Isentropic analysis

Why is this necessary to look at vertical motion in a new way, given that we have already used QG-theory for diagnosis of vertical motion?

Rationale: weaknesses of QG-vertical motion

- Involves computation of Laplacians (∇^2's) and evaluation of vertical derivatives. That's problematic given coarse resolution of observational data. So must rely on model fields
- How to invert Laplacian?
- Applies to synoptic scale ($R_o \ll 1$) and where static stability (N^2) assumed constant. So difficult to account for more mesoscale processes that are non-QG and non-adiabatic (e.g. fronts, convection, friction)

Isentropic analysis allows us to more explicitly consider the non-adiabatic contributions to vertical motion, when considering cross-sections in particular.

Isentropic = lines of constant entropy

Conserved variable is potential temperature

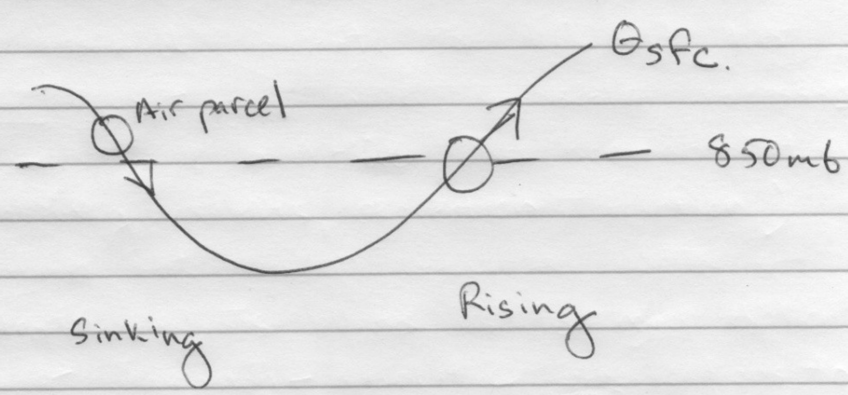
$$\theta = T\left(\frac{P_o}{P}\right)^{R/c_p}$$

<u>Advantages</u>

1) If motion is adiabatic, air is thermodynamically constrained to move on isentropic surfaces. No such restriction for isobaric surface.

Therefore will get a sense of 3-D motion of an air parcel.

Fig 3.1

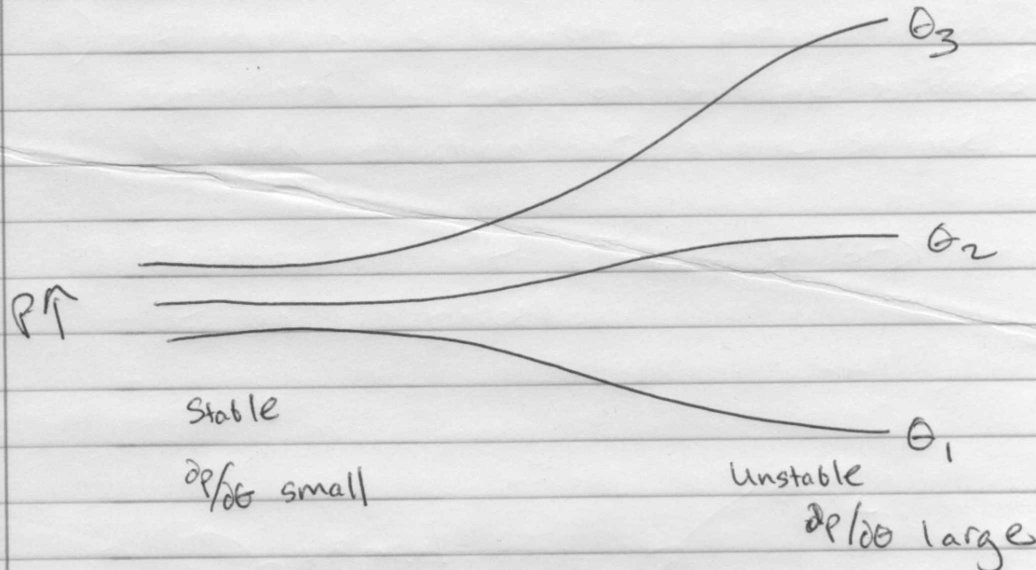


Assuming adiabatic airflow.

