On Quantifying the Exposure to Cloud-to-Ground Lightning

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Abstract: Measuring systems can now determine the number and spatial distribution of cloud-to-ground (CG) lightning flashes under individual thunderstorms and over larger regions on monthly, seasonal, and annual time scales. Here, we show how a measurement of the average area density of strikes, N_g, over a given region, and an assumption of complete spatial randomness, can be used to determine the probability that the *nearest* (and usually the most dangerous) flash will be within any specified distance of an arbitrary point or line segment in that region.

Keywords: Risk assessment, Strike probability

1. Introduction

Gated, wideband lightning sensors [1,2] similar to those used in the U.S. National Lightning Detection Network (NLDN) [3,4] have been monitoring lightning in many countries for many years, and the resulting databases now provide accurate estimates of the average area-density of cloud-to-ground (CG) flashes over large areas on monthly, seasonal, and annual time scales (see, for example, [5] and the references therein). Here, we will show how knowledge of the average area density of CG flashes, N_g, over a given region can be used to estimate the chances that the *closest* (and usually the most deleterious) strike will be within any specified distance of an arbitrary origin in that region. We will also show how this "nearest-neighbor" distribution can be generalized to include higher-order neighbors and strikes that are close to a long line segment.

2. Nearest Strikes to a Point

We begin by supposing that the average area-density of flashes, N_g, over a region is known, and given that knowledge, we want to determine the chances that any strike will be within a distance, R, of any origin (chosen at random) in that region. We will assume that each flash is a random event on the space and time scales of interest and that the spatial pattern of the strike points has a homogeneous Poisson distribution, i.e., N_g has complete spatial randomness. With this assumption, we can use a method outlined by [6,7,8] to determine the probability, *w(r)dr*, that the *nearest* flash is between the radial distance r and $r + dr$ from the origin,

$$w(r)dr = \begin{Bmatrix}\text{Probability that NO} \\ \text{strike is within } r\end{Bmatrix}\begin{Bmatrix}\text{Probability there IS a strike} \\ \text{between r and } r+dr\end{Bmatrix}$$

or

$$\mathrm{w(r)dr} = \left(1 - \int_0^{r} \mathrm{w(r')dr'}\right)\left(2\pi \mathrm{rdrN_g}\right) \qquad [1]$$

when $\int_0^\infty \mathrm{w}\left(\mathrm{r}'\right)\mathrm{dr}' = 1$. Solving equation [1], we obtain the well-known *nearest-neighbor* distribution [9,10],

$$\mathrm{w(r)dr} = 2\pi \mathrm{rdr}\ \mathrm{N_g} \exp\left(-\mathrm{N_g}\pi \mathrm{r}^2\right). \qquad [2]$$

Using equation [2], it is straightforward to show that the *most probable* nearest-neighbor distance, r_{mp}, is

$$\mathrm{r_{mp}} = \frac{1}{\sqrt{2\pi \mathrm{N_g}}};$$

the *mean* is

$$\bar{\mathrm{r}} = \frac{1}{2\sqrt{\mathrm{N_g}}};$$

and the *variance* is

$$\mathrm{VAR}\left(\mathrm{r}\right) = \frac{\left(4-\pi\right)}{4\pi \mathrm{N_g}} = \frac{0.0683}{\mathrm{N_g}}.$$

An integral of equation [2] describes the chances that the *closest* strike will be within the distance, R,

$$P(\le R) = \int_0^R w(r')dr',$$

$$\mathrm{P}(\le \mathrm{R}) = 1 - \exp(-\mathrm{N_g}\pi \mathrm{R}^2), \qquad [3]$$

and, if the probability, P, is specified, then equation [3] can be solved for R,

$$R = \left[-\ln(1-P)/\pi N_g\right]^{1/2}.$$

It should be noted that when $N_g \pi R^2 << 1$, equation [3] reduces to

$$P(\leq R) \approx N_g \pi R^2 .$$

Figure 1 shows plots of P vs. R computed using equation [3] for 8 different values of N_g.

