Dynamic pressure and its importance on the mesoscale, non-hydrostatic modeling

Simplified equations of motion for convection on mesoscale

	Full equation	Linearize, Boussinesq approx.
U momentum	$\frac{Du}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$	$\frac{Du}{Dt} = -\frac{1}{\bar{\bar{\rho}}}\frac{\partial p'}{\partial x}$
V momentum	$\frac{Dv}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial y}$	$\frac{Dv}{Dt} = -\frac{1}{\bar{\bar{\rho}}}\frac{\partial p'}{\partial y}$
W momentum	$\frac{Dw}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g$	$\frac{Dw}{Dt} = -\frac{1}{\bar{\bar{\rho}}}\frac{\partial p'}{\partial z} + B$

ASIDE: Absolutely no Coriolis effects here! Dynamic pressure effects relevant mostly at meso-gamma scale (order km) for situations of deep convection

$$B = -\frac{\rho'}{\bar{\bar{\rho}}} g$$

Buoyancy
Can also be written in terms of potential temperature (Boussinesq approx.)

Differentiate u and v momentum equations with respect to x and y, respectively, then add together..

Add w momentum equation, differentiated with respect to z and assuming constant mean density, to get a **divergence equation:**

$$\frac{D}{Dt}(\nabla \bullet \vec{v})+\frac{1}{\bar{\rho}}\nabla^2 p' = \left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2+2\frac{\partial u}{\partial y}\frac{\partial v}{\partial x}+2\frac{\partial u}{\partial z}\frac{\partial w}{\partial x}+2\frac{\partial v}{\partial z}\frac{\partial w}{\partial y}\right]+\frac{\partial B}{\partial z}$$

Invoking a shallow water assumption, our first term on LHS vanishes because:

$$\nabla \bullet \vec{v} = \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z} = 0$$

Linearize about the following state:

$$u = \bar{u}(z) + u'(x,y,z,t)$$

$$v = \bar{v}(z) + v'(x,y,z,t)$$

$$w = w'(x,y,z,t)$$

Assume a linearly varying mean vertical wind profile of u and v, for simplicity.

Laplacian equation for pressure perturbation = **dynamic pressure + buoyancy**

$$\frac{1}{\rho}\nabla^2 p' = -\left[\left(\frac{\partial u'}{\partial x}\right)^2 + \left(\frac{\partial v'}{\partial y}\right)^2 + \left(\frac{\partial w'}{\partial z}\right)^2 + 2\left(\frac{\partial u'}{\partial y}\frac{\partial v'}{\partial x} + \frac{\partial u'}{\partial z}\frac{\partial w'}{\partial x} + \frac{\partial v'}{\partial z}\frac{\partial w'}{\partial y} + \frac{\partial \bar{u}}{\partial z}\frac{\partial w'}{\partial x} + \frac{\partial \bar{v}}{\partial z}\frac{\partial w'}{\partial y}\right)\right] + \frac{\partial B}{\partial z}$$

$$p' = p'_{dyn} + p'_B = p'_{dynL} + p'_{dynNL} + p'_B$$

<u>Linear dynamic contribution:</u> Interaction of environmental shear with updraft vertical velocity (i.e. rotation of horizontal environmental vorticity into vertical)

$$\nabla^2 p'_{dynL} \propto -\left[\frac{\partial \bar{u}}{\partial z}\frac{\partial w'}{\partial x} + \frac{\partial \bar{v}}{\partial z}\frac{\partial w'}{\partial y}\right] = -\frac{\partial \vec{v}_H}{\partial z}\cdot \nabla_H w'$$

<u>Non-linear dynamic contributions:</u> Fluid extension terms and shear terms

$$\nabla^2 p'_{dynNL} \propto -\left[\left(\frac{\partial u'}{\partial x}\right)^2 + \left(\frac{\partial v'}{\partial y}\right)^2 + \left(\frac{\partial w'}{\partial z}\right)^2 + 2\left(\frac{\partial u'}{\partial y}\frac{\partial v'}{\partial x} + \frac{\partial u'}{\partial z}\frac{\partial w'}{\partial x} + \frac{\partial v'}{\partial z}\frac{\partial w'}{\partial y}\right)\right]$$

