

Brief review of thermodynamics

Part I: The dry atmosphere

Note:

The "boxed" variables are what we need for Skew-T, log P diagram later.

Equation of state

$$P = \rho R T$$

P = pressure $\frac{\text{N}}{\text{m}^2}$ Pa

R = gas constant $287 \frac{\text{J}}{\text{kgK}}$ for dry air

T = Temperature K

First law of thermodynamics

$$\Delta Q = \Delta u + \Delta W$$

↑
Heat exchanged with environment

↑
Change in internal energy

↓
Work done

For the atmosphere, ΔQ is:

- latent heating, cooling due to phase change of water vapor
- radiation
- Chemical changes.

* This is of primary concern for synoptic, mesoscale, met. perspective.

Left hand side terms

$$\frac{dT}{dt} = \frac{dT}{dz} \frac{dz}{dt}$$

Right hand side terms

$$\frac{dp}{dt} = \frac{dp}{dz} \frac{dz}{dt}$$

$$\frac{dp}{dz} = -\rho g = \frac{-g}{\alpha}$$

Hydrostatic

So...

$$c_p \frac{dT}{dt} = -g \frac{dz}{dt}$$

$$-\frac{dT}{dz} = \frac{g}{c_p} = \Gamma_d \quad \text{dry adiabatic lapse rate.}$$

(-9.8 K/m)

Can also derive Poisson's equation for potential temperature (see supplemental H0)

$$\Theta = T \left(\frac{P_0}{P} \right)^k$$

$$k = \frac{R}{c_p} = 0.286$$

$$P_0 = 1000\text{-mb.}$$

Potential temperature (Θ)

\Rightarrow Temperature air would have accounting for adiabatic displacement, referenced to 1000-mb.

Θ and not T is the thermodynamically conserved variable for an adiabatic process, because the atmosphere is hydrostatically balanced and p and ρ decrease exponentially with height.

So another way to express conservation of energy is

$$\frac{d\Theta}{dt} = \Delta Q$$

Note: Lagrangian derivative

$$\frac{d\Theta}{dt} = 0$$

for adiabatic process.

⇒ Again, for all intents and purposes the only diabatic process that we need to worry about for mesoscale is latent heating and cooling due to phase changes of water.

Vapor - liquid: Evaporation, condensation

Vapor - ice: Sublimation, deposition

Liquid - ice: Freezing, melting

Brief review of thermodynamics

Part II: The moist atmosphere

Equation of state for water vapor

$$e = \rho_v R_v T$$

e = vapor pressure

SI units

Pa

ρ_v = density of water vapor

kg/m^3

T = temperature

K

R_v = gas constant
for H_2O vapor

461 J/kgK

Total pressure is then:

$$p = p_d + e$$

→ Sum of dry atmosphere (p_d) + water vapor pressure

Water vapor mixing ratio (r_v or q)

⇒ The ratio of water vapor content to mass of dry air. Typically expressed in g/kg .

$$r_v = \frac{\rho_v}{\rho_d} \stackrel{\text{use gas law}}{\downarrow} = \frac{\frac{e}{R_v T}}{\frac{p_d}{R_d T}} = \frac{R_d}{R_v} \frac{e}{p_d}$$

"d" = dry

Defining ratio $\epsilon = R_v / R_d \sim 0.622$

$$r_v = \frac{\epsilon e}{p - e}$$

\Rightarrow Conversion of vapor pressure to mixing ratio.

If there is a phase change of water vapor

$$\frac{dr_v}{dt} = S_i \rightarrow \text{sources and sinks}$$

Note: Similar equations for other water species in liquid, ice phase (e.g. in a microphysical parameterization)

Any latent heating or cooling would ~~be~~ necessarily also be associated with a source or sink to r_v , (e.g. condensation, evaporation)

Relative humidity (RH)

⇒ Ratio of how much water vapor air is holding to how much water vapor air can potentially hold at a given temperature.

$$RH = \frac{e}{e_s} \quad \text{or} \quad \frac{r_v}{r_s} \left(\frac{q}{q_s} \right)$$

That is, can be defined by ratio of vapor pressures or mixing ratios.

e_s = saturation ~~vapor~~ vapor pressure
 r_s, q_s = saturation mixing ratio.

When $e = e_s$ (or $q, r_v = q_s, r_s$) the air is fully saturated and $RH = 100\%$

Clausius - Clapeyron equation (CC eqn)

Governs the relationship of saturation vapor pressure to temperature.

$$\frac{de_s}{dT} = \frac{L_v e_s}{R_v T^2}$$

L_v = latent heat of vaporization.

The CC eqn. is non-linear, so saturation vapor pressure increases exponentially with temperature.