2) The density of the isentropes is inversely related to the static stability.

$$\sigma = -\frac{1}{g}\frac{\partial p}{\partial \theta} \sim \frac{1}{N^2}$$

So the packing of the isentropes on a cross-section is an indicator of stable and unstable air.



The tropopause is very easy to spot, since stability rapidly increases there.

Derving an adiabatic vertical velocity

Consider cross-sectional analysis (non-saturated)

First law of thermodynamics

Constant volume form $\quad dQ = c_v dT + P d\alpha \qquad P d\alpha = d(P\alpha) - \alpha dp.$

$\underset{d(RT)}{\uparrow}$

Constant pressure form $\quad dQ = c_p dT - \alpha dp.$

Express in terms of temperature tendency

$$\frac{dT}{dt} = \frac{1}{c_p}\frac{dQ}{dt} + \frac{\alpha}{c_p}\frac{dp}{dt} \qquad \frac{\alpha}{c_p} = \frac{dT}{dp} = \gamma_d$$

Dry adiabatic lapse rate in p-coord.

Expand total derivative for T

$$\frac{\partial T}{\partial t} + \underset{\text{3-D}}{\vec{V}\cdot\nabla T} = \frac{1}{c_p}\frac{dQ}{dt} + \frac{\alpha}{c_p}\frac{dp}{dt}$$

$$\frac{\partial T}{\partial t} + \vec{V}\cdot\nabla T = \frac{1}{c_p}\frac{dQ}{dt} + \gamma_d\,\omega$$

Note

$$\vec{V} \cdot \nabla T = \vec{V}_H \cdot \nabla T + \omega \frac{\partial T}{\partial p}$$

$$= \vec{V}_H \cdot \nabla T + \omega \gamma$$

So...

$$\underbrace{\frac{\partial T}{\partial t}}_{\text{Local time rate of change of } T} + \underbrace{\vec{V}_H \cdot \nabla_{\cancel{H}} T + \omega \gamma}_{\text{Horizontal and vertical advection of } T} = \underbrace{\frac{1}{c_p} \frac{\partial Q}{\partial t}}_{\text{Diabatic heating}} + \underbrace{\gamma_d \omega}_{\text{Adiabatic term}}$$

Ignoring diabatic heating → get adiabatic vertical velocity

$$\boxed{\omega = \frac{\partial T/\partial t + \vec{V}_H \cdot \nabla T}{\gamma_d - \gamma}}$$

pressure form

Using conversion factor of $\omega = -g\rho w$

$$\boxed{w = \frac{-\left(\frac{\partial T}{\partial t} + \vec{V}_H \cdot \nabla T\right)}{\Gamma_d - \Gamma}}$$

Cartesian form

Contributions to adiabatic vertical motion

	Subsidence	Ascent
Advection	cold	warm
Local ΔT	Cooling	Warming
Stability	Stable	Unstable

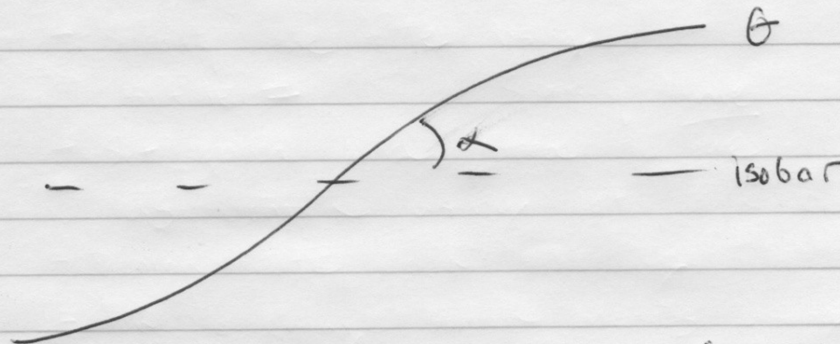
Depending on whether there is rising or sinking motion on θ_{sfc}, different nomenclature for fronts

Katafront: Downsloping winds (subsidence) (e.g. cold front)

Anafront: Upsloping winds (overrunning) (e.g. warm front)

In practice, can estimate the adiabatic vertical velocity by assuming $\partial T / \partial t$ is zero and then all that is left is advection.