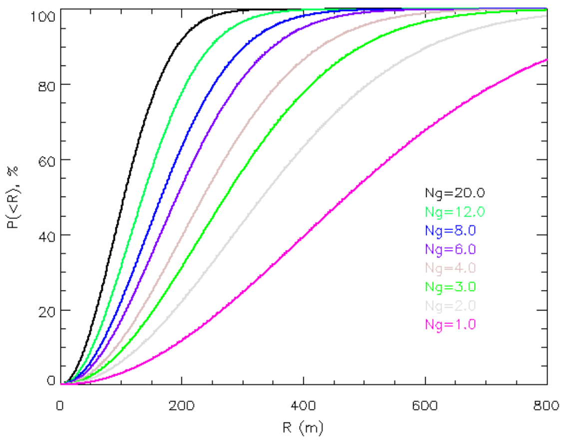

Figure 1: Probability that the nearest flash will be within a distance, R, for various values of N_g in Fl km^{-2}.

Now, it is straightforward to generalize the above to obtain the probability distribution for the distance to the nth nearest-flash, namely, the chances that the nth nearest-neighbor is between r_n and $r_n + dr_n$ from the origin is

$$w(r_n) dr_n = \frac{2(\pi N_g)^n}{(n-1)!} r_n^{2n-1} \exp(-N_g \pi r_n^2) dr_n \qquad [4]$$

where n=1,2,3…[11]. Using equation [4], it is straightforward to show that the *most probable* distance to the nth nearest-neighbor is

$$r_{mp,n} = \sqrt{\frac{2n-1}{2\pi N_g}} ;$$

the *mean* distance is

$$\overline{r}_n = \frac{1}{\sqrt{N_g}} \frac{(2n)! n}{(2^n n!)^2} ;$$

and the *variance* is

$$VAR(r_n) = \frac{n}{\pi N_g} - \left(\frac{1}{\sqrt{N_g}} \frac{(2n)! n}{(2^n n!)^2} \right)^2 .$$

An integral of [4] describes the chances that the nth nearest-strike will be within a distance, R_n,

$$P(\leq R_n) = \int_0^{R_n} w(r_n') dr_n'$$

$$P(\leq R_n) = 1 - \exp(-X_n) \sum_{i=1}^{n} \frac{X_n^{\,i-1}}{(i-1)!} \qquad [5]$$

where $X_n = \pi R_n^{\,2} N_g$.

3. Nearest Strikes to a Long Line

One can use the same formalism to describe flashes that are close to a long line segment of length, L. In this case, the probability that the nth nearest-neighbor lies between h_n and $h_n + dh_n$ (on either side) of the line segment is given by:

$$p(h_n) dh_n = \frac{(2N_g L)^n}{(n-1)!} h_n^{\,n-1} \exp(-2N_g L h_n)\, dh_n \qquad [6]$$

where n=1,2,3,…and $h_n << L$. Using equation [6], it is straightforward to show that the *most probable* distance is

$$h_{n,mp} = \frac{(n-1)}{2LN_g} ;$$

the *mean* is

$$\overline{h}_n = \frac{n}{2LN_g} ;$$

and the *variance* is

$$VAR(h_n) = \frac{n}{(2LN_g)^2} .$$

An integral of [6] describes the chances that the nth nearest flash is between the line and the horizontal distance, H_n,

$$P(h_n \leq H_n) = \int_0^{H_n} p(h_n') dh_n'$$

$$P(\leq H_n) = 1 - \exp(-Y_n) \sum_{i=1}^{n} \frac{Y_n^{\,i-1}}{(i-1)!}$$

[7]

where $Y_n = 2LH_n N_g$ and $H_n << L$.