Fluid extension Shear

Contributions to dynamic pressure by linear dynamics

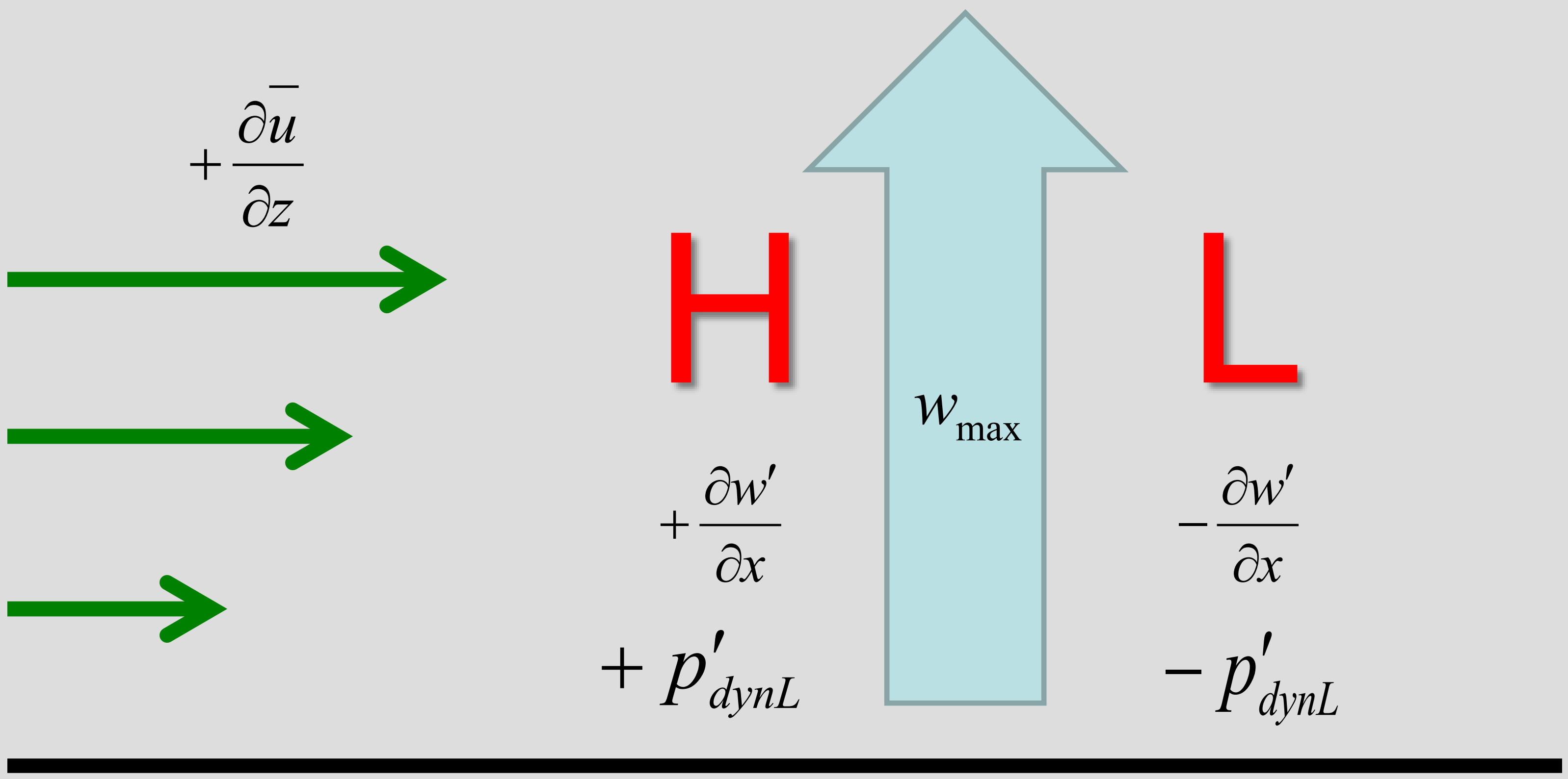
Since a Laplacian tends to change the sign of the variable on which it operates:

$$p'_{dynL} \propto \frac{\partial \vec{v}_H}{\partial z} \cdot \nabla_H w'$$

For an updraft in an environment of positive unidirectional zonal shear,

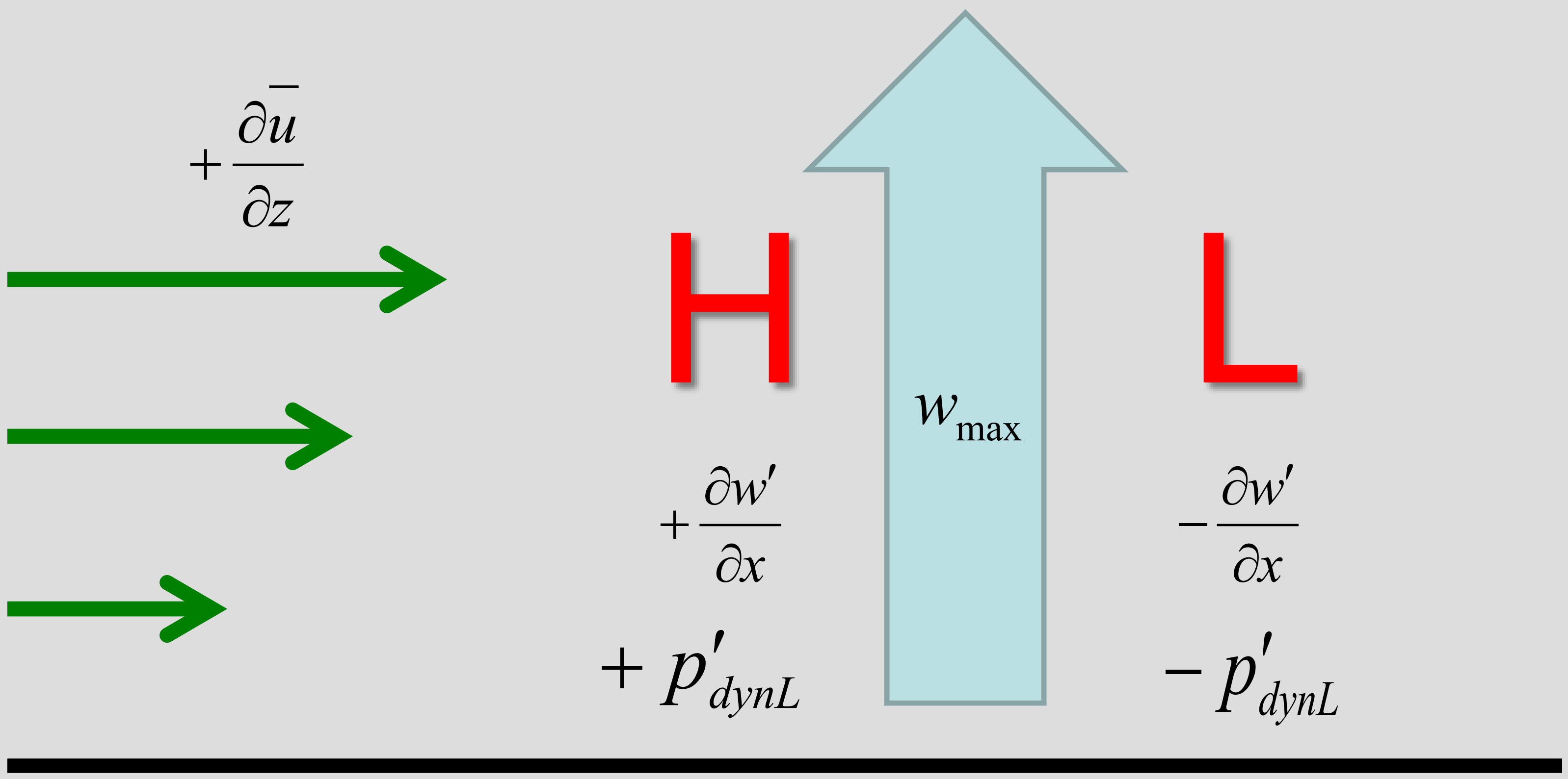
Positive perturbation pressure on upshear side of updraft

