Prognostic Use of QG theory

Ultimately, what we want is to relate QG concepts to short-term prognosis of how the synoptic-scale circulation features are going to evolve.

QG Height Tendency

Going back to derivation of QG omega equation:

Vorticity:

$$\nabla^2 \chi = -f_0 \vec{V}_g \cdot \nabla \left(\frac{1}{f_0} \nabla^2 \Phi + f \right) + f_0^2 \frac{\partial \omega}{\partial p}$$

↑

Vorticity tendency ($\partial \zeta / \partial t$)

Thermodynamic:

$$\frac{\partial \chi}{\partial p} = -\vec{V}_g \cdot \nabla \left(\frac{\partial \Phi}{\partial p} \right) - \sigma \omega$$

To get QG Height tendency equation, ~~eliminate~~ eliminate ω by: 1) Take $\partial/\partial p$ of thermodynamic 2) Multiply result by f_0^2/σ, 3) Add to Vorticity eqn.

Result:

$$\left[\nabla^2 + \frac{\partial}{\partial p}\left(\frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\right)\right]\chi =$$

(A)

$$-f_0 \vec{V}_g \cdot \nabla \left(\frac{1}{f_0}\nabla^2 \Phi + f_0\right)$$

(B)

$$-\frac{\partial}{\partial p}\left[\frac{-f_0^2}{\sigma}\vec{V}_g \cdot \nabla\left(-\frac{\partial \Phi}{\partial p}\right)\right]$$

(C)

A: 3-D Laplacian of height tendency $\chi = \partial\Phi/\partial t$

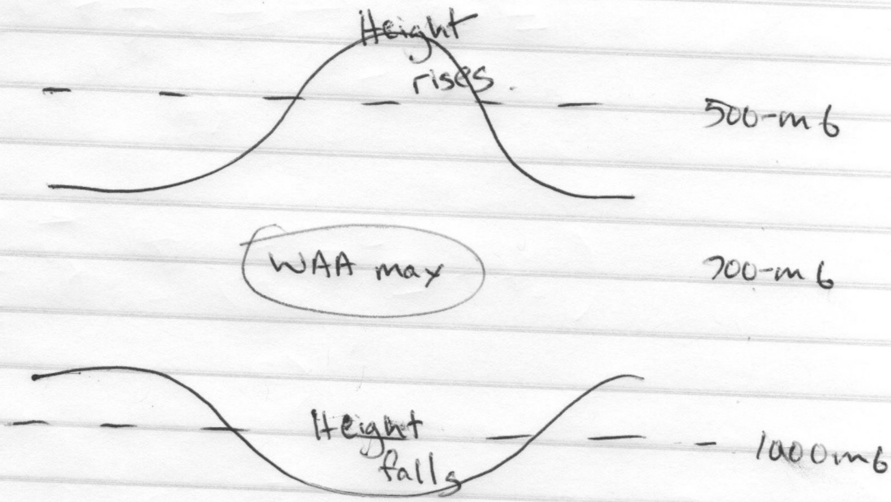
B: Geostrophic vorticity advection

C: Differential thickness advection.

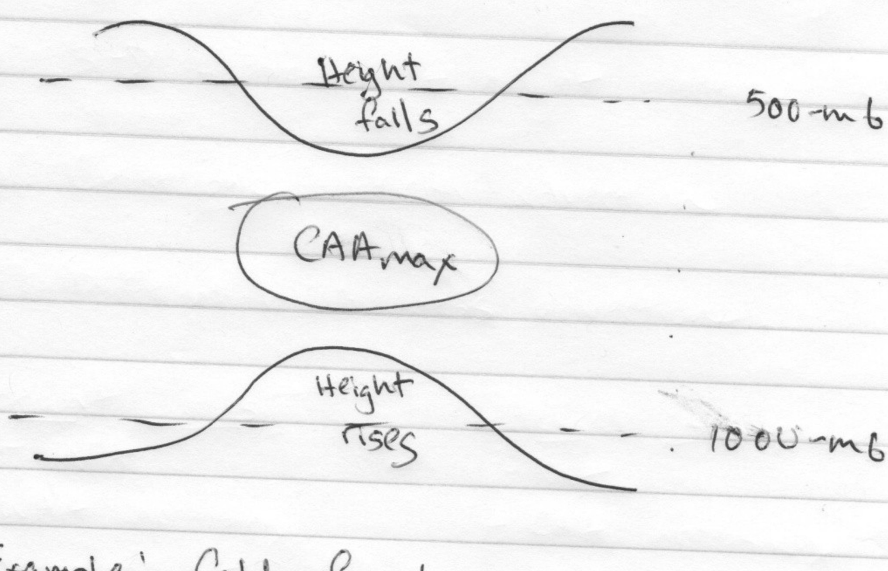
Height falls
- PVA
- WAA increasing with height
- CAA decreasing with height