Can also crudely estimate vertical velocity by the projection of the wind that is cross-isobaric.



Estimate vertical motion by

$$|V| \sin\alpha \simeq w$$

* Should yield estimate on order of $cm\, s^{-1}$

V = wind in plane of cross-section (in horizontal direction)
Zonal → u Meridional → v

α = Angle of slope of θ_{sfc} relative to isobaric surface
Function of $\Delta Z / \Delta S$ where
ΔZ = change in vertical
ΔS = synoptic-scale distance (~ hundreds of km)

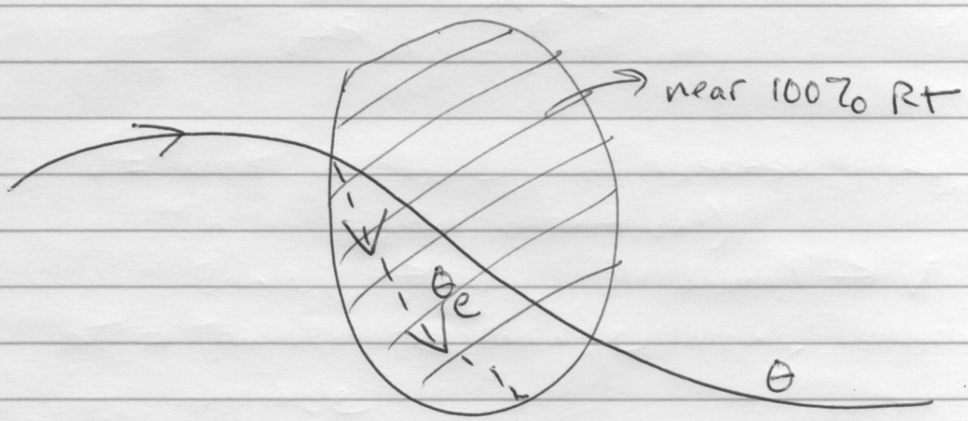
What about saturated conditions?

If the air is saturated, then there is an additional component of diabatic heating due to latent heat release.

In this case the trajectory of the air parcel will follow θ_e (equivalent potential temperature), since that accounts for latent heat release.

Can assume make this change when RH approaches 70% or above, or something that is close to saturation.

Arrows indicate parcel trajectory



Trajectory will follow θ_e when near saturation. In this case, then, vertical motion equation should use saturation adiabatic lapse rate instead of dry adiabatic lapse rate.

Estimating ω from constant θ map.

$$\omega = \frac{\partial p}{\partial t} + \vec{V}\cdot\nabla_\theta p + \frac{dp}{d\theta}\frac{d\theta}{dt}$$

(A) (B) (C)

(A) → local pressure tendency

(B) → pressure advection (dominant)
Cross isobar wind component

(C) → diabatic heating
Causes isentropic surfaces to move (expand / contract)

Lackmann notes that in practice we must estimate a storm-relative motion and then