Negative perturbation pressure on downshear side of updraft



$$p'_{dynL} \propto \frac{\partial \vec{v}_H}{\partial z} \cdot \nabla_H w'$$

Physical interpretation: Vertical advection by an updraft of horizontal momentum associated with environmental shear is balanced by pressure gradient force.



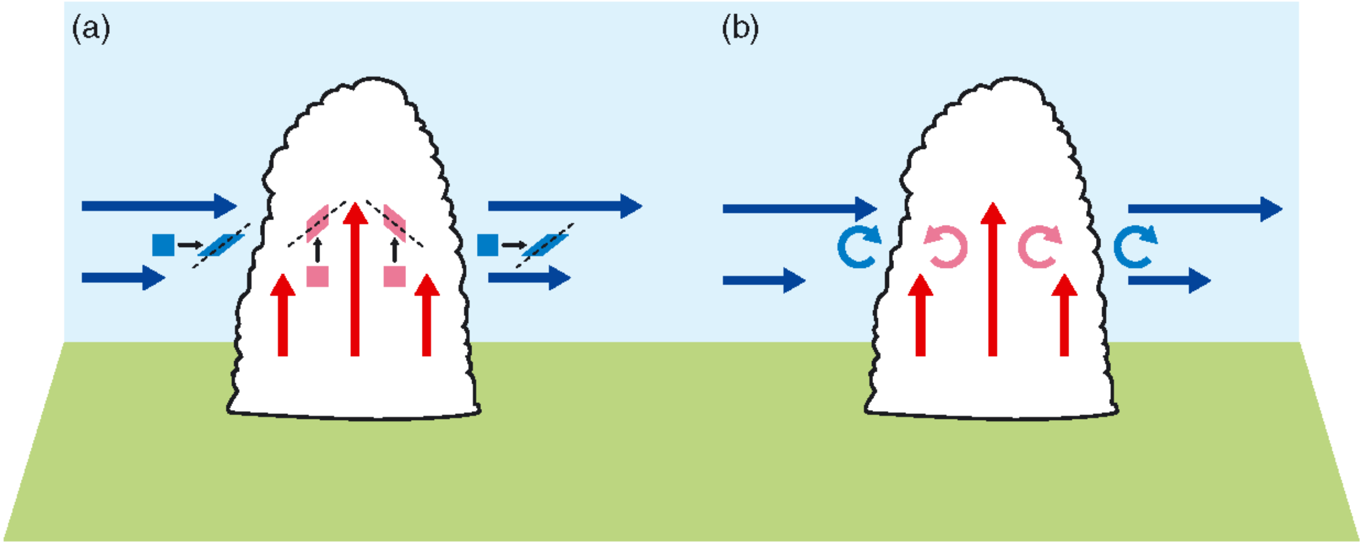


Figure 2.8 (a) Illustration showing how the deformation associated with the mean vertical wind shear acts in the same (opposite) sense as that associated with the gradient of w' on the upshear (downshear) flank of an updraft. The blue vectors give a sense of the increase in westerly winds with height (the shear points toward the right, thus the upshear flank is the left flank, and the downshear flank is the right flank), and the red vectors give a sense of the horizontal gradient of (perturbation) vertical velocity. The sense of how a fluid element would be deformed by the mean and perturbation wind gradients is shown schematically via the light blue and pink polygons, respectively. The black dashed lines indicate the axes along which the fluid elements are elongated by the shear. (b) Illustration showing how the vorticity associated with the mean vertical wind shear points in the opposite (same) direction as that associated with the gradient of w' on the upshear (downshear) flank of an updraft. The light blue curved arrows indicate the sense of rotation associated with the mean wind shear, and the pink curved arrows indicate the sense of rotation associated with the updraft (i.e., the perturbation wind field).