4. Comparison with Lightning Data

Figure 2 shows the spatial pattern of 7318 CG lightning flashes that were recorded by the U.S. NLDN over a 3 year period in a 20 x 20 km^2 region that is centered on the Memphis, TN, airport (see also Table 1). Here, each dot shows the most probable location of the first return stroke in each flash (see the Appendix in [3]), and in the following we will refer to these points as *events*. (Note: we have made no corrections for the imperfect NLDN detection efficiency or for the multiple attachment points that commonly occur in CG flashes [12,13].)

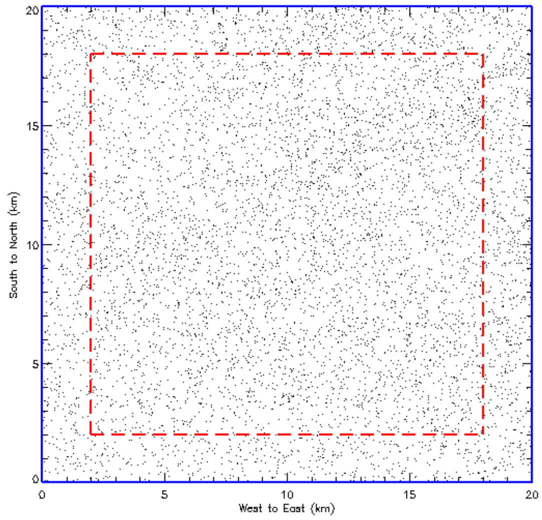

Figure 2: Plot of CG lightning locations over a 3-year period centered on the Memphis Airport (January, 1997, through December, 1999). There are a total of 4160 strikes in the analysis sub-region shown by the dashed red line.

The data in Figure 2 have a numerical precision of four decimal digits in latitude and longitude, which translates to a spatial resolution of about 10 m, but random and systematic errors in the NLDN typically produce location errors that are approximately 0.5 to 1 km [3,4]. There were 4160 events within the 16x16 km^2 analysis sub-region (to avoid edge effects) that is shown by the dashed red line in Figure 2, so N_g is a total of 16.17 flashes (Fl) per km^2 in that region over the 3 years.

Table 1:

Annual values of the average area density of CG flashes within the red sub-region of figure 2 for 1997-1999.

Year	CG Fl km^{-2}
1997	6.41
1998	5.36
1999	4.40
Mean 3-Year Total	5.4 ± 1.0 Fl km^{-2} yr^{-1} 16.17 Fl km^{-2}

Figure 3 shows the measured distribution of the event-to-nearest-event distances within the analysis sub-region of Figure 2 together with the cumulative distribution, and the red and blue curves show plots of equations [2] and [3], respectively, for N_g = 16.17 Fl km^{-2}. The mean and variance of the experimental data are 125 m and 4345 m^2, respectively, and the corresponding values predicted by equation [2] are 124 m and 4223 m^2.

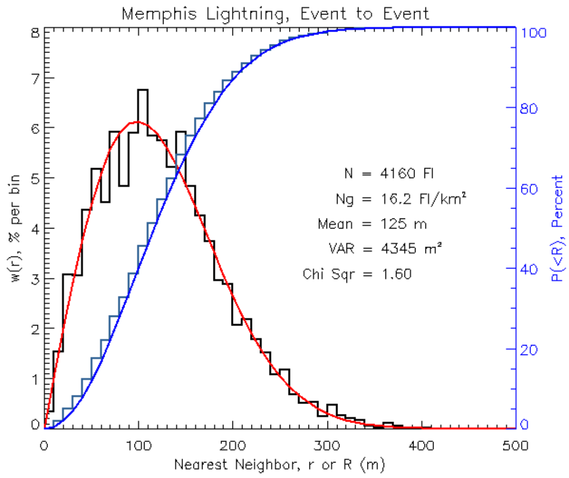

Figure 3: Event-to-nearest-event distribution (using 10 m bins) for the events within the red analysis area shown in Figure 2.