Saturation vapor pressure (e_s) * OR sat. mixing ratio

⇒ The maximum amount of water vapor the air can hold at a given temperature.

Can approximate e_s using eqn.:

$$e_s \approx 6.112 \exp\left(\frac{17.67 T}{T + 243.5}\right) \text{ (mb)}$$

Assuming $L_v = 2.501 \times 10^6 \text{ J/kg}$
(value at 0°C)

Virtual temperature (T_v)

⇒ An adjusted temperature that accounts for presence of water vapor in atmosphere.

$$P = \rho R_d T_v$$

$$T_v = T (1 + 0.61 r_v)$$

↑ Accounts for ratio of moist to dry gas constant

Similarly, there is a virtual potential temperature (θ_v)

Aside: Water vapor content does also affect c_p, c_v , but its impact usually neglected.

Dewpoint temperature (T_d)

\Rightarrow Temperature to which air must be cooled at constant pressure to achieve saturation.

A good approximation . . .

$$T_d \approx \frac{243.5}{\left[\frac{17.67}{\ln(e/6.112)} - 1 \right]} \quad e \text{ in mb.}$$

Wet bulb temperature (T_w)

\Rightarrow Temperature to which air can be cooled at constant pressure by evaporating water into air until air becomes saturated.

T_w is easiest to compute on a thermodynamic diagram, or else requires an iterative solution (eq. 2.26 of textbook)

Similarly, a potential wet bulb temperature (Θ_w)

Pseudoadiabatic process

Conceptually . . .

- Rising parcel of air hits saturation at the end of adiabatic rise
- Cooling and expansion continues after the saturation point reached as parcel rises.
- Condensation occurs and released latent heat tends to warm the air.

⇒ Result: Temperature will decrease at slower rate than Γ_d as the parcel rises. This is the moist adiabatic or pseudo-adiabatic lapse rate (Γ_m)