Contribution to pressure perturbation by non-linear dynamics

Shearing terms:

$$\nabla^2 p'_{dynNLshear} \propto -2\left(\frac{\partial u'}{\partial y}\frac{\partial v'}{\partial x} + \frac{\partial u'}{\partial z}\frac{\partial w'}{\partial x} + \frac{\partial v'}{\partial z}\frac{\partial w'}{\partial y}\right)$$

Can express with deformation and vorticity terms in all directions:

$$= -\frac{1}{2}\left[\left(\frac{\partial v'}{\partial x} + \frac{\partial u'}{\partial y}\right)^2 - \left(\frac{\partial v'}{\partial x} - \frac{\partial u'}{\partial y}\right)^2 + \left(\frac{\partial u'}{\partial z} + \frac{\partial w'}{\partial x}\right)^2 - \left(\frac{\partial u'}{\partial z} - \frac{\partial w'}{\partial x}\right)^2 + \left(\frac{\partial w'}{\partial y} + \frac{\partial v'}{\partial z}\right)^2 - \left(\frac{\partial w'}{\partial y} - \frac{\partial v'}{\partial z}\right)^2\right]$$

Shearing terms:

$$\nabla^2 p'_{dynNLshear} \propto -2\left(\frac{\partial u'}{\partial y}\frac{\partial v'}{\partial x} + \frac{\partial u'}{\partial z}\frac{\partial w'}{\partial x} + \frac{\partial v'}{\partial z}\frac{\partial w'}{\partial y} \right)$$

Can express with deformation and vorticity terms in all directions:

$$= -\frac{1}{2}\left[\left(\frac{\partial v'}{\partial x} + \frac{\partial u'}{\partial y} \right)^2 - \left(\frac{\partial v'}{\partial x} - \frac{\partial u'}{\partial y} \right)^2 + \left(\frac{\partial u'}{\partial z} + \frac{\partial w'}{\partial x} \right)^2 - \left(\frac{\partial u'}{\partial z} - \frac{\partial w'}{\partial x} \right)^2 + \left(\frac{\partial w'}{\partial y} + \frac{\partial v'}{\partial z} \right)^2 - \left(\frac{\partial w'}{\partial y} - \frac{\partial v'}{\partial z} \right)^2 \right]$$

Consider tilting of unidirectional shear by an updraft, such that all deformation terms and horizontal vorticity terms (i.e. crossed out terms) are zero. **All that is left is vertical vorticity.**

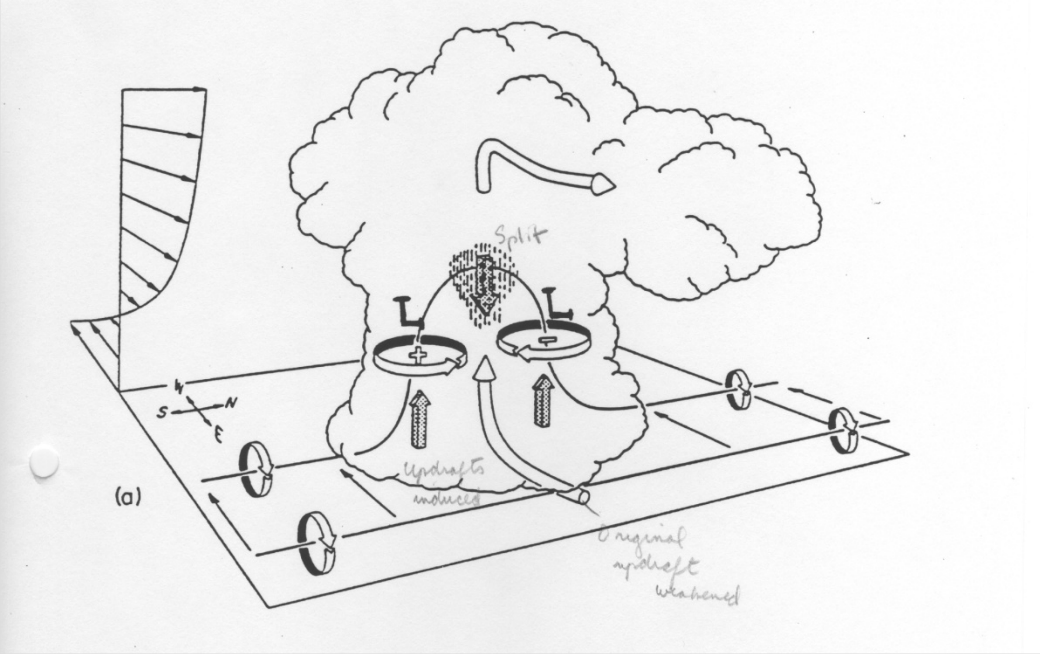
$$\nabla^2 p'_{dynNLshear} \propto \left(\frac{\partial v'}{\partial x} - \frac{\partial u'}{\partial y} \right)^2 = \zeta'^2$$

$$p'_{dynNLshear} \propto -\zeta'^2$$

Again, sign changes with Laplacian inversion

<u>Result:</u> Non-linear shearing terms produce low perturbation pressure in the vicinity of mid-level anticyclonic and cyclonic vorticity induced by the updraft.

