp}{dz} = -\rho g$$

Note: At the synoptic scale, assuming hydrostatic balance in vertical equation of motion is okay. Not so great for mesoscale, though.

Use ideal gas law. $P = \rho R T$

Substitute for ρ

$$\frac{dp}{dz} = \frac{-Pg}{RT}$$

By definition of geopotential height

$$d\Phi = g\,dz$$

$$d\Phi = g\,dz = -\frac{RT\,dp}{P}$$

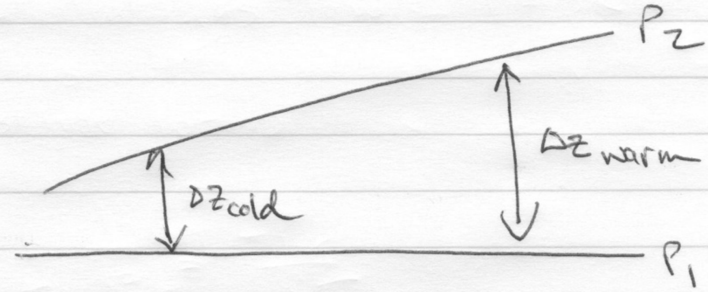
Integrating

$$z_2 - z_1 = \frac{R}{g}\int_{P_2}^{P_1} T\,\frac{dp}{P}$$

$$\boxed{\Delta z = z_2 - z_1 = \frac{RT}{g} \ln \frac{P_1}{P_2}}$$ Hypsometric equation

Aside! $H = RT/g$ is the scale height where atmospheric pressure falls to $1/e$ of its surface value (~8km for US standard atmosphere)

Physically → Thickness (Δz) is related to mean temperature of air between 2 pressure surfaces.



In forecasting, typically look at thickness from 1000-mb to 500-mb. 540 dm line is roughly rain/snow line in eastern 2/3 of U.S.

Thermal Wind and Baroclinicity

Physically! Vertical shear, or change in wind speed and direction with height of geostrophic wind is related to the horizontal temperature gradient

Consider geostrophic wind written in terms of geopotential

$$v_g = \frac{1}{f}\frac{\partial \Phi}{\partial x} \qquad u_g = -\frac{1}{f}\frac{\partial \Phi}{\partial y} \qquad \Phi = gz$$

Differentiate geostrophic wind with respect to pressure (consider v_g)

$$\frac{\partial v_g}{\partial p} = \frac{1}{f}\frac{\partial\left(\partial \Phi/\partial p\right)}{\partial x}$$

By hydrostatic $\partial \Phi/\partial p = -RT/p$

$$p\,\frac{\partial v_g}{\partial p} = -\frac{R}{f}\left(\frac{\partial \bar{T}}{\partial x}\right)_p$$

$\bar{T}$ = mean temp in a layer

$$\frac{\partial v_g}{\partial \ln p} = -\frac{R}{f}\left(\frac{\partial \bar{T}}{\partial x}\right)_p$$

$$\partial v_g = -\frac{R}{f}\int_{p_1}^{p_2}\frac{\partial \bar{T}}{\partial x}\,\partial \ln p$$

Thermal wind (v_T)

$$\boxed{v_T = v_{g2} - v_{g1} = \frac{R}{f}\left(\frac{\partial \bar{T}}{\partial x}\right)\ln\left(p_1/p_2\right)}$$

Shear of geostrophic wind

Horizontal temp. gradient

Pressure level surfaces term.

Similarly for u_T

$$\boxed{u_T = -\frac{R}{f}\frac{\partial \bar{T}}{\partial y}\ln\left(\frac{p_1}{p_2}\right)}$$

Vector form of thermal wind

$$v_T = \cdot\frac{R}{f}\hat{k}\times\nabla_p T \ln\left(p_1/p_2\right)$$

Can also express in terms of vertical thickness gradient, given hypsometric equation

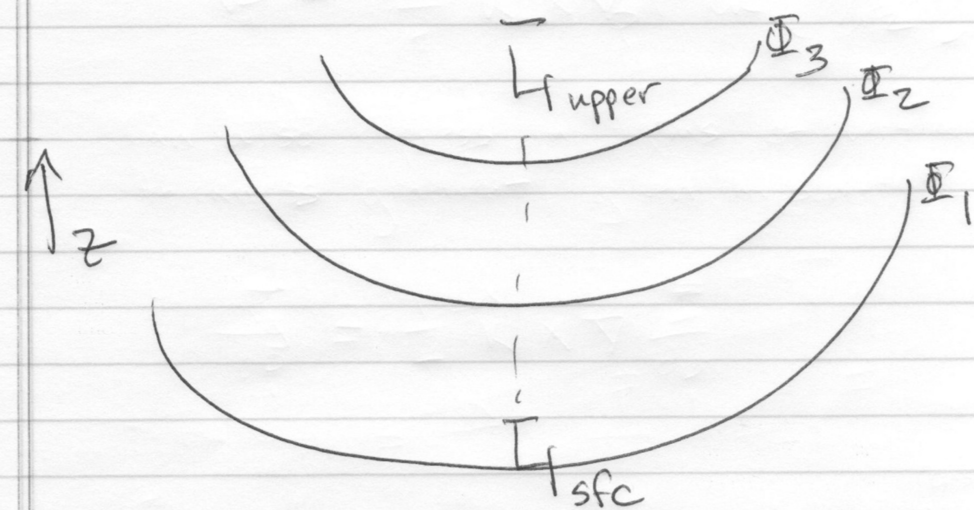
$$\frac{\partial \vec{v}_g}{\partial p} = \frac{1}{f}\hat{k}\times\nabla\frac{\partial \Phi}{\partial p}$$

Thermal wind oriented parallel to constant thickness lines, as geostrophic wind parallel to ~~geo~~ constant geopotential lines.

Physical implications of v_T

1) Barotropic atmosphere (pure): No horizontal temperature gradients. Atmosphere is typically not geostrophically balanced in terms of flows
   → situation in tropics

2) Equivalent barotropic: Highs and lows are vertically stacked, so the direction of the geostrophic wind doesn't change. Orientation of isotherms (thickness lines) parallel to height contours. No temperature advection.



This what happens to mid-latitude cyclones in the occlusion phase. Die out because no baroclincity to help them keep strengthening

3) Fully baroclinic: Highs and lows tilt with height, such that warm and cold advection can occur. Component of $\nabla_h T$ that is normal to $\nabla_h \Phi$, or isotherms (thickness lines) and geopotential height lines cross each other.

Example of Baroclinic Atmosphere (for intensifying mid-lat. cyclone)

Highs and Lows tilt with height, causing temperature advection, intensifying $\nabla_h T$.