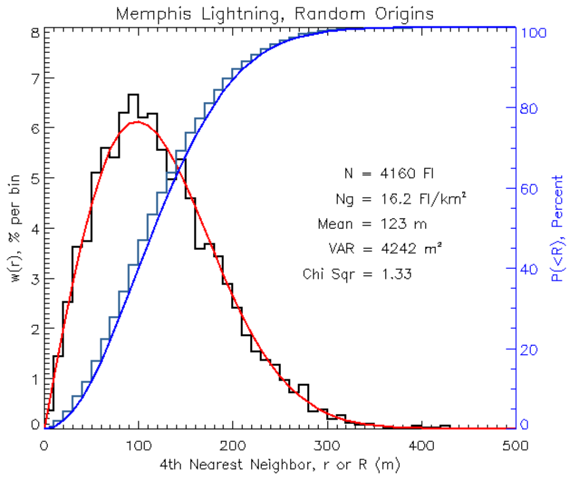

Figure 4: Random-origin-to-nearest-event distribution (using 10 m bins) for the events shown in Figures 2.

Another way to characterize a spatial pattern of events is to place a series of random origins within the pattern, and then to compute the distribution of the origin-to-nearest-event distances for a large number of random origins. (Note: if the spatial distribution of events is truly random and homogeneous, this distribution will be the same as the event-to-nearest-event distribution that is shown in Figure 3.) Figure 4 shows the random-origin-to-nearest-event distribution that was computed for the same pattern of events that is shown in Figure 2, using the same number of random origins as there are events in Figure 3. The mean and variance of the distances in Figure 4 are 123 m and 4242 m^2, respectively, and again, the values predicted by equation [2] are 124 m and 4223 m^2. It is clear that the mean and variance of the distances from random origins (Figure 4) are very close to the corresponding mean and variance of the event-to-event distances (Figure 3).