Effect of non-linear pressure perturbation terms (shearing)



In unidirectional shear

Have formation of mid-level vorticies at storm flanks. Results from tilting of horizontal vorticity of the environment by updrafts results in perturbation low pressure.

This DOES NOT depend on the direction of rotation!

The upward directed pressure gradient at storm flanks will tend to enhance updrafts at the <u>sides of the storm</u>.

KLEMP-WILHELMSON
3-D NUMERICAL CLOUD MODEL

* Compressible Equations of Motion

* Prognostic Variables: u,v,w,t,p,q_V,q_C,q_R

* Kessler-type parameterization for microphysics (no ice)

* Subgrid scale turbulence parameterization based on a turbulence energy equation

* Open lateral boundary conditions

* Storms initiated by a symmetric warm bubble within a horizontally homogeneous environment, characterized by specified vertical profiles of wind, moisture, and temperature

Simplified Klemp-Wilhelmson model dynamical core

$$\frac{\partial u}{\partial t} = -\vec{V} \cdot \nabla u - c_{pd}\bar{\theta}_v \frac{\partial \pi'}{\partial x} \tag{14.1}$$

$$\frac{\partial w}{\partial t} = -\vec{V} \cdot \nabla w - c_{pd}\bar{\theta}_v \frac{\partial \pi'}{\partial z} + g\frac{\theta'}{\bar{\theta}} \tag{14.2}$$

$$\frac{\partial \theta'}{\partial t} = -\vec{V} \cdot \nabla \theta' - w\frac{d\bar{\theta}}{dz} \tag{14.3}$$

$$\frac{\partial \pi'}{\partial t} = -\frac{\bar{c_s}^2}{\bar{\rho} c_{pd}\bar{\theta}_v^2}\left[\nabla \cdot \bar{\rho}\bar{\theta}_v \vec{V}\right] \tag{14.4}$$

Can be used to simulate a rising warm (positively buoyant) bubble. From Robert Fovell modeling course notes.

A non-hydrostatic numerical model must have a way to represent dynamic pressure, typically through a prognostic equation.

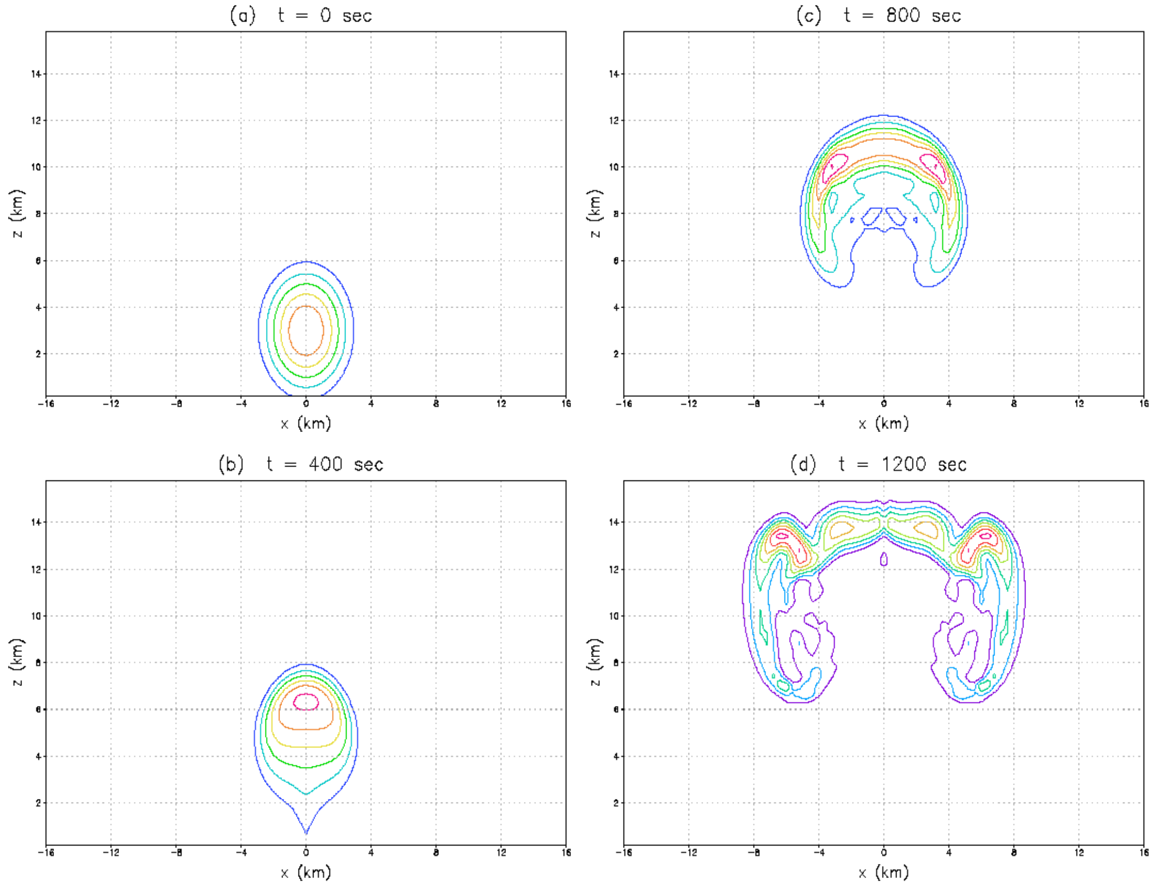


Figure 14.2: Perturbation potential temperature field at initial and three subsequent times. Contour interval 0.5 K; zero contour suppressed.