Height rises
- NVA
- CAA increasing with height
- WAA decreasing with height

Concept of
Differential thickness advection - Fig. 2.16



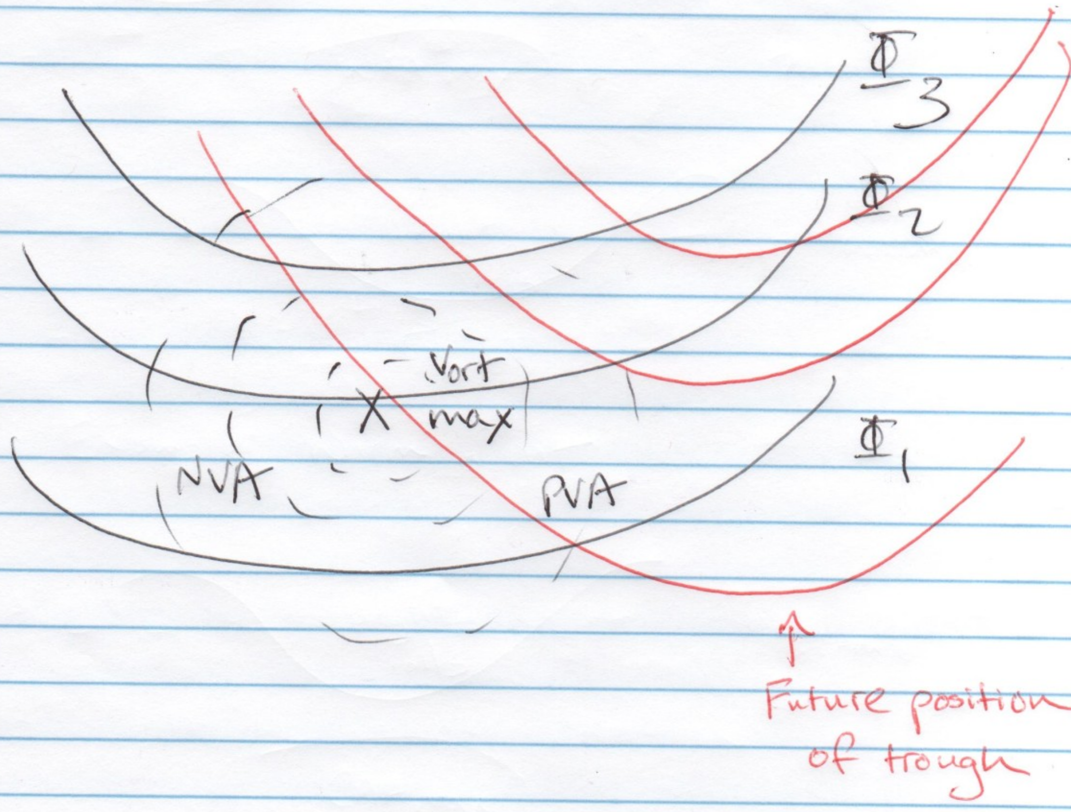
Example: Warm front



Example: Cold front

Digging vs. Progressive Trough

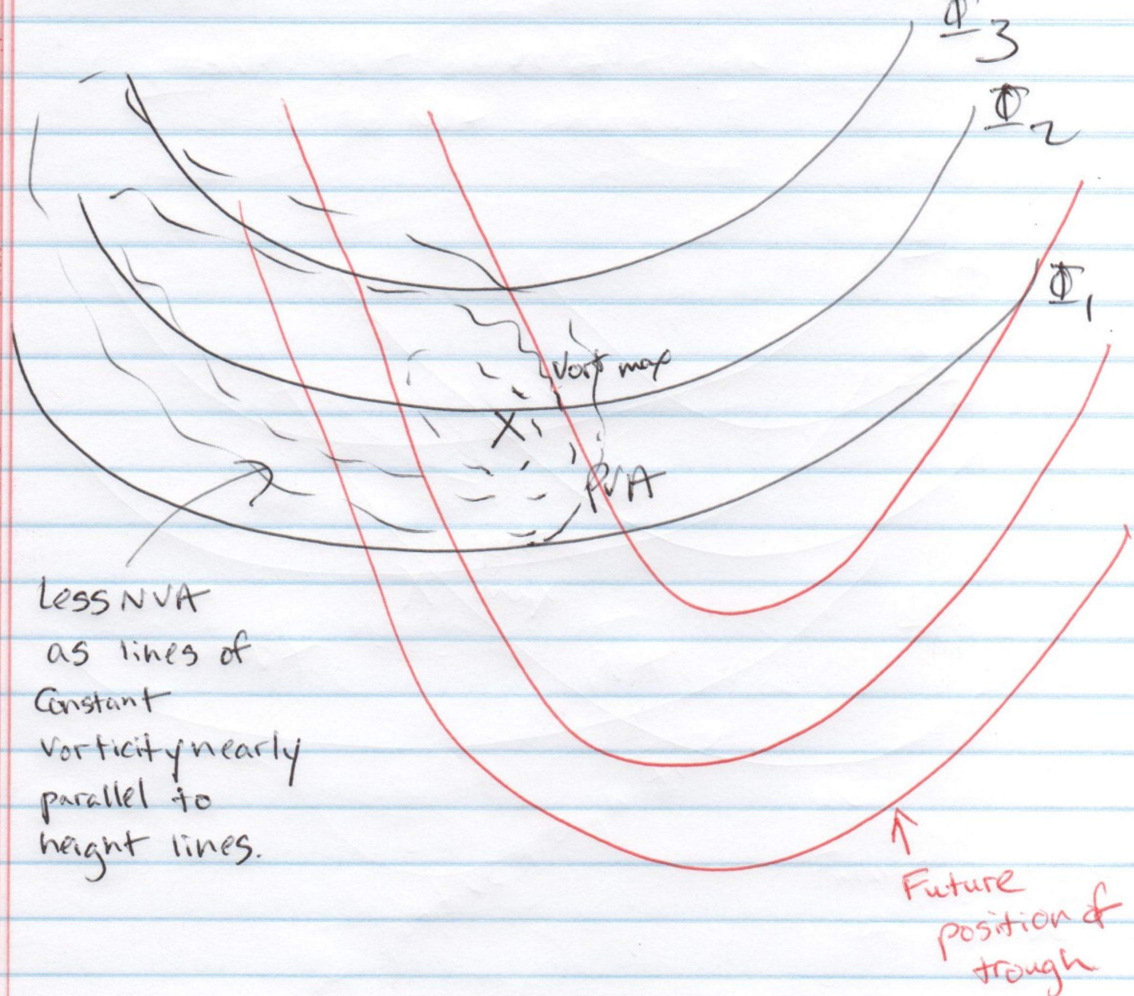
Progressive: PVA ahead of trough, equal to NVA behind.



Result: Disturbance just translates along, but trough doesn't deepend

Characteristic of fast-moving, weak weather systems (e.g. Alberta Clipper) that maintain a positive trough tilt.

Digging: PVA ahead of trough, but less NVA behind.



Result: Trough will deepen and strengthen

Characteristic of intensifying mid-latitude cyclone with well-defined jet streak, that may be occluding with trough tilt going negative.

Pettersen's Cyclone Development Eqn

Prognostic equation for sfc. vorticity tendency. → tells where sfc low will move!

Start with vorticity equation

$$\frac{\partial(\zeta + f_0)}{\partial t} + \vec{V}_g \cdot \nabla(\zeta + f_0) = 0$$

Assume negligible:
- Vertical advection of absolute vorticity
- Tilting term
- Solenoidal term

If wind is in geostrophic balance:

$$\vec{V}_{H500} = \vec{V}_{Hsfc} + \Delta \vec{V}_g$$

$\Delta \vec{V}_g$ = geostrophic wind shear or thermal wind

So....

$$(\zeta + f)_{500} = (\zeta + f)_{sfc} + (\zeta + f)_T$$

↑ Vorticity of 500-mb to 1000-mb thermal wind

Since

$$\nabla \times \vec{V}_{H500} = (\nabla \times \vec{V}_{Hsfc}) + (\nabla \times \Delta \vec{V}_g)$$

Vorticity 500-mb — Vorticity sfc. — Vorticity of thermal wind

$$\frac{\partial(\zeta + f)_{sfc}}{\partial t} = -\vec{V}_{H500} \cdot \nabla(\zeta + f)_{500} - \frac{\partial(\zeta + f)_T}{\partial t}$$

Surface vorticity tendency — Advection of vorticity @ 500-mb — Vorticity tendency of thermal wind

$$(\zeta + f)_T = \nabla \times \Delta \vec{V}_g = \frac{g}{f} \nabla_p^2 (\Delta Z)$$

Because, using thermal wind

$$\Delta \vec{V}_g = \frac{g\hat{k}}{f} \times \nabla_p (\Delta Z)$$

So . . .

$$\frac{\partial(\zeta + f)_T}{\partial t} = \frac{g}{f} \nabla_p^2 \frac{\partial(\Delta Z)}{\partial t}$$

Thermodynamic equation

$$\frac{\partial T}{\partial t} = -V_g \nabla T + \frac{\sigma p}{R}\omega + \frac{J}{c_p}$$

$$\frac{\partial z}{\partial p} = \frac{-R}{g}\frac{T}{P}$$ → from hydrostatic equation