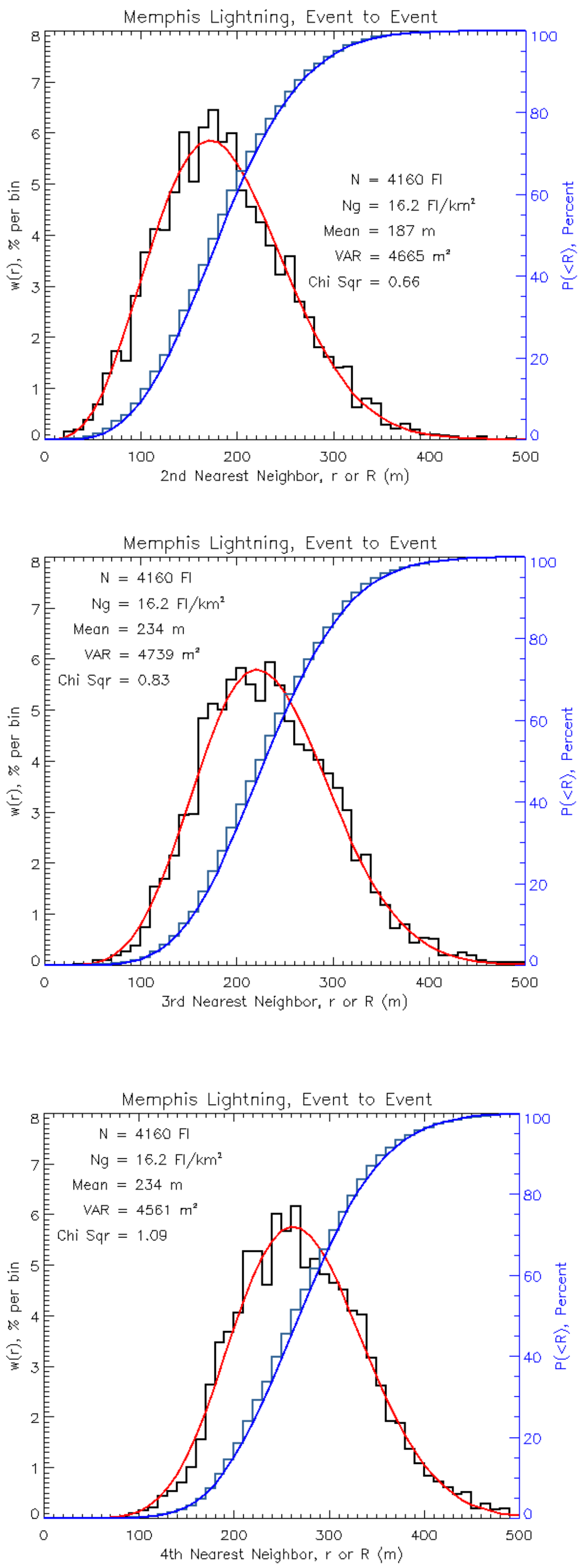

Figure 5: The event-to-event distributions for the 2^{nd}(top), 3^{rd}(middle), and 4^{th}(bottom) nearest events.

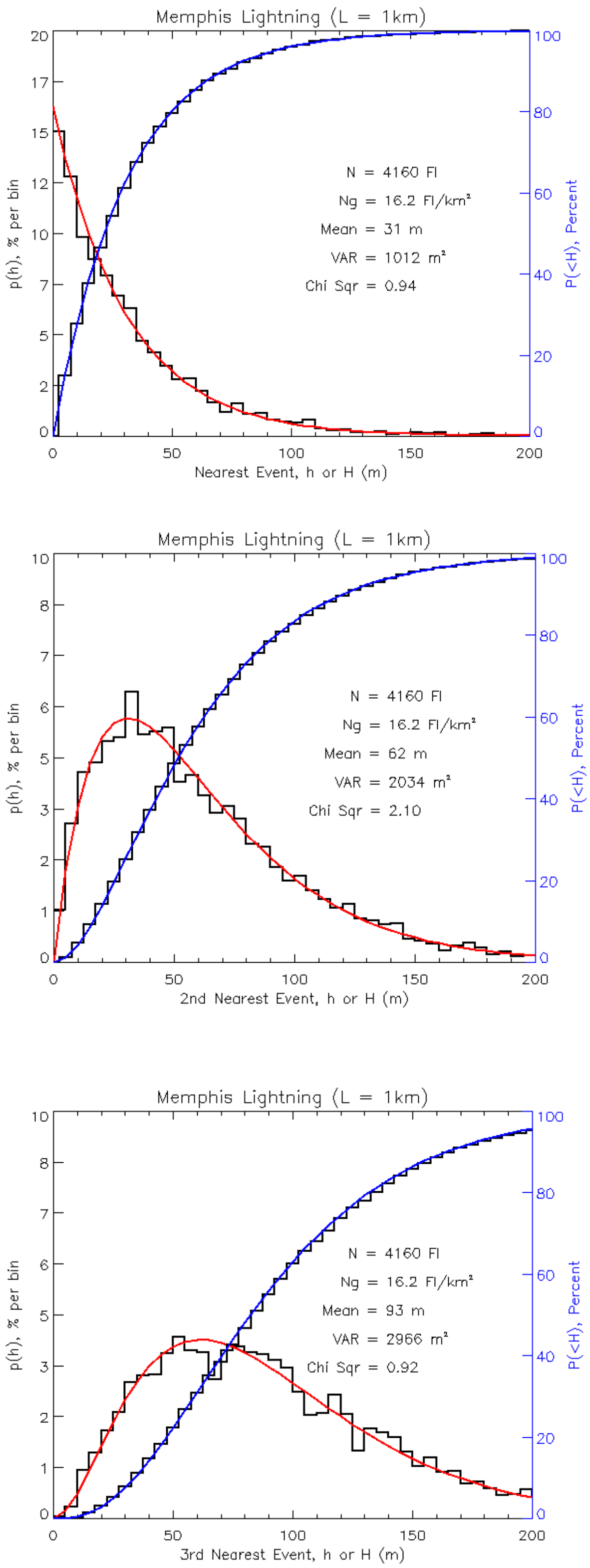

Figure 6: Line-to-event distributions (using 5 m bins) for the nearest (top), 2^{nd} nearest (middle), and 3^{rd} nearest (bottom) for random 1 km line segments.