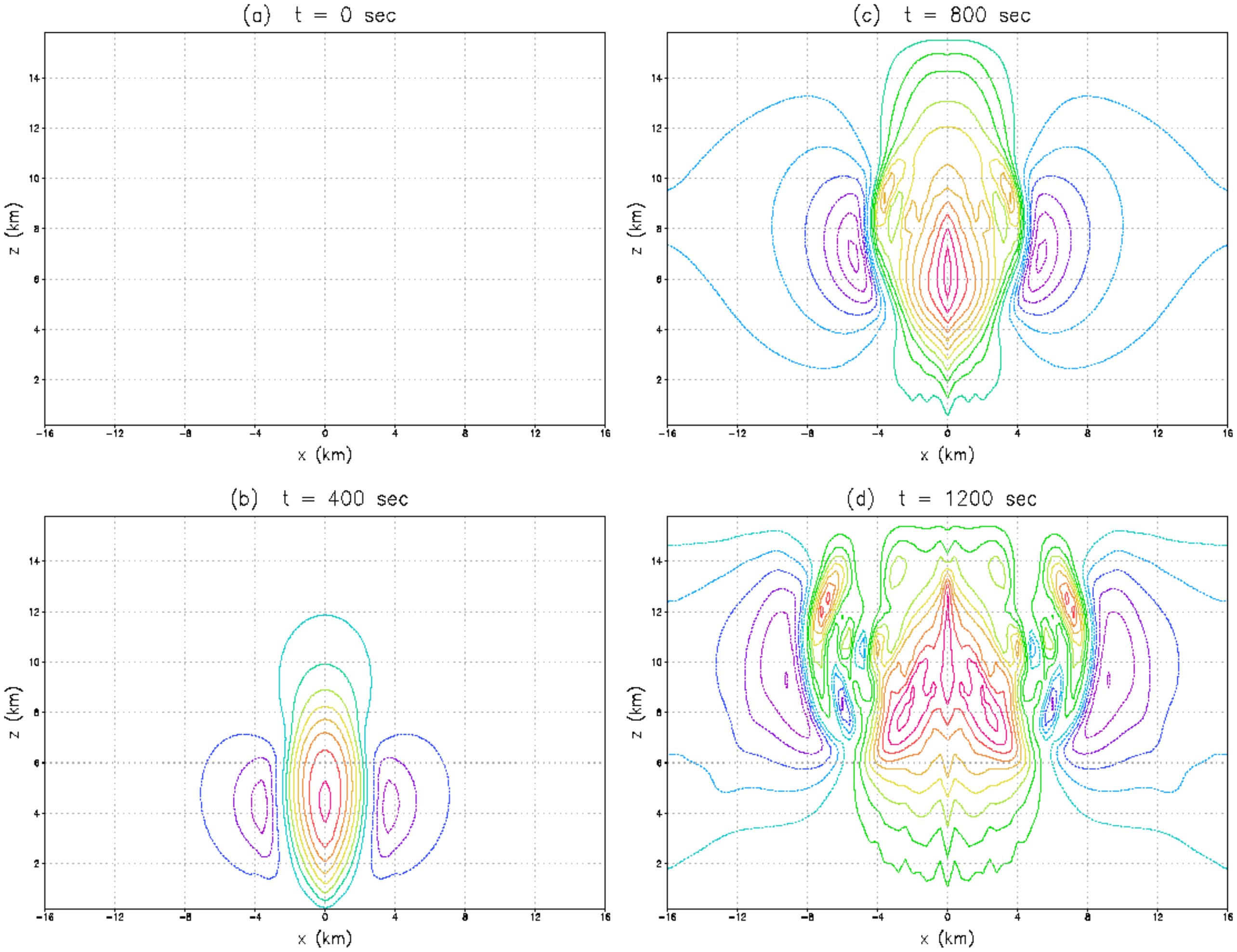


Figure 14.3: Vertical velocity field at initial and three subsequent times. Contour interval 2 m s^{-1}; zero contour suppressed.