$$\Gamma_m \sim -6 \text{ to } -7 \frac{\text{K}}{\text{km}}$$

range

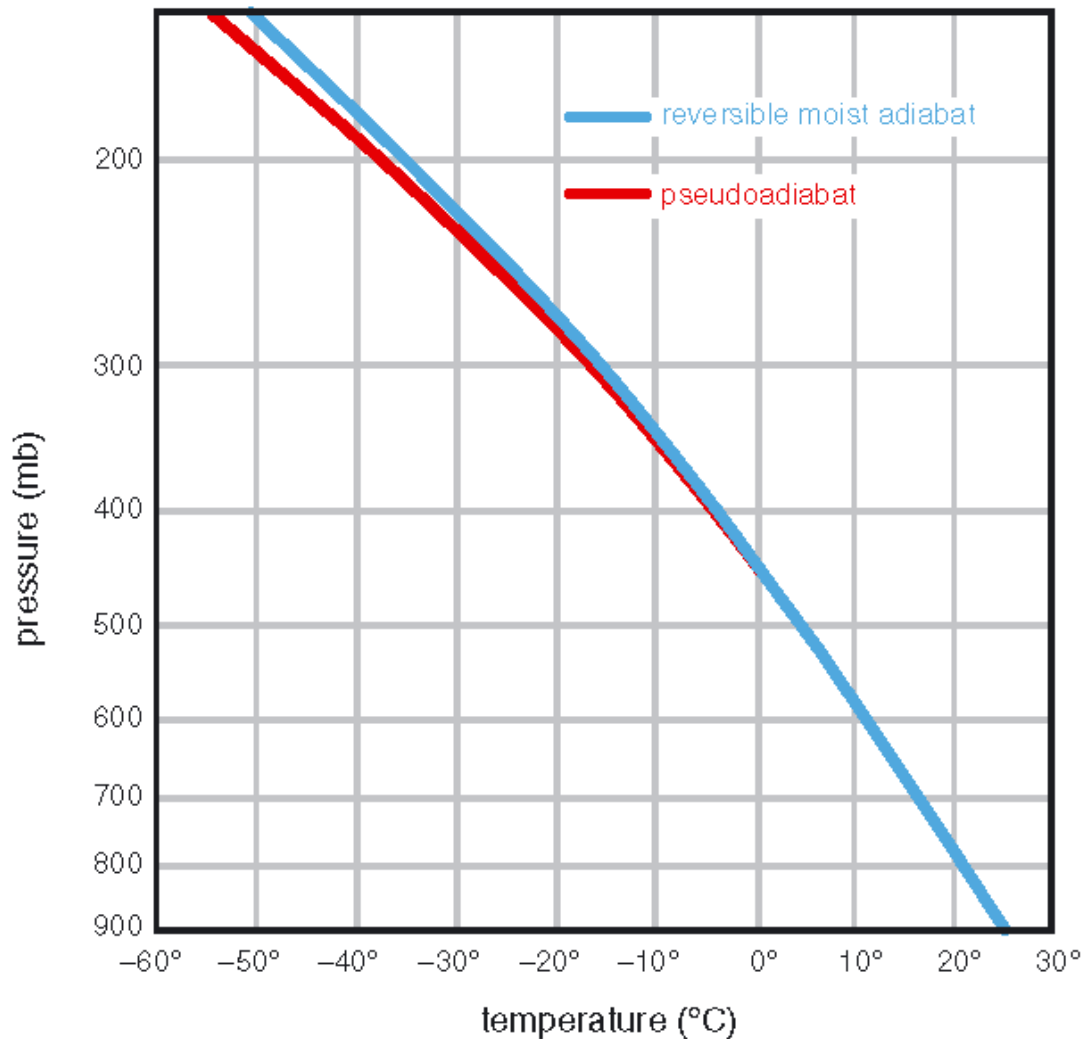


Figure 2.1 Temperature as a function of pressure for reversible moist adiabatic and pseudoadiabatic ascent from saturated initial conditions of $p = 900$ mb and $T = 25^\circ\text{C}$.

For saturated air ($RH = 100\%$)

$$T_e = T \exp\left(\frac{L_v r_v}{c_p T}\right)$$

For unsaturated air ($RH < 100\%$)

$$T_e = T \exp\left(\frac{L_v r_v}{c_p T_e}\right)$$

T_e = isentropic condensation temperature
Essentially the temperature of a lifted parcel to the point at which saturation occurs \rightarrow defines lifting condensation level (LCL)

Similarly, an equivalent potential temperature (Θ_e)

Bring T_e down ~~to~~ adiabatically to reference level of 1000-mb.

* Θ_e is a very important variable when diagnosing potential for convection, as it combines both heat and moisture into one metric.