Figure 5 shows distributions of the 2nd, 3rd, and 4th order event-to-nearest-event distances that were computed for the spatial pattern in Figure 2 together with the predictions of equations [4] and [5]. The higher order distributions using random origins are very similar to those in Figure 5 and are in very good agreement with equations [4] and [5].

We have checked equations [6] and [7] by placing a large number of line segments (L = 1.0 km), at random locations and with random orientations, into the population of events shown in Figure 2, and then computing the horizontal distances from these lines to the nearest event, the second-nearest event, etc. The results are shown in Figure 6. Note in Figure 6 that there is excellent agreement between these Monte Carlo simulations and the predictions of equations [6] and [7].

5. Discussion

Figures 3 to 6 show that nearest-neighbor distributions do describe the measured, long-term patterns of the distances to the nearest-flashes rather well, but of course, such tests are limited by the accuracy of the NLDN data on small spatial scales. The most probable distances are in good agreement with equations [2], [4], and [6], and the values of the reduced chi-square, a "goodness of fit" parameter, are excellent. The sample means and variances are also in very good agreement with model predictions.

As further examples of applications of the above ideas, let us consider a region that has an average area density of 6.0 CG strikes per km^2, a representative value for the *annual* area density over much of the U.S., after corrections are made for the imperfect NLDN detection efficiency and the multiplicity of strike points per CG flash. From equations [2] and [4], we can predict that, in such a region, the *most probable* distances to the nearest- and 2nd-nearest strikes from any origin (or person) chosen at random will be 163 m and 282 m each year, and that the mean (and standard deviation) of the nearest- and 2nd-nearest distances will be 223 ± 107 m and 306 ± 111 m, respectively. In this region, there is a 10% chance of a strike within 75m, a 50-50 chance of a strike within 192 m, and a 90% chance of a strike within 350 m each year (equation [3]). For each 1 km segment of a long, straight power line, fence, or pipeline, the most probable distances to the nearest- and 2nd-nearest strikes will be 0 m and 167 m, respectively, and the mean (and standard deviation) of the nearest- and 2nd-nearest distances will be 83 ± 83 m and 167 ± 118 m, respectively. For each 10 km line segment, the above means will be only 8.3 ± 8.3 m and 16.7 ± 11.8 m, respectively.

In practice, the above estimates will only be valid over spatial scales that range from a few tens of meters on the low end to tens of kilometers on the upper end. At small distances, the primary factors controlling the exposure to lightning strikes (in addition to the proximity of a lightning leader) are the location, size, and geometry of any objects near the strike point(s) and the number and length of the upward connecting discharges; of course, the latter depend on the size and geometry of the strike object(s), the presence and size of other objects in the local vicinity, and the magnitude and polarity of the electric field under the leader channel [14,15,16,17]. At distances greater than about 10 km, N_g may not be spatially uniform [18], but the local strike probability can still be estimated using the average N_g in the region of the strike object.

Measurements of the spatial patterns of CG flashes will often show regions of reduced or enhanced area density, especially if there has been an unusually active storm in the region and/or if the averaging-time is relatively short, and in this case, N_g will not be homogeneous. Even if N_g is not completely uniform, however, the assumption of complete spatial randomness can still be used as the null hypothesis when using various statistical models to identify and quantify the underlying spatial pattern and/or to estimate the optimum value of N_g over the region of interest (see, for example, [9, 10,19,20]).

6. Acknowledgements

The authors appreciate the assistance of Christopher J. Biagi in preparing the figures in this manuscript and for several helpful discussions. The NLDN data were provided by Vaisala, Inc., Tucson, AZ. This research has been supported in part by the NASA Kennedy Space Center under Grant NAG10-302.

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