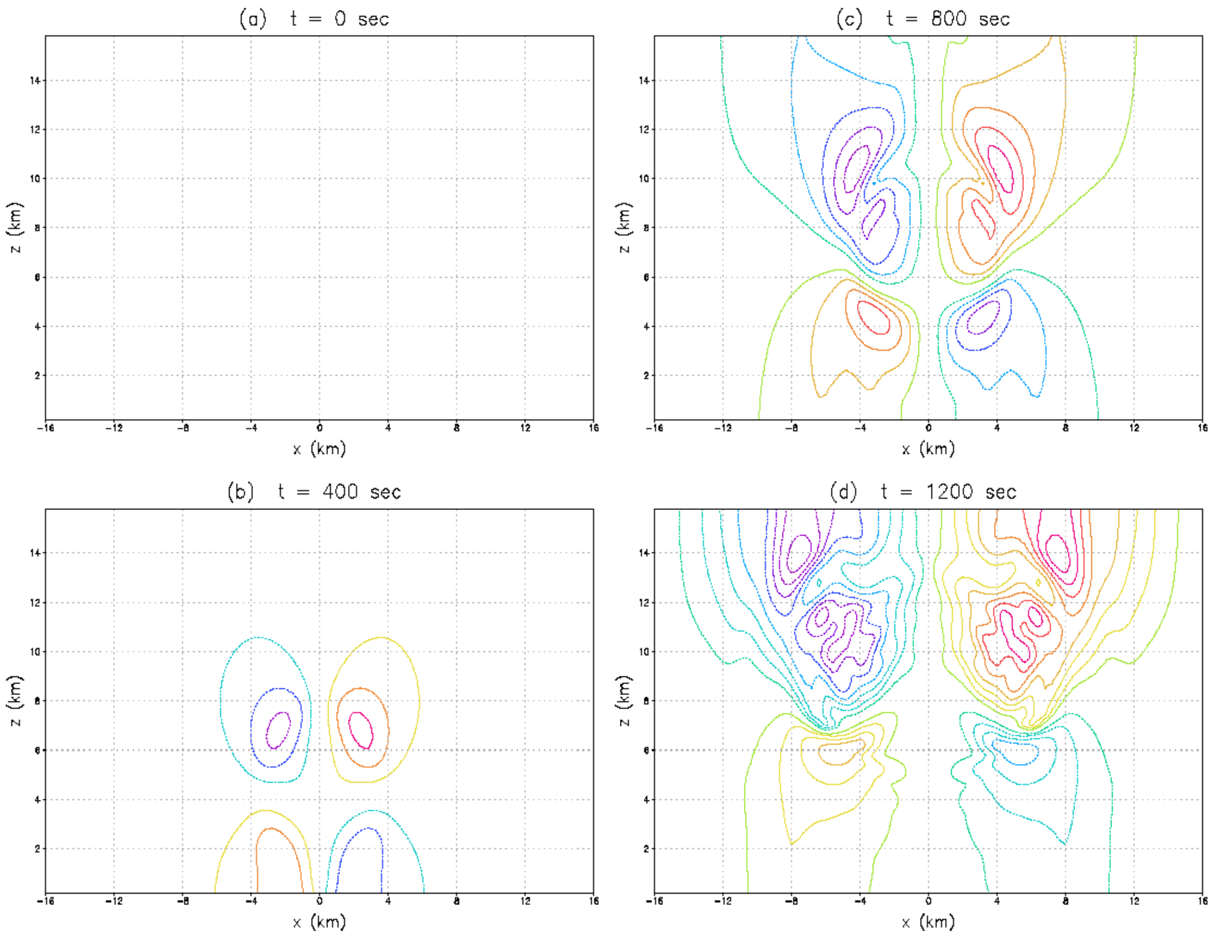


Figure 14.4: Horizontal velocity field at initial and three subsequent times. Contour interval 2.5 m s^{-1}; zero contour suppressed.