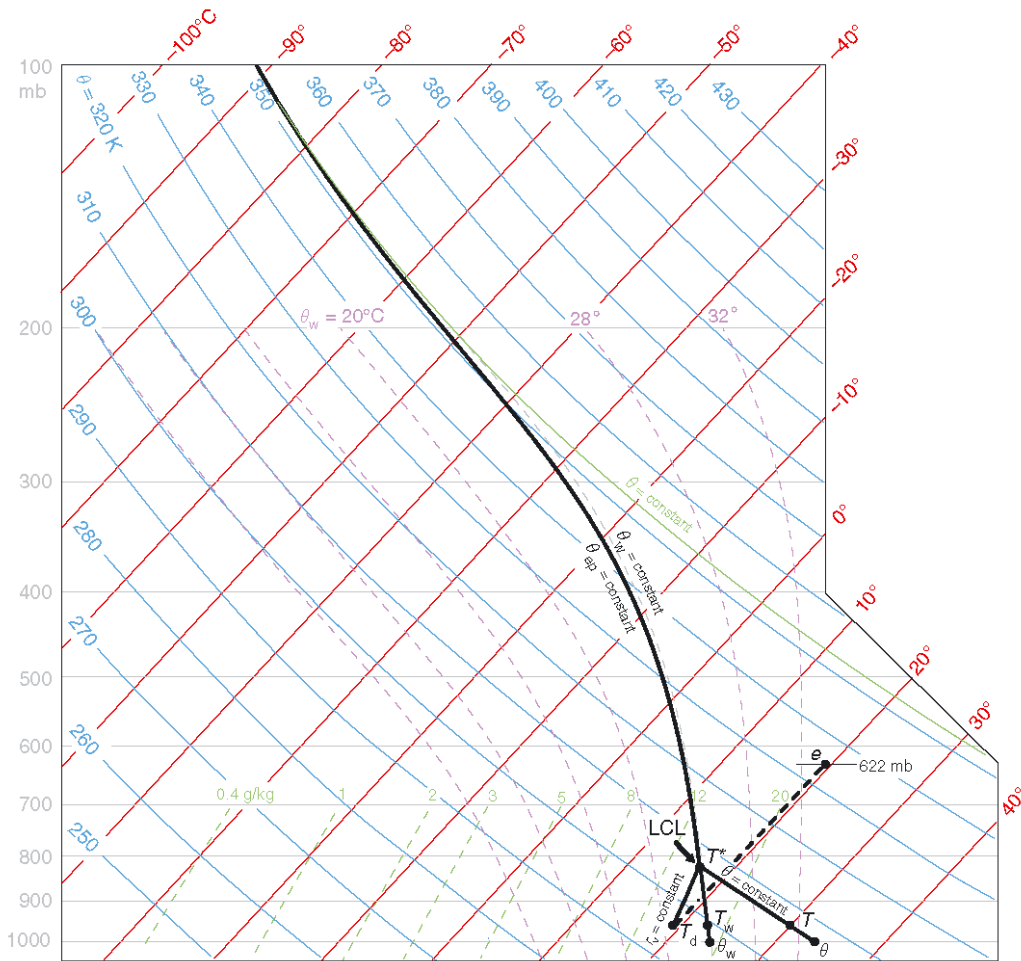


Figure 2.2 Skew T -log p diagram illustrating how to compute θ , θ_w , θ_{ep} , T_w , T^* , e , and the lifting condensation level (LCL) from observations of T and T_d . Isobars are gray (labeled in mb), isentropes (dry adiabats) are blue (labeled in K), isotherms are red (labeled in $^{\circ}\text{C}$), water vapor mixing ratio lines are green (dashed; labeled in g kg^{-1}), and pseudoadiabats are purple (dashed; labels in $^{\circ}\text{C}$ indicate θ_w values corresponding to the pseudoadiabats). The LCL and T^* are found by finding the intersection of the dry adiabat and constant mixing ratio line that pass through the potential temperature and mixing ratio, respectively, of the parcel to be lifted. The potential temperature is found by following a dry adiabat to 1000 mb through the temperature of the parcel. The wet-bulb temperature (wet-bulb potential temperature) is found by following a pseudoadiabat from a parcel's T^* value back to the parcel's pressure (1000 mb). The equivalent potential temperature is found by following a pseudoadiabat from T^* to a pressure (high altitude) where the pseudoadiabat is parallel to the dry adiabats, and then identifying the potential temperature associated with the pseudoadiabat at this pressure (the solid green line identifies the θ_{ep} of a parcel located at the surface). The vapor pressure is found by locating the intersection of the isotherm passing through the parcel's water vapor mixing ratio and the $p = 622$ mb isobar. The water vapor mixing ratio line passing through this intersection point (dashed black line) represents e in millibars. (The saturation vapor pressure can be found via the same exercise but by following the isotherm that passes through the temperature rather than water vapor mixing ratio of the parcel.) For the parcel shown ($T \approx 30^{\circ}\text{C}$, $T_d \approx 19^{\circ}\text{C}$, $r_v \approx 15 \text{ g kg}^{-1}$), the θ , θ_w , θ_{ep} , T_w , T^* , e , and LCL are as follows, respectively: 307.5 K, 23.5 $^{\circ}\text{C}$, 352 K, 22.5 $^{\circ}\text{C}$, 17 $^{\circ}\text{C}$, 22 mb, and 825 mb.

Potential temperature is denoted with the symbol θ and is the standard prognostic variable for heat in (most) mesoscale models. (e.g. RAMS, WRF)

To a definition of heat with θ - - -

Differentiate ideal gas law!

$$P\alpha = RT_V$$

$$P d\alpha + \alpha dp = R dT_V$$

$$P d\alpha = R dT_V - \alpha dp. \quad (P\alpha = RT)$$

$$\frac{RT_V}{\alpha} = R dT_V - \frac{RT_V}{P} dp.$$

$$\left(\frac{1}{\alpha}\right) d\alpha = \frac{1}{T_V} dT_V - \frac{1}{P} dp.$$

Substitute into expression for ds

$$ds = c_\alpha \frac{dT}{T_V} + R_d \frac{dT_V}{T_V} - \frac{R_d}{P} dp.$$

$$ds = (c_\alpha + R_d) \frac{dT_V}{T_V} - \frac{R_d}{P} dp.$$

c_p = specific heat at constant pressure.

~~the~~

$$ds = c_p \frac{dT_v}{T_v} - \frac{R_d}{P} dp. \quad \rightarrow \text{Can this be expressed with one variable } (\theta)?$$

In the situation of no heat lost/gained
(i.e. adiabatic process)

$$\frac{dT_v}{T_v} = \frac{R_d}{c_p} \frac{dp}{P}.$$

Rewrite as:

$$d \ln T_v = \frac{R_d}{c_p} d \ln p.$$

Integrating yields:

$$\ln \left(\frac{T_{v2}}{T_{v1}} \right) = \frac{R_d}{c_p} \ln \left(\frac{P_2}{P_1} \right)$$

Can be rewritten using rules of logs:

$$\frac{T_{v2}}{T_{v1}} = \left(\frac{P_2}{P_1} \right)^{R_d/c_p}.$$

Potential temp (θ) defined as:

$$\theta = T_v \left(\frac{1000}{P} \right)^{R_d/c_p}.$$