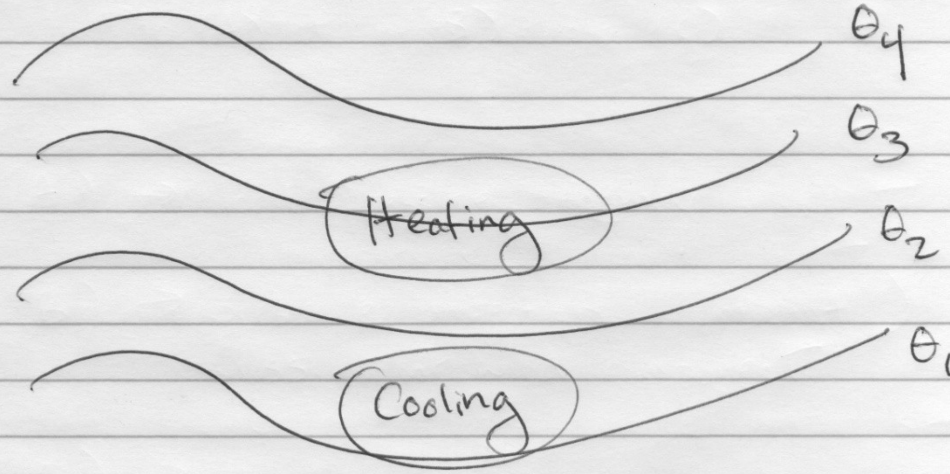
Advecting velocity = Total horiz. wind ($\vec{V}$) − Storm relative motion ($\vec{c}$)

$$\omega \simeq (\vec{V}-\vec{c})\cdot\nabla_\theta p + \frac{\partial p}{\partial\theta}\frac{\partial\theta}{\partial t}$$

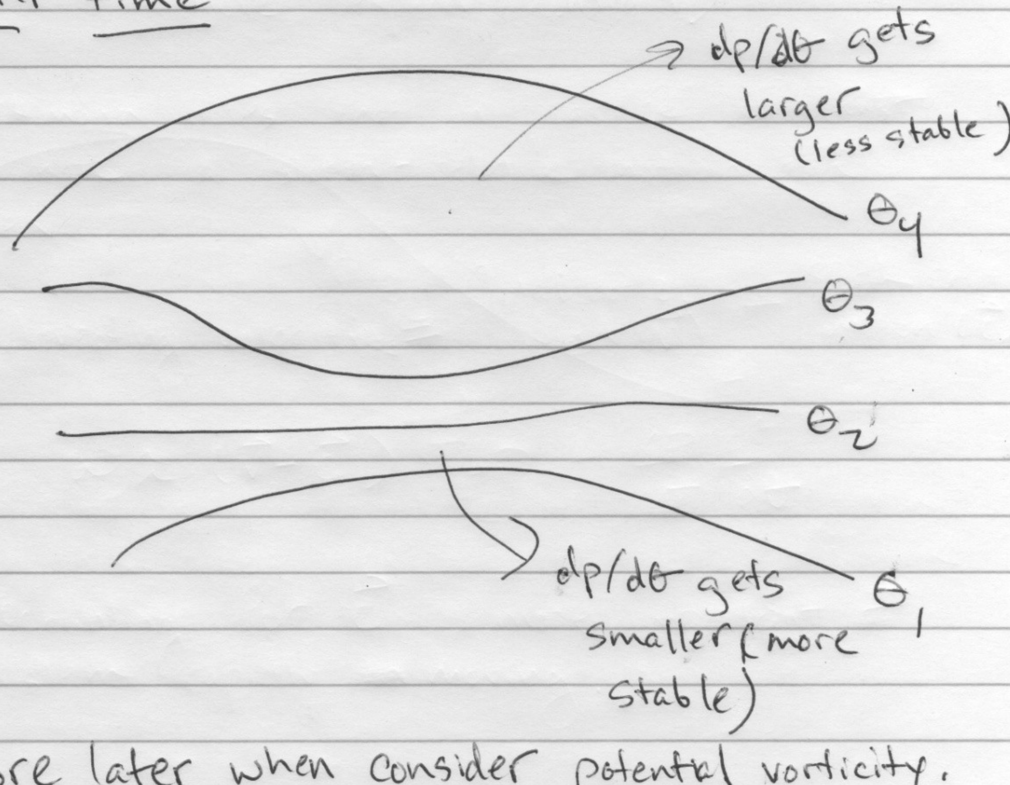
Accounting for $\vec{c}$ is more important if system rapidly ~~is~~ moving.