Using $g\,\partial z/\partial p$ as proxy for temperature, we consider the change of the mean temperature of the 500-mb. to surface layer.

$$-g\frac{\partial(\Delta z)}{\partial t} = \int_{p_{sfc}}^{500\text{-mb}} \left(-\vec{V}_g \cdot \nabla_p\left(-g\frac{\partial z}{\partial p}\right) + \frac{\sigma p}{R}\omega + \frac{J}{c_p}\right) dp.$$

Perform ∇_p^2 on this equation, divide by f and substitute into equation for $\partial(\zeta+f)_T/\partial t$

$$\frac{\partial(\zeta+f)_T}{\partial t} = \frac{g}{f}\nabla_p^2\,\partial(\Delta z)/\partial t$$

<u>Result</u> ! Pettersen's cyclone development eqn.

$$\frac{\partial(\zeta+f)}{\partial t}_{sfc} =$$

$$-\vec{V}_{H500} \nabla_p (\zeta+f)_{500}$$

(A)

$$+ \frac{g}{f} \nabla_p^2 \int_{p_{sfc}}^{500\text{-mb}} \vec{V}_H \cdot \nabla_p \left(\frac{\partial z}{\partial p} \right) dp.$$

(B)

$$+ \frac{\nabla_p^2}{f} \int_{p_{sfc}}^{500\text{-mb}} \frac{\sigma_p}{R} dp \qquad + \frac{\nabla_p^2}{f} \int_{p_{sfc}}^{500\text{-mb}} J/c_p \, dp.$$

(C) (D)

(A) Horizontal vorticity advection @ 500-mb

(B) Pressure weighted temperature advection between sfc and 500-mb.

(C) Vertical motion

(D) Diabatic heating.

Trough tilt: Qualitative arguments

*See more extended discussion in text.

Depending on the tilt of the trough, it will act to extract or export energy to the mean flow.

Mathematically, eddy kinetic energy change is related to the interactions of eddies with the mean flow

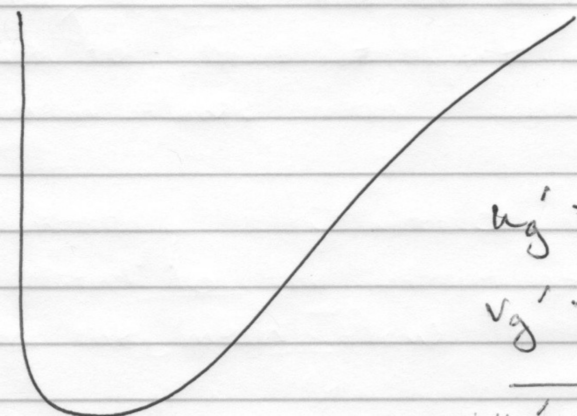
$$\frac{dK_{eddy}}{dt} \approx -\overline{u_g' v_g'} \frac{\partial \bar{u}_g}{\partial y}$$

Where prime terms = departure from mean flow

We can assume that $\partial \bar{u}_g / \partial y$ is positive since we're interested in baroclinic waves that form south of the polar jet stream.

Therefore, for eddy growth we require that $\overline{u_g' v_g'}$ be negative. This will depend on orientation of the trough.

Positively tilted trough:



$u_g' > 0$

$v_g' > 0$

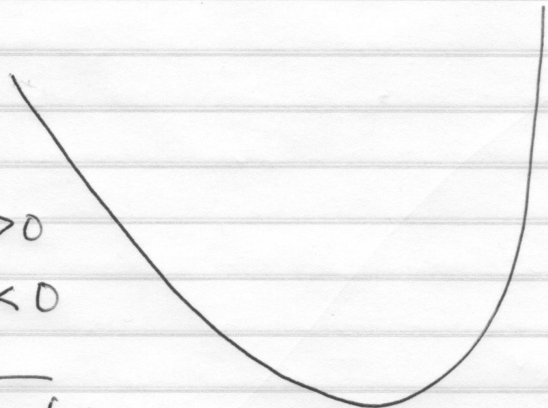
$-\overline{u_g' v_g'} < 0$

- Eddy will not amplify
- Will export its energy back to mean flow
- Export positive planetary vorticity.

Physically: Eddy is leaning with the ambient shear, so can't convert energy of mean flow to eddy kinetic energy.

Examples: Happens with weak, progressive systems. Typical of mid-latitude cyclones in early stages.

Negatively tilted trough



$u_g' > 0$

$v_g' < 0$

$-\overline{u_g' v_g'} > 0$

- Eddy will dig and amplify
- Will extract energy from mean flow
- Import positive planetary vorticity

Physically: Eddy is leaning against the ambient shear, so able to convert energy of mean flow to eddy kinetic energy

Examples: Happens with very strong, digging mid-latitude cyclones in mature to occluded phase of their development.