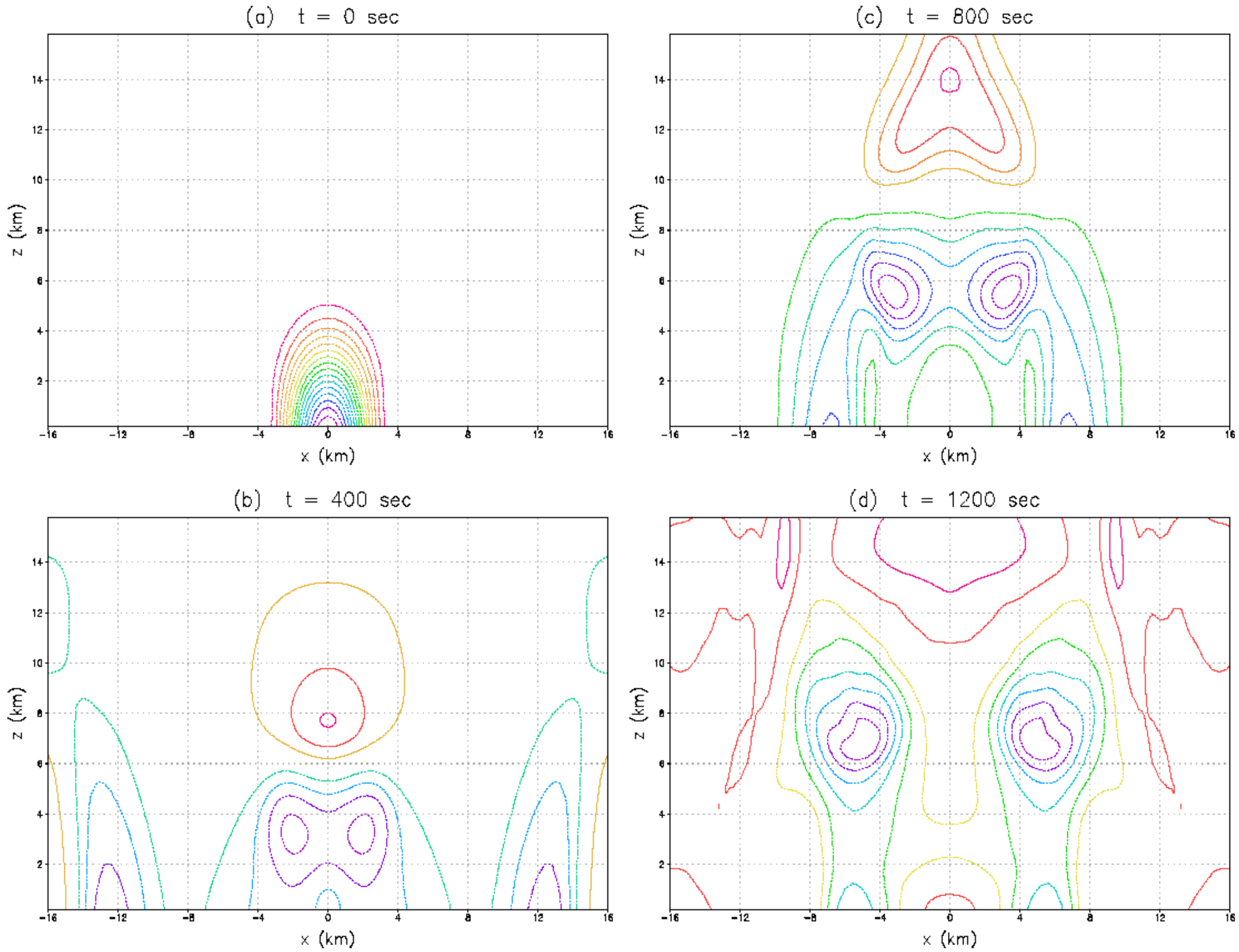


Figure 14.5: Dimensional pressure perturbation field at initial and three subsequent times. Contour interval 0.25 mb; zero contour suppressed.

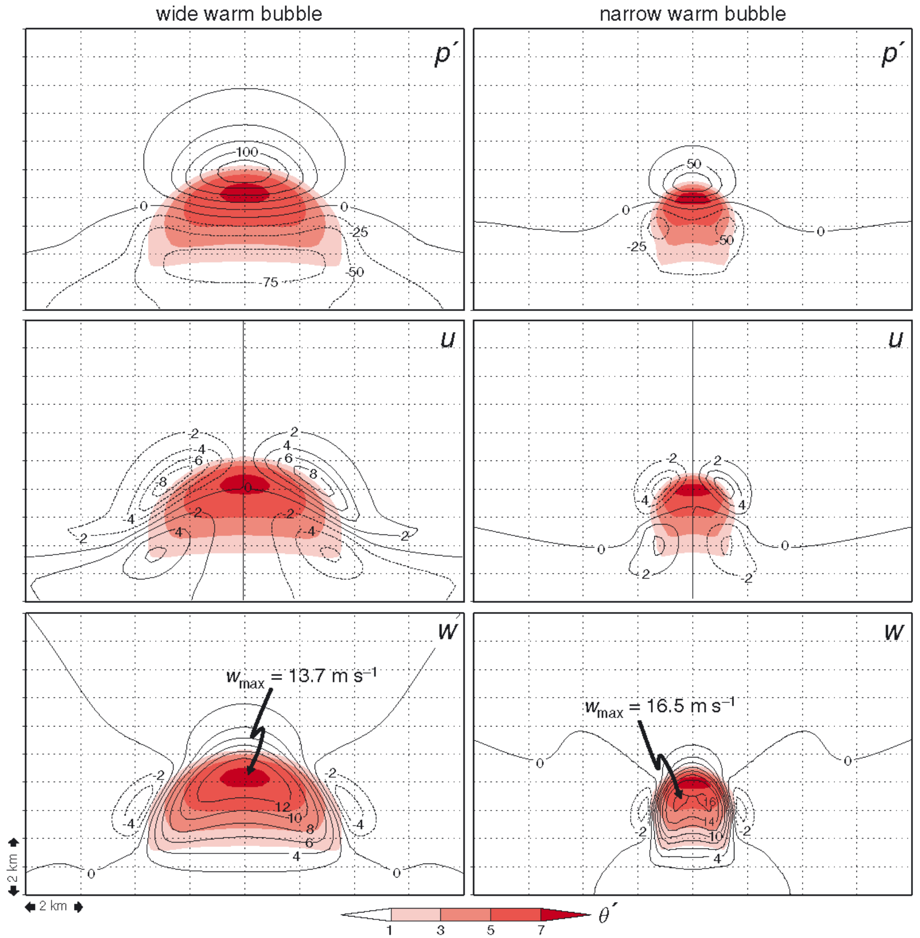


Figure 3.1 A comparison of the perturbation pressure (p') fields and zonal (u) and vertical (w) velocity components for the case of a wide warm bubble (left panels) and a narrow warm bubble (right panels) released in a conditionally unstable atmosphere in a three-dimensional numerical simulation. The contour intervals for p' and the wind components are 25 Pa and 2 m s^{-1}, respectively (dashed contours are used for negative values). Potential temperature perturbations (θ') are shown in each panel (refer to the color scale). The horizontal and vertical grid spacing is 200 m (the domain shown above is much smaller than the actual model domain). Both warm bubbles had an initial potential temperature perturbation of 2 K and a vertical radius of 1.5 km, and were released 1.5 km above the ground. The wide (narrow) bubble had a horizontal radius of 10 km (3 km). In the simulation of the wide (narrow) bubble, the fields are shown 800 s (480 s) after its release. The fields are shown at times when the maximum buoyancies are comparable. Despite the comparable buoyancies, the narrow updraft is 20% stronger owing to the weaker adverse vertical pressure gradient.

At hydrostatic limit, the vertical pressure perturbation → 0, no rising motion