Effects of heating (and cooling) on isentropic surfaces

Initial time



Later time



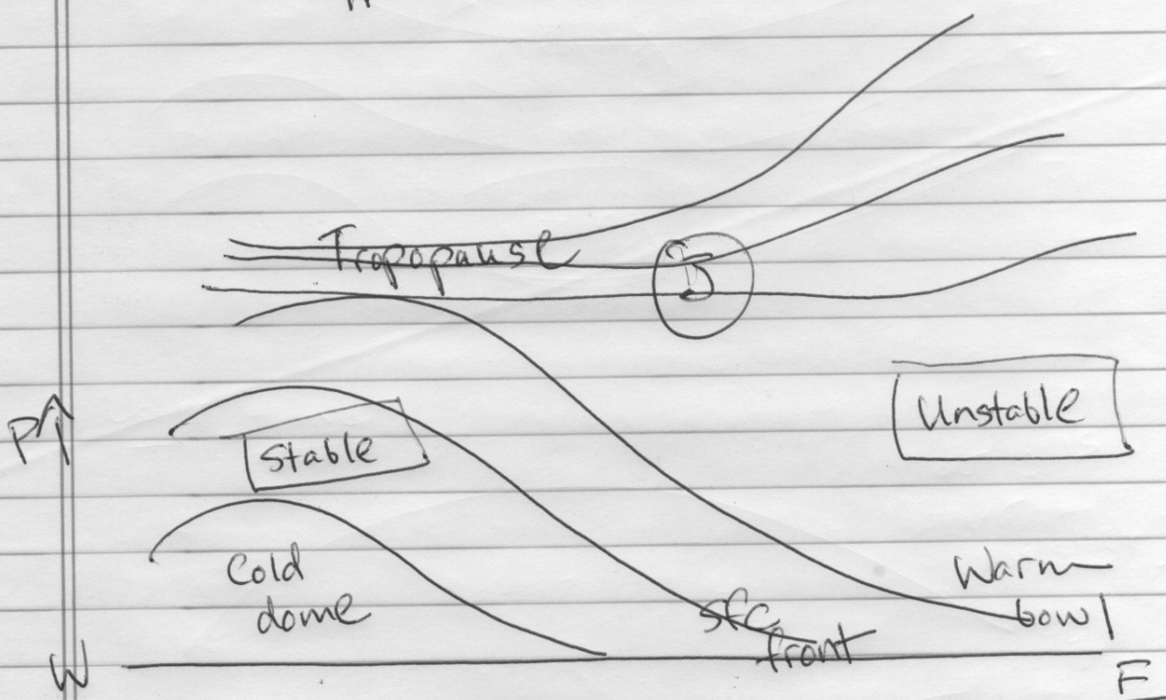
More later when consider potential vorticity.

Cross sections

Useful for

1) Atmospheric stability
2) Frontal positions
3) Vertical motion

Across typical cold front



Tropopause: Abrupt change in packing of θ lines around 300-mb.

Jet stream: Occurs where slope in tropopause greatest → vertically integrated $\nabla_H T$ is large. Tropopause sinks on cold side.

Polar front: Look just below jet stream where θ lines tightly packed. Can trace ~~sa~~ it to surface to define surface front

Cold domes: Inverted parabolas

Warm bowls: Parabolic bowls.

Advantages of isentropic analysis

1) Visual depiction of 3-D airflow, including ω and moisture transport
2) Conceptual simplicity to ω
3) Adiabatic assumption is good, and switch to θ_e surface if not!
4) Don't need uniform N^2 or $R_o \ll 1$

Disadvantages

1) Diabatic processes are important
2) θ does not always increase with height (e.g. tropopause folding)
3) Must interpolate data to θ_{sfcs}
4) Not very dynamically insightful as to cause of ω.
5) What particular surface in θ to consider varies depending on time and space.